precalc final project Documentation

Release 1.0

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Contents

Luis Naranjo's project

PDF Version available here.

Precalculus Portfolio

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CHAPTER 1

Function Composition And Inverse Functions

Portfolio Entry Reflection Sheet

<table>
<tr><td colspan="12">Name: Function Composition and Inverse Functions</td></tr>
<tr><td colspan="12">Entry Title: Function Composition and Inverse Functions</td></tr>
<tr><td colspan="12">Class Objective # (highlight one)</td></tr>
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</table>

Explanation:

This artifact demonstrates function composition and inherited domain and range.

I present two different funtions, and I give their individual domain and range.

Then, I compose them ($(f \circ g)(x)$), and show the inherited domain and range of the new function.

The domain of $(f \circ g)(x)$ is usually the domain of $f(x)$ and the range of $g(x)$,

but in this case, it is only the positive portion because $(f \circ g)(x)$ has a radical, and the domain of any radical is $x \geq 0$

Independent Thinking:

This artifact also demonstrates independent thinking because its' source code uses quite a few LaTeX expressions that I didn't know how to use at the time.

I went through the source code of another LaTeX document I found online and figured out how to use latex expressions in my code.

Artifact:

Find $(f \circ g)(x)$

$$f(x) = \sqrt{x}$$
$$\text{Domain: } [0, \infty)$$
$$\text{Range: } [0, \infty)$$
$$g(x) = \frac{1}{(x-1)}$$
$$\text{Domain: } (-\infty, 1), (1, \infty)$$
$$\text{Range: } (-\infty, 0), (0, \infty)$$
$$(f \circ g)(x) = f(g(x))$$
$$f(\frac{1}{(x-1)}) = \sqrt{\frac{1}{(x-1)}}$$
$$\text{Domain: } (1, \infty)$$
$$\text{Range: } (0, \infty)$$

1.2 Function decomposition

Source: I made this problem up.

Explanation:

In this artifact I demonstrate how a function can be decomposed into two different functions.

It's not easy for me to show my work on this problem, but basically what I did is compose two new function that would evaluate to $(f \circ g)(x) = \sqrt{x^2 + 3}$.

I can still check my work by plugging in g(x) into f(x):

$f(g(x)) = \sqrt{x^2 + 3}$

Awareness and appreciation:

There is potentially an infinite amount of solutions to this problem, but this is probably the best solution for two reasons.

- It is the simplest solution

- Because it is the simplest solution, it is also the easiest to re-compose into the original function.

Artifact:

Decompose $(f \circ g)(x)$

$$(f \circ g)(x) = \sqrt{x^2 + 3}$$
$$f(x) = \sqrt{x}$$
$$g(x) = x^2 + 3$$

1.3 Parametric functions and how they relate to function composition

Source: Question #4 from 1.5

Explanation:

The first example demonstrates: How to find an (x, y) pair when given a parametric function and a t value.

The second example demonstrates: How a linear function can be composed from a parametric function by eliminating the parameter.

All I have to do is solve for t and plug t back into one of the original equations.

Artifact:

Find the (x,y) pair for the value of the parameter

$x = t + 3$ and $y = \frac{1}{t}$ for $t = -8$:

$$(x, y)$$
$$x = t + 3$$
$$x = -8 + 3$$
$$x = 5$$
$$y = \frac{1}{t}$$
$$y = \frac{1}{-8}$$
$$(5, -\frac{1}{8})$$

Eliminate the parameter:

$$x = t + 3$$
$$t = x - 3$$
$$y = \frac{1}{t}$$
$$y = \frac{1}{(x - 3)}$$

1.4 Inverse functions and inherited domain and range

Source: Section 1.5: Example 4

Explanation:

This artifact demonstrates inverse functions and inherited domain and range.

I present a regular function, and I show its' domain and range.

Then I invert it, and show that the domain and range of the result is the inverse of the original function.

Appropriate Use of Technology

I used an online graphing calculator to generate the graph below.

Once I generated it:

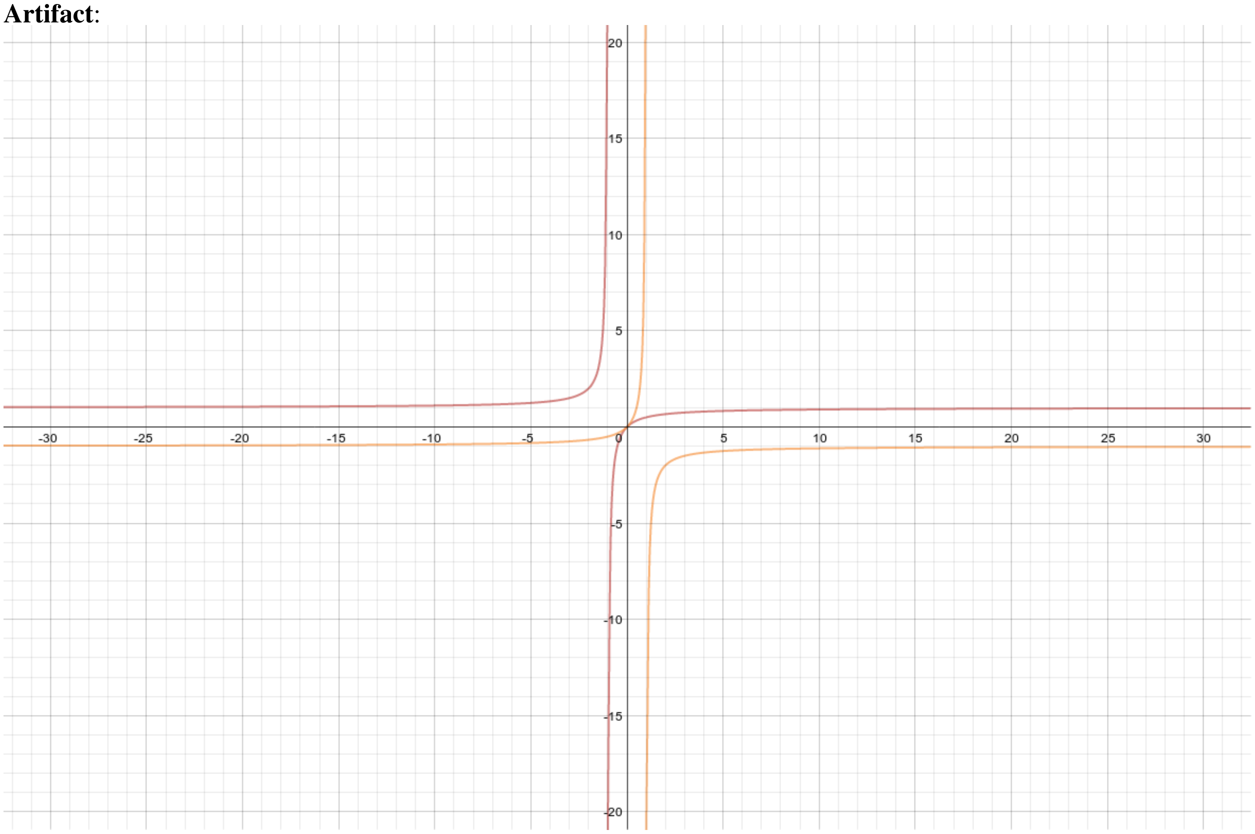
- I took a screenshot of the online graph
- I cropped the screenshot
- I added the image to my local code repository
- I included the image in my source code
- I uploaded the image to my code repository (https://github.com/doubledubba/precalc) and updated my code
- I synchronized my readthedocs.org project with my repo

Numeric Algebraic Graphic Connection (N.A.G.)

I used a graph to show the N.A.G. connection between the original function and the inverse function.

The red one is the original function, and the yellow one is the inverse function.

Artifact:



Find an equation for $f^-1(x)$ if $f(x) = \frac{x}{(x+1)}$.

Domain: $(-\infty, -1), (-1, \infty)$

Range: $(-\infty, 1), (1, \infty)$

$$x = \frac{y}{(y+1)}$$
$$x(y+1) = y$$
$$xy + x = y$$
$$xy - y = -x$$
$$y(x-1) = -x$$
$$y = \frac{-x}{(x-1)}$$
$$y = \frac{x}{(1-x)}$$
$$f^{-}1(x) = \frac{x}{(1-x)}$$

The domain and range are flipped because x and y were flipped.

Domain: $(-\infty, 1), (1, \infty)$

Range: $(-\infty, -1), (-1, \infty)$

CHAPTER 2

Transformations Of All Functions

Portfolio Entry Reflection Sheet

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<tr><td colspan="12">Name: Transformations of all functions</td></tr>
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</table>

Explanation:

In this artifact, I compare f(x) and g(x), and then list the graphical transformations required to get from f(x) to g(x).

