# tess Documentation

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A 3D cell-based Voronoi library based on voro++

This library includes Python bindings, using Cython. Code available on Github. Documentation available at Read the Docs.

# Description

Tess is a library to calculate Voronoi (and Laguerre) tessellations in 3D and analyze their structure. The tessellation is calculated as a list of Cell objects, each of which can give information on its volume, centroid, number of faces, surface area, etc. The library is made with packings of spherical particles in mind, possibly with variable sizes.

### 2.1 voro++

The Tess library is a set of Python bindings to the Voro++ library. Voro++ provides all the algorithms, and Tess provides an easy to use interface to the voro++ library for Python, using Cython to do so.

Original work on voro++ by Chris H. Rycroft (UC Berkeley / Lawrence Berkeley Laboratory).

### Quick Start

### 3.1 Installation

To install, use pip (or easy\_install):

pip install --user tess

Or to install from Github:

pip install --user git+git://github.com/wackywendell/tess@master

### 3.2 Usage

The first step is to create a Container:

```
>>> from tess import Container
>>> cntr = Container([[1,1,1], [2,2,2]], limits=(3,3,3), periodic=False)
```

A container is a list of *Cell* objects, representing Voronoi cells:

```
>>> [round(v.volume(), 3) for v in cntr]
[13.5, 13.5]
```

*Cell* objects have many methods. Here are a few:

```
>>> [v.pos for v in cntr]
[(1.0, 1.0, 1.0), (2.0, 2.0, 2.0)]
>>> [v.centroid() for v in cntr]
[(1.09375, 1.09375, 1.09375), (1.90625, 1.90625, 1.90625)]
```

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```
>>> [v.neighbors() for v in cntr]
[[-5, -2, -3, -1, -4, 1, -6], [0, -3, -6, -4, -5, -2, -1]]
>>> [v.face_areas() for v in cntr]
[[7.875, 1.125, 7.875, 7.875, 1.125, 11.691342951089922, 1.125],
[11.691342951089922, 1.125, 7.875, 7.875, 1.125, 7.875, 1.125]]
>>> [v.normals() for v in cntr]
[[(0.0, 0.0, -1.0)],
  (1.0, 0.0, 0.0),
  (0.0, -1.0, 0.0),
  (-1.0, 0.0, 0.0),
  (0.0, 1.0, 0.0),
  (0.5773502691896257, 0.5773502691896257, 0.5773502691896257),
  (0.0, 0.0, 1.0)],
 [(-0.5773502691896257, -0.5773502691896257, -0.5773502691896257),
  (-0.0, -1.0, -0.0),
  (0.0, 0.0, 1.0),
  (0.0, 1.0, -0.0),
  (0.0, 0.0, -1.0),
  (1.0, 0.0, -0.0),
  (-1.0, -0.0, -0.0)]]
```

See the Reference for more methods, or just use a Python interpreter or IPython notebook to find them on your own!

## Voro++ Copyright And Acknowledgments

## 4.1 Copyright Notice

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## 4.2 Acknowledgments

This work (voro++) was supported by the Director, Office of Science, Computational and Technology Research, U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

### **Full Contents**

### 5.1 Reference

This is a library to calculate Voronoi cells and access their information.

### 5.1.1 Basic Process

- Create a *Container* object, using information about your system.
  - a Container is a list of Cell objects
- Access the Cell methods to get information about them

Example

```
>>> from tess import Container
>>> c = Container([[1,1,1], [2,2,2]], limits=(3,3,3), periodic=False)
>>> [round(v.volume(), 3) for v in c]
[13.5, 13.5]
```

**class** tess.**Container** (*points*, *limits=1.0*, *periodic=False*, *radii=None*, *blocks=None*) A container (*list*) of Voronoi cells.

This is the main entry point into the *tess* module. After creation, this will be a *list* of *Cell* objects.

The *Container* must be rectilinear, and can have solid boundary conditions, periodic boundary conditions, or a mix of the two.

```
>>> from tess import Container
>>> c = Container([[1,1,1], [2,2,2]], limits=(3,3,3), periodic=False)
>>> [round(v.volume(), 3) for v in c]
[13.5, 13.5]
```

#### Parameters

- points (iterable of *float*) The coordinates of the points, size Nx3.
- **limits** (*float*, 3-tuple of float, or two 3-tuples of float) The box limits. If given a float *L*, then the box limits are [0, 0, 0] to [L, L, L]. If given a 3-tuple (*Lx*, *Ly*, *Lz*), limits are [0, 0, 0] to [Lx, Ly, Lz]. If given two 3-tuples (*x0*, *y0*, *z0*), (*x1*, *y1*, *z1*), limits are [x0, y0, *z0*] to [x1, y1, z1].
- periodic (bool or 3-tuple of bool, optional) Periodicity of the x, y, and z walls
- **radii** (iterable of *float*, optional) for unequally sized particles, for generating a Laguerre transformation.

Returns A list of Cell objects

Return type Container

### **Notes**

Voronoi Tesselation

A point  $\vec{x}$  is part of a Voronoi cell *i* with nucleus  $\vec{r_i}$  iff

$$\left|\vec{x} - \vec{r}_i\right|^2 < \left|\vec{x} - \vec{r}_j\right|^2 \forall j \neq i$$

Laguerre Tesselation, also known as Radical Voronoi Tesselation

A point  $\vec{x}$  is part of a Laguerre cell *i* with nucleus  $\vec{r_i}$  and radius  $R_i$  iff

$$|\vec{x} - \vec{r}_i|^2 - R_i^2 < |\vec{x} - \vec{r}_j|^2 - R_j^2 \forall j \neq i$$

### get\_widths()

Get the size of the box.