I got the solution by referencing the formula: $a * f(b(x-c)) + d$

$g(x)$ fits the formula like so: $3 * f(1(x-1)) + 2$

Artifact:

Describe how the graph of $f(x) = \sqrt{x}$ can be transformed into $g(x) = 3 * \sqrt{(x-1)} + 2$

1. Horizontal shift of f(x) by **one** unit beacuse c shifts the graph horizontally by d units.
2. Vertical shift of f(x) by **two** units because d shifts the graph vertically by d units.
3. Vertical stretch by magnitude of **three** because $a > 1$

2.2 Graphical transformations by rewriting a function from a list of transformations

Source: From my notes

Explanation:

This artifact demonstrates graphical transformations by rewriting a function from a list of transformations.

Like the previous proficiency, I got the solution by referencing the formula: $a * f(b(x-c)) + d$

Here are the steps I took to get from $\sqrt{x}$ to $-4 * \sqrt{3(x-2)} + 5$, in order.

1. $-\sqrt{x}$
2. $-4\sqrt{x}$
3. $-4\sqrt{3x}$
4. $-4\sqrt{3(x-2)}$
5. $-4\sqrt{3(x-2)+5}$

Artifact:

Transform $f(x) = \sqrt{x}$ into $g(x)$

1. Reflect over the x-axis
2. Vertical stretch by a magnitude of **four**
3. Horizontal shrink by a magnitude of $\frac{1}{3}$
4. Horizontal shift by **two** units.
5. Vertical shift by **five** units.

$$g(x) = -4 * \sqrt{3(x-2)} + 5$$

2.3 Graphical transformations by transforming a graph given transformations

Source: Made it up

Explanation:

This artifact demonstrates graphical transformations by transforming a graph given transformations.

Like the previous proficiencies, I got the solution by referencing the formula: $a * f(b(x - c)) + d$

Here are the steps I took to get from $f(x) = \sqrt{x}$ to $g(x)$.

1. $4\sqrt{x}$
2. $4\sqrt{3x}$
3. $4\sqrt{3(x-2)}$
4. $4\sqrt{3(x-2)+3}$

Numeric Algebraic Graphic Connection

I've included a graph of the functions described in this artifact. This graph backs up my claims. It is the visual/numerical representation of my algebraic formulas.

Appropriate Use of Technology

I used an online graphing calculator to generate the graph below.

Once I generated it:

- I took a screenshot of the online graph
- I cropped the screenshot
- I added the image to my local code repository
- I included the image in my source code
- I uploaded the image to my code repository (https://github.com/doubledubba/precalc) and updated my code
- I synchronized my readthedocs.org project with my repo

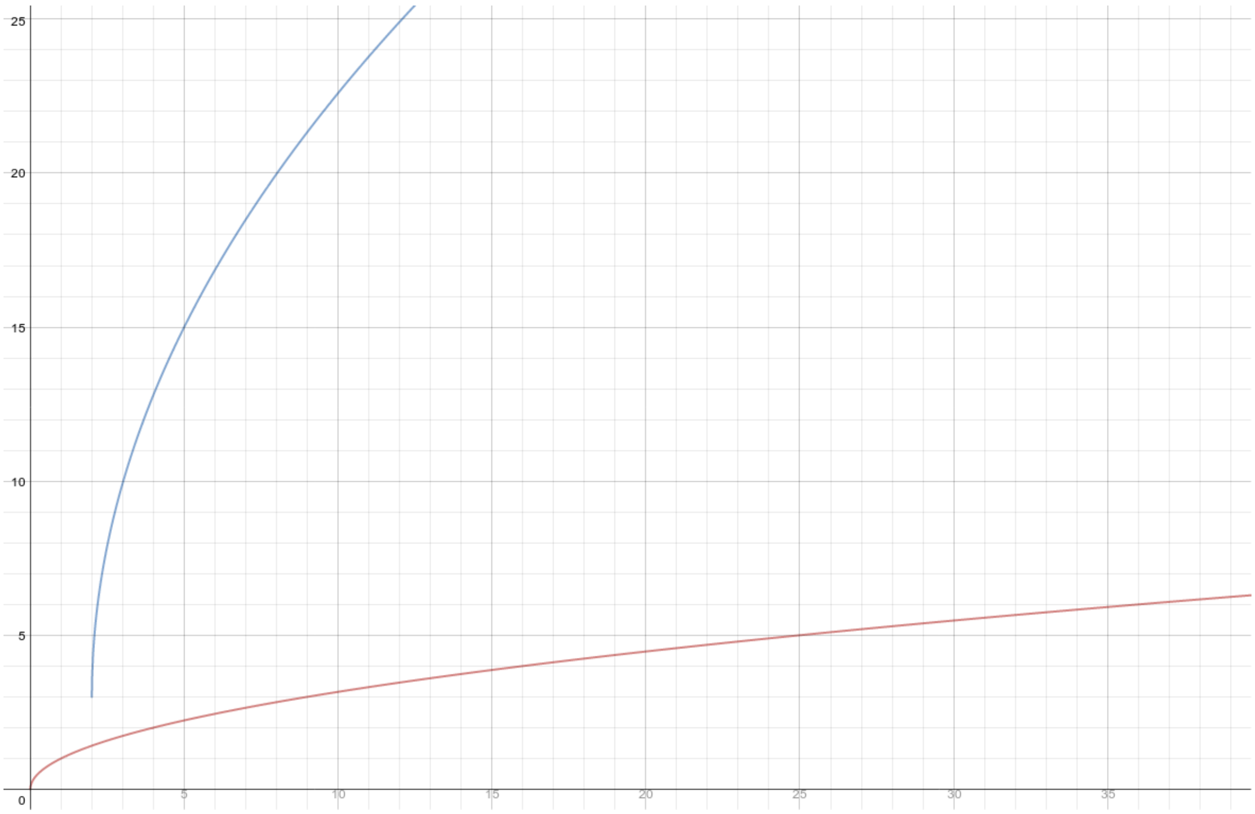
Artifact:

Transform $f(x) = \sqrt{x}$ into $g(x)$ with the following transformations:

1. Vertical stretch by magnitude of **four**
2. Horizontal shrink by magnitude of $\frac{1}{3}$
3. Horizontal shift of **two** units
4. Vertical shift of **three** units.

$f(x) = \sqrt{x}$ (red)

$g(x) = 4\sqrt{3(x-2)} + 3$ (blue)



2.4 All graphical transformations by using each type of transformation

Source: I made it up.

Explanation:

This artifact demonstrates all graphical transformations by using each type of transformation.

It shows proficiency in:

- Reflection
- Translation
- Stretches and shrinks

Artifact:

Transform $f(x) = |x|$ into $g(x) = -3|4(x+4)| - 7$

1. Reflect over x-axis
2. Vertical stretch by magnitude of 3
3. Horizontal shrink by magnitude of $\frac{1}{4}$
4. Horizontal shift by **-4** units.
5. Vertical shift by **-7** units.

CHAPTER 3

Power Functions, Their Graphs And Applications

Portfolio Entry Reflection Sheet

<table>
<tr><td colspan="6">Name: Power functions, their graphs, and applications</td></tr>
<tr><td colspan="6">Entry Title: Power functions, their graphs, and applications</td></tr>
<tr><td colspan="6">Class Objective # (highlight one)</td></tr>
<tr><td colspan="6">1 2 3 4 5 6 7 8 9 10 11 12</td></tr>
<tr><td colspan="6">Class Goals
(Circle all the apply)</td></tr>
<tr><td>Awareness & Appreciation</td><td>Application</td><td>Independent Thinking</td><td>Appropriate Use of Technology</td><td>Group Work</td><td>Numeric Algebraic Graphical Connection</td></tr>
</table>

Explanation:

This artifact demonstrates graphs of different power functions ($y = a * x^n$).

There are really four different ways that power functions will look like.

The orange function is one where $n < 0$.

The red on is where $n > 0$

The green one is where $n = 1$

The purple on is where $0 < n < 1$

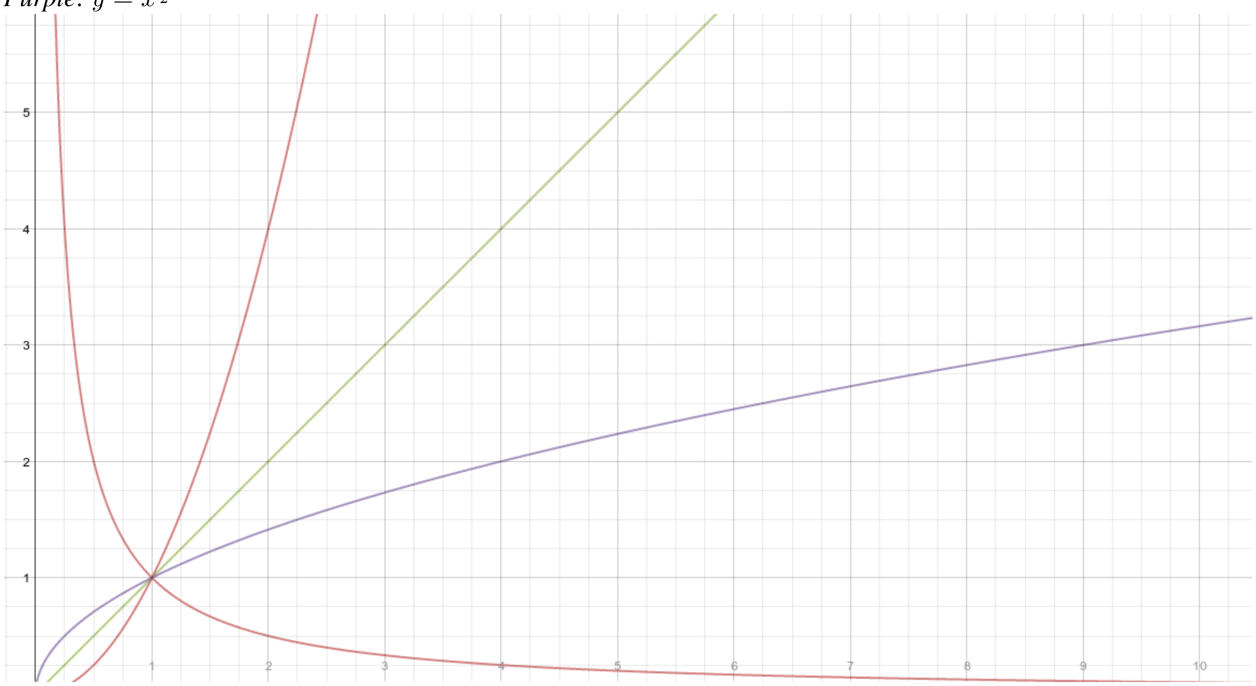
Artifact:

Orange: $y = x^-1$

Red: $y = x^2$

Green: $y = x^1$

Purple: $y = x^{\frac{1}{2}}$



3.2 Writing a power function from a list of data

Source: Chapter 2 Test Non Calculator #1, and python code

Explanation:

This artifact demonstrates writing a power function from a list of data.

I got the first statement ($2^n = \frac{1}{4}$) from the first two points in the data set.

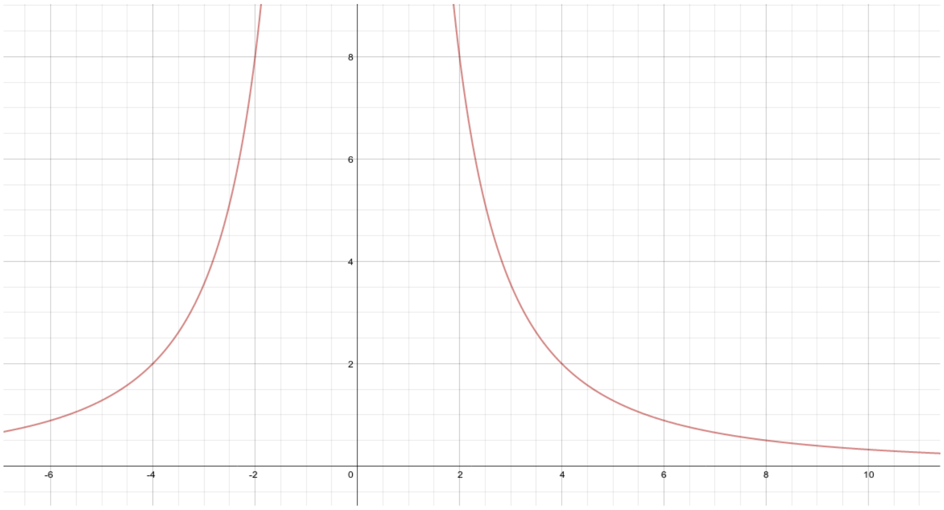
In the x, $1x = 2$
$x = \frac{2}{1} = 2$

In the y, $32x = 8$
$x = \frac{8}{32} = \frac{1}{4}$

From there I infer that $2^n = \frac{1}{4}$, solve for n, and compose a power function.

Numeric Algebraic Graphic Connection

My function is backed up by this graph (look at 2,8 - it matches the data set):



Application and Independent Thinking

I wrote a python script to validate my function.

This demonstrates Application because I was able to apply this math problem into a program, which is a very usefull skill.

It also demonstrates independent thinking because I taught myself how to write python code.

It runs successfully:

```
points = (  # dataset
     (1.0, 32.0),
     (2.0, 8.0),
     (6.0, 8.0/9.0),
     (8.0, 0.5),
     (10.0, 0.32),
)

def f(x):  # This is a function. A x value gets plugged in, and a y value comes out.
    return 32.0*x**-2  # same as 32*x^-2

for x,y in points:  # This is a recursive loop. x and y are variables points from our dataset
    assert y == f(x)  # If y is not equal to f(x), then the program will fail.
```

Artifact:

x	y
1	32
2	8
6	8/9
8	0.5
20	0.32

$2^n = \frac{1}{4}$
$n = -2$
$8 = a2^{-2}$

$a = 32$
$y = 32x^{-2}$

3.3 Applications of power functions through direct or indirect variation

Source: Chapter 2 Test (calculator portion) #2

Explanation:

This artifact demonstrates applications of power functions through direct or indirect variation.

From reading the problem, I could infer that (0.6, 14) was a point on the graph of the power function being described.

I understand as well that the function would be a function of length squared, and that is how I got the algebraic function.

Using my first observation of the point on the line, I plugged in what I knew about the previously defined algebraic function (x and y).

At that point, I was able to solve for the only remaining variable, the constant of variation.

Having been provided the power of the x in the problem, and having solved for the constant of variation, I was able to compose the formula $s(l) = 39.89 * l^2$

From there, it was only a matter of solving for s(0.65) to answer the question.

Application

This problem demonstrates the application of a classroom math problem to a real life situation.

Artifact:

The top speed at which a person can sprint varies *directly* as the *square* of their stride length.

If a person can run at a top speed of 14 mph with a stride length of 0.6m,

how fast can she run if she increases her stride length to 0.65 meters?

$s(l) = a * l^2$
$14 = a * 0.6^2$
$a = 38.89$
$s(l) = 39.89l^2$
$s(0.6) = 13.969$ mph
$s(0.65) = 16.393$ mph

CHAPTER 4

Polynomial Functions, Their Graphs And Applications

Portfolio Entry Reflection Sheet

Name: Polynomial functions, their graphs and applications					
Entry Title: Polynomial functions, their graphs and applications					
Class Objective # (highlight one)					
1 2 3 **4** 5 6 7 8 9 10 11 12					
Class Goals (Highlight all that apply)					
Awareness & Appreciation	**Application**	**Independent Thinking**	**Appropriate Use of Technology**	Group Work	**Numeric Algebraic Graphical Connection**

Explanation:

This artifact demonstrates graphs of polynomial functions by graphing a polynomial that shows comprehension of how multiplicity and end behavior affect the graph.

The graph below has two zeros (5 and -2) and a multiplicity of 3.

While the zeroes overlap and stay the same, changing the exponents of these linear factors changes the end behavior of the graph.