```
order (l=6, local=False, weighted=True)
Returns crystalline order parameter Q_l (such as Q_6).
```

Requires numpy and scipy.

#### Parameters

- 1 (*int*, *optional*) Defines which Q<sub>l</sub> you want (6 is standard, for detecting hexagonal lattices)
- local (bool, optional) Calculate Local  $Q_6$  (true) or Global  $Q_6$
- weighted (bool, optional) Whether or not to weight by area the faces of each polygonal side

#### Notes

For local=False, this calculates

$$Q_{l} = \sqrt{\frac{4\pi}{2l+1} \sum_{m=-l}^{l} \left| \sum_{i=1}^{N_{b}} w_{i} Y_{lm} \left(\theta_{i}, \phi_{i}\right) \right|^{2}}$$

where:

 $N_b$  is the number of bonds

 $\theta_i$  and  $\phi_i$  are the angles of each bond *i*, in spherical coordinates

 $Y_{lm}\left( heta_{i},\phi_{i}
ight)$  is the spherical harmonic function

 $w_i$  is the weighting factor, either proportional to the area (for *weighted*) or all equal  $(\frac{1}{N_b})$ 

For local=True, this calculates

$$Q_{l,\text{local}} = \sum_{j=1}^{N} \sqrt{\frac{4\pi}{2l+1} \sum_{m=-l}^{l} \left| \sum_{i=1}^{n_{b}^{j}} w_{i} Y_{lm} \left(\theta_{i}, \phi_{i}\right) \right|^{2}}$$

where variables are as above, and each *cell* is weighted equally but each *bond* for each cell is weighted:  $\sum_{i=1}^{n_b^i} w_i = 1$ 

#### Returns

#### Return type float

#### tess.cart\_to\_spher(xyz)

Converts 3D cartesian coordinates to the angular portion of spherical coordinates, (theta, phi).

Requires numpy.

**Parameters xyz** (array-like, Nx3) – Column 0: the "elevation" angle, 0 to  $\pi$ 

Column 1: the "azimuthal" angle, 0 to  $2\pi$ 

Returns

#### Return type array, Nx2

tess.orderQ(l, xyz, weights=1)

Returns  $Q_l$ , for a given l (int) and a set of Cartesian coordinates xyz.

Requires numpy and scipy.

For global  $Q_6$ , use l = 6, and pass xyz of all the bonds.

For local  $Q_6$ , use l = 6, and the bonds have to be averaged slightly differently.

### Parameters

- l(int) The order of  $Q_l$
- **xyz** (array-like Nx3) The bond vectors  $\vec{r_j} \vec{r_i}$
- weights (array-like, optional) How to weight the bonds; weighting by Voronoi face area is common.

### **Notes**

This calculates

$$Q_{l} = \sqrt{\frac{4\pi}{2l+1} \sum_{m=-l}^{l} \left| \sum_{i=1}^{N_{b}} w_{i} Y_{lm} \left(\theta_{i}, \phi_{i}\right) \right|^{2}}$$

where:

 $N_b$  is the number of bonds

 $\theta_i$  and  $\phi_i$  are the angles of each bond *i*, in spherical coordinates

 $Y_{lm}\left(\theta_{i},\phi_{i}\right)$  is the spherical harmonic function

 $w_i$  are the *weights*, defaulting to uniform:  $(\frac{1}{N_i})$ 

#### class tess.Cell

A basic voronoi cell, usually created by Container.

A Voronoi cell has polygonal faces, connected by edges and vertices.

The various methods of a Cell allow access to the geometry and neighbor information.

\_\_\_repr\_\_\_

\_\_str\_\_

centroid()

#### face\_areas()

A list of the areas of each face.

Returns

Return type A list of floats. Each inner list corresponds to a face.

face\_freq\_table()

face\_perimeters()

#### face\_vertices()

A list of the indices of the vertices of each face.

Returns

• A list of lists of ints. Each inner list corresponds to a face, and each index corresponds

• to a vertex from *vertices()*.

#### id

The id of the cell, which should generally correspond to its index in the Container.

#### max\_radius\_squared()

Maximum distance from pos() to outer edge of the cell (I think, see voro++ documentation.)

#### neighbors()

Return a list of the neighbors of the current Cell.

This is a list of indices, which correspond to the input points. The exception to this is the walls: walls are numbered -1 to -6, so an index less than 0 in the list of *neighbors()* indicates that a *Cell* is neighbors with a wall.

#### normals()

A list of the areas of each face.

Returns

Return type A list of 3-tuples of floats. Each tuple corresponds to a face.

```
number_of_edges()
```

```
number_of_faces()
```

pos

The position of the initial point around which this cell was created.

### radius

The radius of the particle around which this cell was created.

Defaults to 0.

### surface\_area()

total\_edge\_distance()

### vertex\_orders()

### **vertices**()

A list of all the locations of the vertices of each face.

### Returns

Return type A list of 3-tuples of floats. Each tuple corresponds to a single vertex.

### volume()

Cell volume

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