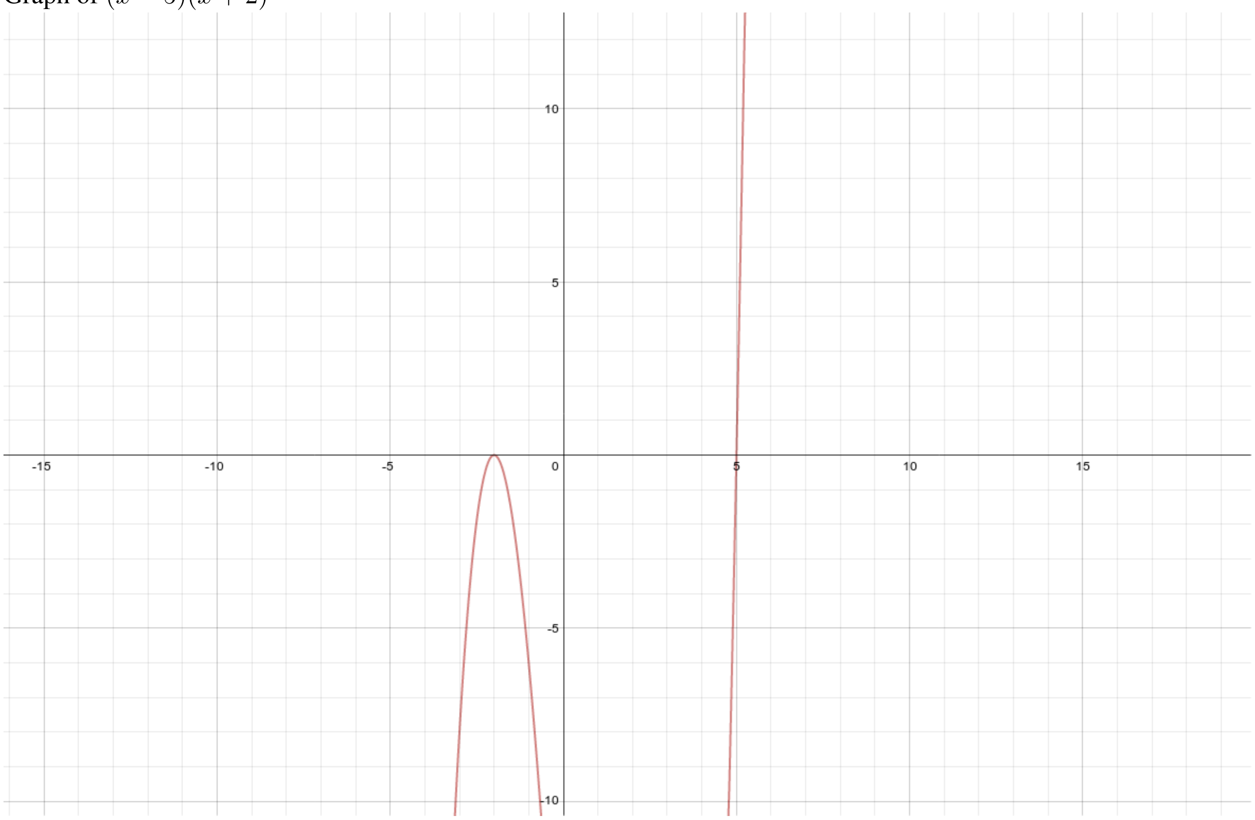
The exponents currently add up to 3 (multiplicity), which is an odd number.

Because the multiplicity is odd, the graph looks the way it does. If you were to change the multiplicity to any odd number, the graph would have the same end behavior and look essentially the same.

If the multiplicity were even, the end behavior would be different (parabolic).

Artifact:

Graph of $(x-5)(x+2)^2$



4.2 Factoring a higher degree polynomial with and without complex zeros

Source: Notes

Explanation:

This artifact demonstrates factoring a higher degree polynomial with and without complex zeros.

Using the rational zero theorem, I knew that $x^4 - 3x^3 - 6x^2 + 6x + 8$ is divisible by one of $\pm 8 \pm 4 \pm 2 \pm 1$

After trying quite a few options, I eventually tried dividing by (x+1), which worked.

Since the original function had a leading degree of 4, the result of that division was a function with a leading degree of 3 and a linear factor (x+1).

So I ran synthetic division one more time.

This time the result was a quadratic and two linear factors.

I then factored the quadratic into two more linear factors (4 total).

Artifact:

Reduce $x^4 - 3x^3 - 6x^2 + 6x + 8$

Synthetic division (divided by -1)

1	-3	-6	6	8
0	-1	4	2	-8
1	-4	-2	8	0

$(x + 1)(x^3 - 4x^2 - 2x + 8)$

Synthetic Division (divided by 4)

1	-4	-2	8
0	4	0	-8
1	0	-2	0

$(x + 1)(x - 4)(x^2 - 2)$

$(x + 1)(x - 4)(x - \sqrt{2})(x + \sqrt{2})$

4.3 Factoring a higher degree polynomial that has a leading coefficient that is not one

Source: Notes

Explanation:

This artifact demonstrates factoring a higher degree polynomial that has a leading coefficiant that is not one.

This one was a lot like the last one, except that I had to do a little bit more work at the end because you aren't supposed to have fractions in the linear factors.

I had to multiply two of them by their denominators to clear the fraction, then I had to multiply the other two by the reciprocal of that.

Artifact:

Reduce $3x^4 + 8x^3 + 6x^2 + 3x - 2$ to linear factors.

Synthetic Division (divided by -2)

3	8	6	3	-2
0	-6	-4	-4	2
3	2	2	-1	0

$(x + 2)(3x^3 + 2x^2 + 2x - 1)$

Synthetic Division (divided by $\frac{1}{3}$)

3	2	2	-1
0	1	1	1
3	3	3	0

$(x+2)(x-\frac{1}{3})(3x^2+3x+3)$

$(x+2)(x-\frac{1}{3})3(x^2+x+1)$

$(x+2)(3x-1)(x^2+x+1)$

from (x^2+x+1) x = $\frac{-1}{2} \pm \frac{i\sqrt{3}}{2}$

$(x+2)(3x-1)(x-(\frac{-1}{2}-\frac{i\sqrt{3}}{2}))(x-(\frac{-1}{2}+\frac{i\sqrt{3}}{2}))$

$(x+2)(3x-1)(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})(x+\frac{1}{2}+\frac{i\sqrt{3}}{2})$

$\frac{1}{2}(x+2)\frac{1}{2}(3x-1)2(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})2(x+\frac{1}{2}+\frac{i\sqrt{3}}{2})$

$\frac{1}{2}(x+2)\frac{1}{2}(3x-1)(2x+1-i\sqrt{3})(2x+1+i\sqrt{3})$

4.4 Solving polynomial equations and inequalities

Source: Chapter 2 Test Non Calculator #2

Explanation:

This artifact demonstrates solving polynomial equations and inequalities.

This problem was not too different from the previous.

Initially, I just had to reduce it to linear factors. The difference between the others comes with the inequality part.

I made a sign chart to make it easier to test the different portions of the function's potential behavior changes for it's positivity/negativity (polarity) in order to solve the original inequality.

Appropriate Use of Technology

I wanted to show a sign chart somehow.

I realized that a LaTeX expression would not have been appropriate, because it would be monstruous.

I concluded that the most appropriate use of technology (in this case) was to draw a sign chart my self, and include it on the page.

Numeric Algebraic Graphic Connection

The original problem was algebraic, but I solved it numerically using a graph.

I used the one-dimensional graph as a reference point.

I plugged in numbers in between the sign changes on the graph to evaluate the possible solutions for the problem.

Awareness

I'm aware that the question was specifically less than **or equal to** 0.

This is why I included the number 3 as one of the answers (it evaluates to 0).

Artifact:

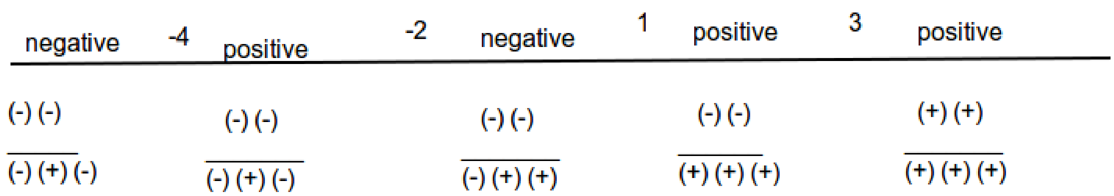
Solve $\frac{x^2-6x+9}{x^3+5x^2+2x-8} \leq 0$

Synthetic Division ($\frac{x^3+5x^2+2x-8}{(x-1)}$)

1	5	2	-8
0	1	6	8
1	6	8	0

$(x-1)(x^2+6x+8)$

$(x-1)(x+2)(x+4)$



$\frac{x^2-6x+9}{x^3+5x^2+2x-8} \leq 0 = (-\infty,-4),(-2,1),3$

4.5 Applications of polynomial functions

Source: section 2.3 Homework question #65.b

Application:

This is a prime example of how math can be applied in our lives.

Even though we may rarely use precalculus level math in our day to day lives, there are situations where math is very important, like the one in this artifact.

Explanation:

This artifact demonstrates applications of polynomial functions.

The main hurdle in this problem was really understanding what the question was asking.

In reality, the only math required in this problem is solving for r using a square root, and knowing that the answer is (cm).

Independent thinking

The final answer is actually one of two because it is the result of a square root, which always yields two answers.

Where the independent thinking comes in is the part where I take the absolute value of the square root.

I knew that it wouldn't make sense to have a blood cell that is a negative distance away from the center of an artery.

So I decided to discard the negative answer by taking the absolute value of the square root.

Artifact:

Research conducted at a national health research project shows that the speed at which a blood cell travels in an artery depends on its distance from the center of the artery.

The function $v = 1.19 - 1.87r^2$ models the velocity (in centimeters per second) of a cell that is *r* centimeteres from the center of an artery.

If a blood cell is traveling at $0.975\frac{cm}{sec}$, estimate the distance the blood cell is from the center of the artery.

$$\begin{gathered}
0.975 = 1.19 - 1.87r^2 \\
-0.215 = -1.87r^2 \\
0.215 = 1.87r^2 \\
r^2 \approx 0.1149 \\
r \approx \pm\sqrt{0.1149} \approx |\pm 0.339| \approx 0.339\text{cm}
\end{gathered}$$

CHAPTER 5

Rational Functions, Their Graphs, And Applications

Portfolio Entry Reflection Sheet

<table>
<tr><td colspan="12">Name: Rational functions, their graphs, and applications</td></tr>
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<tr><td colspan="12">Class Objective # (highlight one)</td></tr>
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<tr><td colspan="2">Awareness & Appreciation</td><td colspan="2">Application</td><td colspan="2">Independent Thinking</td><td colspan="2">Appropriate Use of Technology</td><td colspan="2">Group Work</td><td colspan="2">Numeric Algebraic Graphical Connection</td></tr>
</table>

Explanation:

This artifact demonstrates graphs of rational functions including all intercepts and asymptotes.

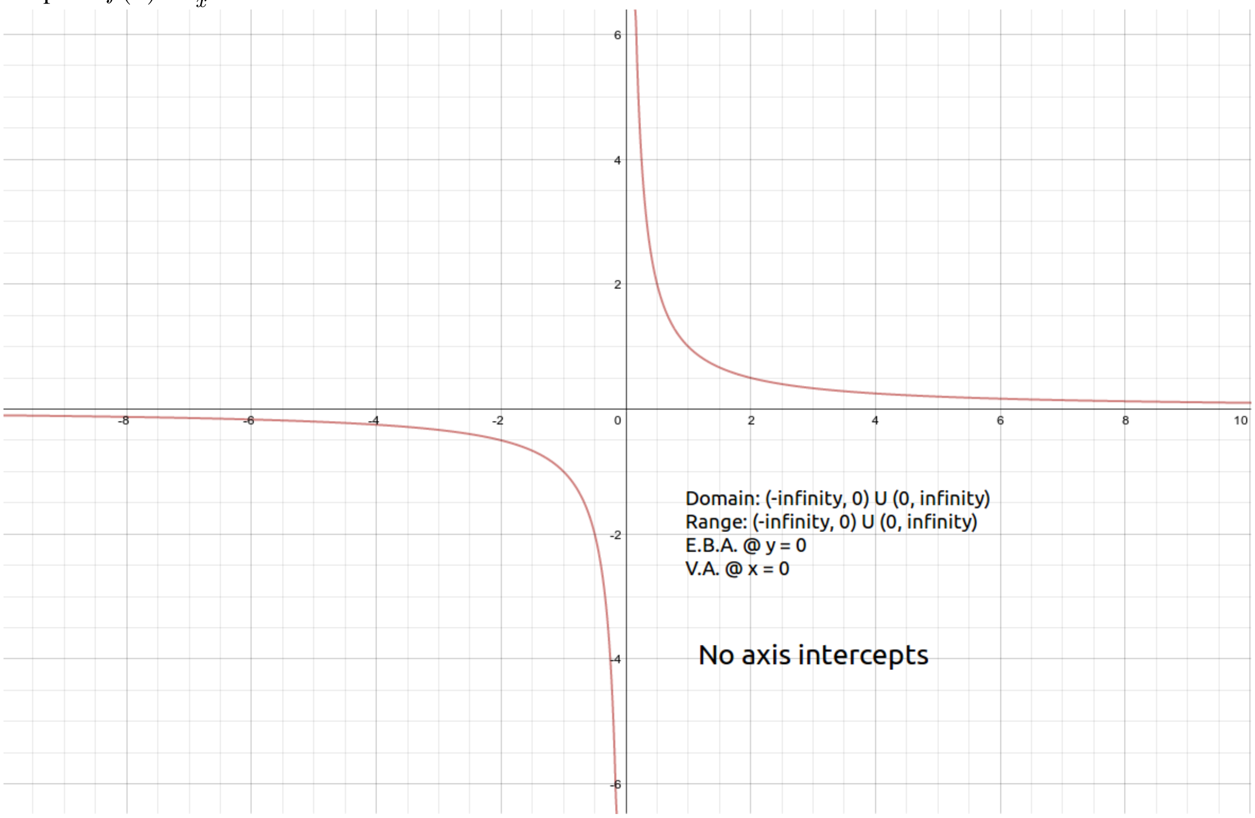
There is an asymptote at $x = 0$ because if x were ever 0 the function would be undefined (you can't divide by zero).

There is an asymptote at $y = 0$ because the numerator is 1. One slice of the pie is larger than 0 slices of the pie.

There is no x-intercept and there is no y-intercept because of the asymptotes (y = 0, and x = 0).

Artifact:

Graph of $f(x) = \frac{1}{x}$



5.2 Algebraic manipulation of rational functions

Source: Notes

Explanation:

This artifact demonstrates algebraic manipulation of rational functions.

Here I demonstrate that I find rational zeroes by manipulating a rational function into linear factors.

Artifact:

Find all of the rational zeroes for $t(x) = 3x^3 + 4x^2 - 5x - 2$

Using the rational zero theorem: $\frac{\pm 1, \pm 2}{\pm 1, \pm 3}$

Synthetic Division ($\frac{3x^3+4x^2-5x-2}{(x-1)}$)

3	4	-5	-2
0 3	3 7	7 2	2 0

$(x-1)(3x^2+7x+2)$

$(x-1)(3x+1)(x+2)$

5.3 Utilizing rational functions through applications

Source: Online

Application:

I don't think that this formula is reliable, but it is still a good example of how rational functions might be applied in our day to day life.

This type of formula may have been useful to me a few months ago, when I was trying to numerically compare my phone's battery life with and without wifi.

The "task" could be draining the battery completely. One "worker" could be my phone without wifi, and the other could be my phone with wifi.

Awareness and appreciation

A few months ago I went on the Junior Retreat. We did a lot of gardening. I worked on planting new plants, which was a repetitive process.

I counted the time it took my to plant a new plant (I can't remember what the time exactly).

But I remember being curious how long it would take me if I had a friend help me. A rational function like this may have helped me figure this out.

This kind of goes along with the application objective as well, but it does show that I am aware of opportunities to apply what I've learned in math class, and I can appreciate how cool that is.

Explanation:

This artifact demonstrates utilizing rational functions through applications.

Artifact:

T=(AB)/(A+B), gives the time T, it takes for two workers to complete a particular task.

A+B represents the time it would take for each individual worker to complete the identical task.

It takes Joe 2 hours to weed the garden, and it takes Joe's older brother twice as long.

Estimate how long it would take for the two of them to complete the task together.

A = 2, B = 4

$T = \frac{2*4}{2+4}$
$T = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3} =$ 1 hour and 20 minutes

5.4 Solving rational functions inequalities

Source: I don't know.

Explanation:

This artifact demonstrates solving rational functions inequalities.

The first thing I did was simplify the rational function.

Then I set up a sign chart with divisions at the zeroes, because those are the places where the sign of the graph could potentially change.

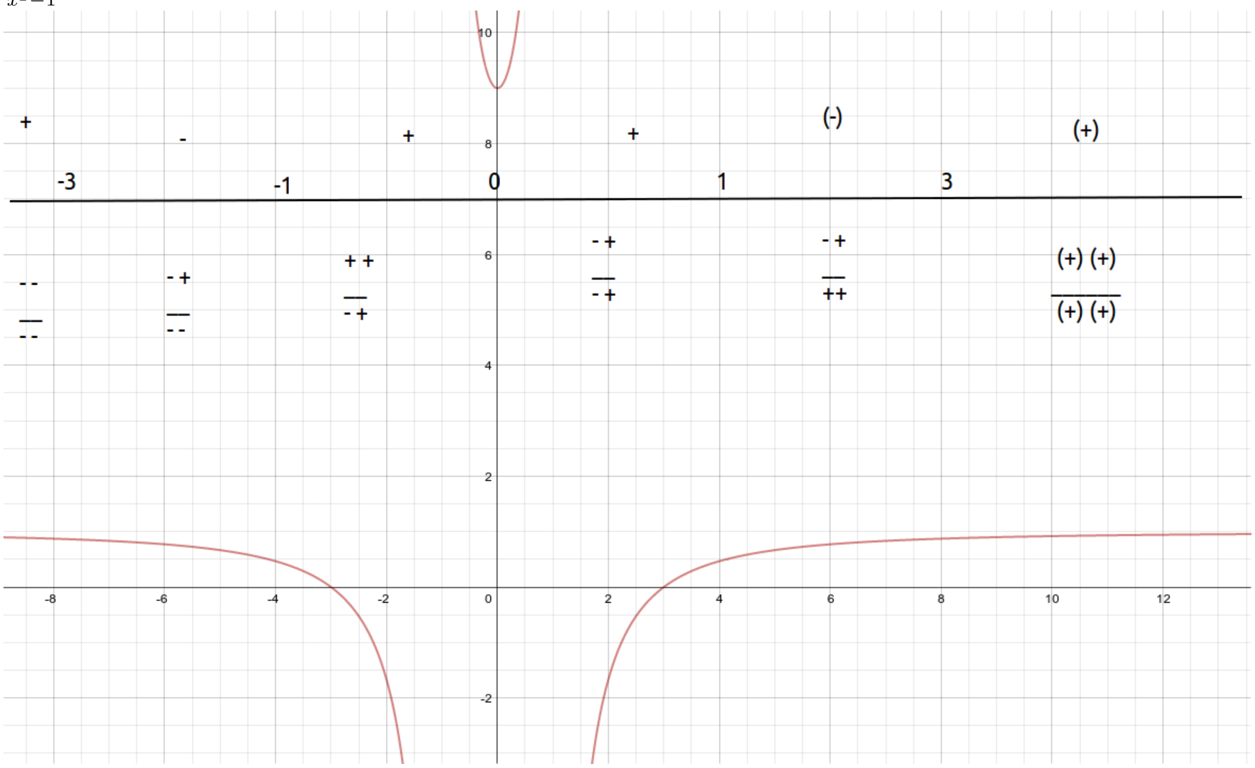
It was really only a matter of plugging in values in between the zeroes and recording them.

Artifact:

$\frac{x^2-9}{x^2-1} < 0$

$\frac{x^2-9}{x^2-1} == \frac{(x-3)(x+3)}{(x-1)(x+1)}$

$\frac{x^2-9}{x^2-1} < 0$ when $-3 < x < -1$ or $1 < x < 3$



CHAPTER 6

Exponential And Logistic Functions, Their Graphs And Applications

Portfolio Entry Reflection Sheet

Name: Exponential and logistic functions, their graphs and applications					
Entry Title: Exponential and logistic functions, their graphs and applicationss					
Class Objective # (highlight one)					
1 2 3 4 5 **6** 7 8 9 10 11 12					
Class Goals (Highlight all that apply)					
Awareness & Appreciation	**Application**	Independent Thinking	Appropriate Use of Technology	Group Work	Numeric Algebraic Graphical Connection

Explanation:

This artifact demonstrates writing exponential models.

I used the exponential formula $y = ab^x$ to solve this problem.

a stands for the initial amount.

b stands for the rate.

Since the rate is a percentage that is growing, I need to express it as 1 + 0.031, or else my function would be decaying and inaccurate.

Artifact:

My height is growing exponential by 3.1% every year. Right now he I am 68.4 inches tall.

An exponential model for my height would be

$$y = 68.4(1 + 0.031)^{\frac{1}{3}}$$

$$y = 68.4(1.031)^{\frac{x}{3}} \text{ x is in years.}$$

6.2 Writing logistic models

Source: Chapter 3 Test Non Calculator Section #3

Explanation:

This artifact demonstrates writing logistic models.

This problem makes use of the logistic formula: $y = \frac{c}{1+ab^x}$.

I only get the limit, the initial value, and a point - I need to solve for a so I can solve for b.

I get started by filling in what I know (the limit growth and the given x,y point).

Then I solve for a, which I plug back into the formula so I can solve for b.

Once I've got b, I can drop the x,y coordinate that I was using before so I can re-use my function with different variables.

Artifact:

Find the logistic function that has a limit to growth of 24, and initial value of 6, and goes through the point $(8, 14)$.

$$
\begin{aligned}
6 &= \frac{24}{a + ab^x} \\
6 &= \frac{24}{1 + a(1)} \\
6 + 6a &= 24 \\
6a &= 18 \\
a &= 3 \\
14 &= \frac{24}{1 + 3(b)^8} \\
14(1 + 3b^8) &= 24 \\
14 + 42b^8 &= 24 \\
42b^8 &= 10 \\
b^8 &= \frac{10}{42} = \frac{5}{21} \\
b &= \frac{5}{21}^{\frac{1}{8}} \\
y &= \frac{24}{1 + 3(\frac{5}{21})^{\frac{x}{8}}}
\end{aligned}
$$

6.3 Graphing exponential functions

Source:

Explanation:

This artifact demonstrates graphing exponential functions.

All this problem had me do was create an x,y table for evaluating f(x) with different x values.

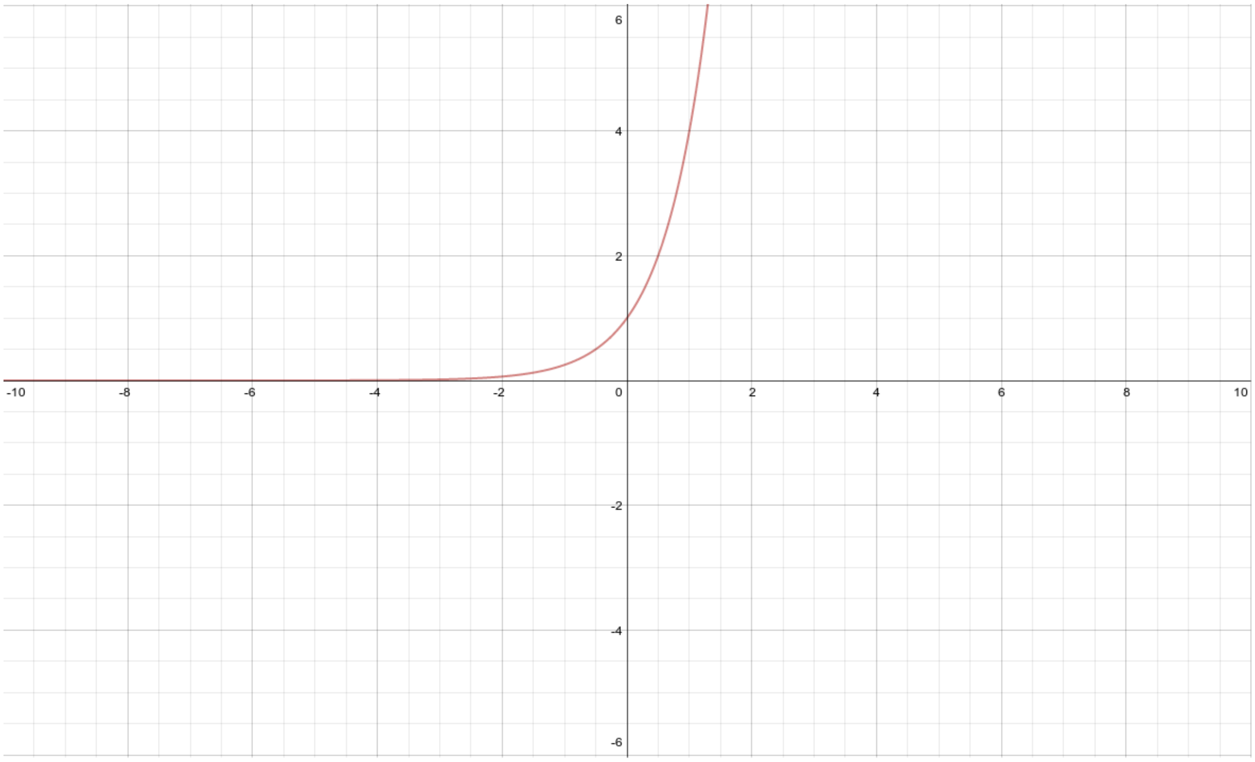
Then I could plot the points on the line and graph the function accurately based on my knowledge of that the end behavior of an exponential graph should look like.

Artifact:

Graph $f(x) = 4^x$

x	y
0	1
1	4
2	16
3	64

$\lim_{x \to -\infty} = 0$



6.4 Applying exponential models

Source: Made it up.

Explanation:

This artifact demonstrates applying exponential models.

Since I am currently 17, I plug in 2 for x because that is the difference between 19 and 17.

Then all I have to do is solve the exponential equation to get my answer.

Artifact:

Using the previously defined model for my height ($y = 68.4(1.031)^{\frac{x}{3}}$ x is in years.).

I can try and guess what my height will be when I am 19.

$$y = 68.4(1.031)^{\frac{x}{3}}$$
$$y = 68.4(1.031)^{\frac{2}{3}}$$
$$y \approx 69.80639 \text{ inches}$$

The 3.1% statistic is obviously incorrect, so my answer is a bit skewed.

6.5 Applying logistic models

Source:

Explanation:

This artifact demonstrates applying logistic models.

The logistic formula is given to me here. All I have to do is solve for the y value of 1 million.

I start by multiplying by $1 + 21.602e^{-0.05054t}$ and dividing by 1,000,000 to get the t in an easier postion.

I keep cutting down the side of the equation that has t on it until I've got it ready to use the logarithmic power rule.

Then I just have to some more basic simplification and I'm done!

Application:

This is a very real-life application. Every 4 years the government runs a national census.

Individual counties and districts all over the country run censuses too.

Knowing how to use logistic models is crucial, because they are good for population modeling.

This is because they have a limit growth value, which makes them more realistic functions than others that are more unbounded.

Artifact:

Based on recent census data, a logistic model for the population of Dallas, *t* years after 1900, is as follows:

$P(t) = \frac{1{,}301{,}642}{1+21.602e^{-0.05054t}}$

According to this model, when was the population 1 million?

$$\frac{1,301,642}{1 + 21.602e^{-0.05054t}} = 1,000,000$$
$$\frac{1,301,642}{1,000,000} = 1 + 21.602e^{-0.05054t}$$
$$1.301642 = 1 + 21.602e^{-0.05054t}$$
$$0.301642 = 21.602e^{-0.05054t}$$
$$\log(0.0139636145) = \log(e^{-0.05054t})$$
$$\log(0.0139636145) = \log(e) * -0.05054t$$
$$\frac{\log(0.0139636145)}{\log(e)} = -0.05054t$$
$$-0.8056171977 = -0.05054t$$
$$t \approx 15.9401 \text{ years (1915)}$$

CHAPTER 7

Logarithmic Functions Their Graphs And Applications

Portfolio Entry Reflection Sheet

<table>
<tr><td colspan="12">**Name:** Logarithmic functions their graphs and applications</td></tr>
<tr><td colspan="12">**Entry Title:** Logarithmic functions their graphs and applications</td></tr>
<tr><td colspan="12">Class Objective # (highlight one)</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>**7**</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td colspan="12">Class Goals
(Highlight all that apply)</td></tr>
<tr><td colspan="2">Awareness & Appreciation</td><td colspan="2">Application</td><td colspan="2">Independent Thinking</td><td colspan="2">Appropriate Use of Technology</td><td colspan="2">Group Work</td><td colspan="2">Numeric Algebraic Graphical Connection</td></tr>
</table>

Explanation:

This artifact demonstrates rewriting exponentials into logarithms and logarithms into exponentials using common log, natural log, and logarithms of other bases.

1. Exponential → Common Log
2. Commong log → Exponential
3. Exponential → Natural Log
4. Natural log → exponential

Artifact:

1. $4^x = 6 \rightarrow log_4 6 = x$
2. $log_2 2 = x \rightarrow 2^x = 2$
3. $e^x = e \rightarrow ln(e)$
4. $ln(8) \rightarrow e^x = 8$

7.2 Properties of logarithms

Source: Notes

Explanation:

This artifact demonstrates properties of logarithms.

1. This problem demonstrates the Product Rule.

$log_b rs = log_b r + log_b s$

2. This problem demonstrates the Quotient Rule.

$log_b \frac{r}{s} = log_b r - log_b s$

3. This problem demonstrates the Power Rule.

$log_b r^c = log_b r * c$

Artifact:

1. $log(x + 6) + log(x - 2) = 2$

$$log((x + 6)(x - 2)) = 2$$
$$log(x^2 + 4x - 12) = 2$$
$$10^2 = x^2 + 4x - 12$$
$$x^2 + 4x - 112 = 0$$
$$x \approx 8.77 \text{ or } x \approx -12.770$$

2. $log(x + 6) - log(x - 2) = 2$

$$log(\frac{(x + 6)}{(x - 2)}) = 2$$
$$10^2 = \frac{(x + 6)}{(x - 2)}$$
$$x \approx \frac{26}{9}$$

3. Solve for $log_4 117$

$$4^x = 117$$
$$log(4^x) = log(117)$$
$$x * log(4) = log(117)$$
$$x = \frac{log(117)}{log(4)} = 3.435$$

7.3 Graphs of logarithms

Source: Made it up.

Explanation:

This artifact demonstrates graphs of logarithms.

I started with the base function $y = log(x)$ and manipulated it into $log(x - 3) + 1$.

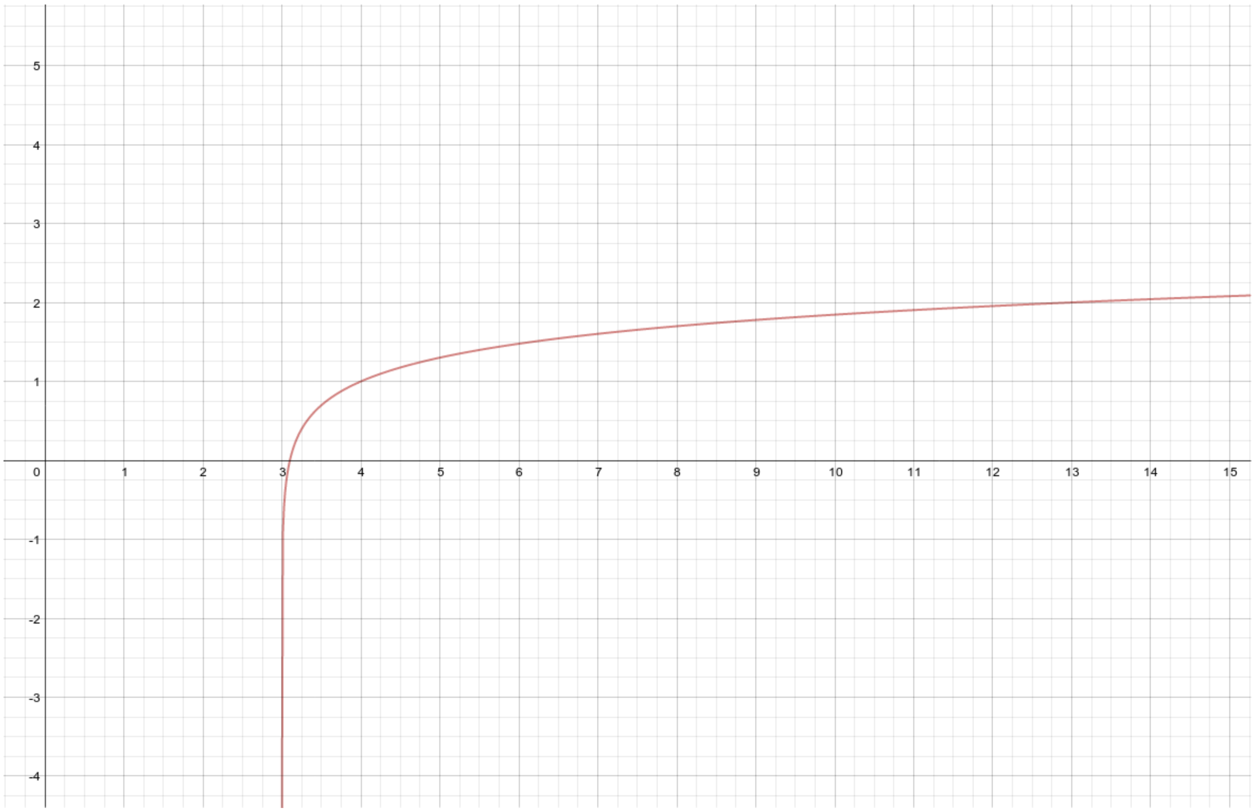
The formula $y = log(x)$ is the same as $10^y = x$, which is easier to evaluate (for y).

Artifact:

Graph $log(x - 3) + 1$

x	y
0.01	-2
0.1	-1
1	0
10	1

x+3	y+1
3.01	-1
3.1	0
4	1
13	2



7.4 Applications of logarithms

Source: #53 from Section 3.4

Explanation:

This artifact demonstrates applications of logarithms.

In the first step I demonstrate that I know how to re-write common logs into exponential form.

After that, I can plug in the given x value (40 ft) and the equation becomes linear and easy to solve.

Awareness and Appreciation:

In this artifact I demonstrate that I am aware that I am error-prone even if the problem is easy, and that I can appreciate double-checking my answers.

Artifact:

The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Superior is given by $log(\frac{I}{12}) = -0.00235x$

What is the intensity at a depth of 40 ft?

$$10^{-0.00235x} = \frac{I}{12}$$
$$0.805358 = \frac{I}{12}$$
$$I = 9.664541294 \text{ lumens}$$
$$\text{Checking my work...}$$
$$log(\frac{9.664541294}{12}) = -0.94$$
$$-0.00235 * 40 = -0.94$$

CHAPTER 8

Vectors And Their Applications

Portfolio Entry Reflection Sheet

Name: Vectors and their applications					
Entry Title: Vectors and their applications					
Class Objective # (highlight one)					
1 2 3 4 5 6 7 **8** 9 10 11 12					
Class Goals (Highlight all that apply)					
Awareness & Appreciation	Application	Independent Thinking	Appropriate Use of Technology	Group Work	Numeric Algebraic Graphical Connection

Explanation:

This artifact demonstrates the different forms of vectors.

In the problem, the vector is in coordinate form.

I convert it to component form using the Head Minus Tail Rule, and then I convert it to magnitude form by taking its' magnitude.

Artifact:

Find the magnitude of the vector v represented by $\overrightarrow{PQ}$ where P = (-3, 4) and Q = (-5, 2)

$(2-4, -5-(-3)) = (-2, -2)$ Using the Head Minus Tail Rule
$|v| = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$ Component form to magnitude

8.2 Vector application

Source: #2 from quiz 6.1 & 6.2

Explanation:

This artifact demonstrates vector application.

Artifact:

A boat is on a bearing of $210°$ traveling at 32 mph.

If it is in a 10 mph current that is on a bearing of $273°$, what is the boats ground speed and direction?

Vector for the boat: $32 < \cos 240, \sin 240 > \ = \ < -16, -27.713 >$

Vector for the current: $10 < \cos 177, \sin 177 > \ = \ < -9.85, 1.736 >$

Sum of the two vectors = $< -25.986, -27.1894 >$

Speed = $| < -25.986, -27.1894 > | =$ 37.611 mph

$tan^{-1}(\frac{-27.1894}{-25.85}) = 46.296$

Bearing = $90° - 46.296° = 43.5538°$

8.3 Finding the angle between two vectors

Source: #3 from quiz 6.1 & 6.2

Explanation:

This artifact demonstrates finding the angle between two vectors.

I found the answer to the problem using the following formulas:

- Angle between two vectors v and u: $\cos^{-1}(\frac{v*u}{|v|*|u|})$
- $< u_1, u_2 > * < v_1, v_2 >= u_1 * v_1 + u_2 * v_2$

First I found the dot product of the two vectors, their I found their individual magnitudes.

Then, all I had to do was plug into the equation and solve for the angle.

Artifact:

Find the angle between the vectors $< 6, -4 >$ and $< -2, 5 >$

$$\text{Dot : } 6 * -2 + -4 * 5 = -32$$
$$\text{Magnitude of vector 1: } \sqrt{6^2 + -4^2} = \sqrt{52}$$
$$\text{Magnitude of vector 2: } \sqrt{-2^2 + 5^2} = \sqrt{29}$$
$$\cos^{-1}\left(\frac{-32}{\sqrt{52} * \sqrt{29}}\right) = 145.491^\circ$$

CHAPTER 9

Polar Coordinates And Equations

Portfolio Entry Reflection Sheet

Name: Polar coordinates and equations											
Entry Title: Polar coordinates and equations											
Class Objective # (highlight one)											
1	2	3	4	5	6	7	8	**9**	10	11	12
Class Goals (Highlight all that apply)											
Awareness & Appreciation	Application	Independent Thinking	Appropriate Use of Technology	Group Work	Numeric Algebraic Graphical Connection						

Explanation:

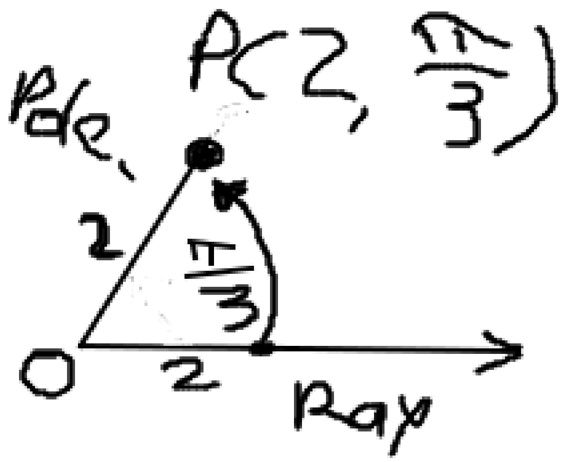
This artifact demonstrates how to graph polar coordinates.

In the polar coordinate $P(2, \frac{\pi}{3})$, the directed distance is 2, and the directed angle is $\frac{\pi}{3}$.

So the pole starts at 2 on the O ray, and swivels out by $\frac{\pi}{3}$ degrees.

Artifact:

Plot the point $P(2, \frac{\pi}{3})$



9.2 Converting polar coordinates to rectangular coordinates and rectangular to polar

Source: Notes, Section 6.4 example 3A

Explanation:

This artifact demonstrates converting polar coordinates to rectangular coordinates and rectangular to polar.

1. In this example I use the equation $r^2 = x^2 + y^2$ to solve for the directed distance (x) and $tan^{-1}(\frac{y}{x})$ to solve for the directed angle (y).
2. In this example I use the formulas $x = r\cos\theta$ and $y = r\sin\theta$, and my knowledge of the unit circle, to calculate the approximate values of x and y.

Artifact:

1. Convert the rectangular coordinate (2,7) into a polar coordinate.

$r = \sqrt{2^2 + 7^2} = \sqrt{53}$
$tan^{-1}(\frac{7}{2}) = 74°$

Polar coordinate = $(\sqrt{53}, 74°)$

1. Convert the polar coordinate $(3, \frac{5\pi}{6})$ into a rectangular coordinate.

$x = r\cos\theta$
$x = 3\cos\frac{5\pi}{6}$
$x = 3(-\frac{\sqrt{3}}{2}) \approx -2.60$

$y = r \sin \theta$
$y = 3 \sin \frac{5\pi}{6}$
$y = 3(\frac{1}{2}) \approx 1.5$

Rectangular coordinate = $(-2.60, 1.5)$

9.3 Converting polar equations to rectangular equations and rectangular to polar

Source: Section 6.4 Example 5 and Example 6

Explanation:

This artifact demonstrates converting polar equations to rectangular equations and rectangular to polar.

$r^2 = x^2 + y^2$
$x = r \cos \theta$
$y = r \sin \theta$

1. Here I simplify $r = 4 \sec \theta$ into $r \cos \theta$ so I can substitute for x using the formula $x = r \cos \theta$. The answer is the line $x = 4$.
2. Here I simplify the original equation into x's and y's in both the second and first degree because I can subsitute them with the conversion formulas
   $x = r \cos \theta$ and $y = r \sin \theta$ to convert them from rectangular form into polar form.

Artifact:

1. Convert $r = 4 \sec \theta$ to rectangular form.

$$r = 4 \sec \theta$$
$$\frac{r}{\sec \theta} = 4$$
$$r \cos \theta = 4$$
$$x = 4$$

1. Convert $(x-3)^2 + (y-2)^2 = 13$ to polar form.

$$(x-3)^2 + (y-2)^2 = 13$$
$$x^2 - 6x + 9 + y^2 - 4y + 4 = 13$$
$$x^2 + y^2 - 6x - 4y = 0$$
$$r^2 - 6r \cos \theta - 4r \sin \theta = 0$$
$$r(r - 6 \cos \theta - 4 \sin \theta) = 0$$
$$r = 0 \text{ or } r - 6 \cos \theta - 4 \sin \theta = 0$$

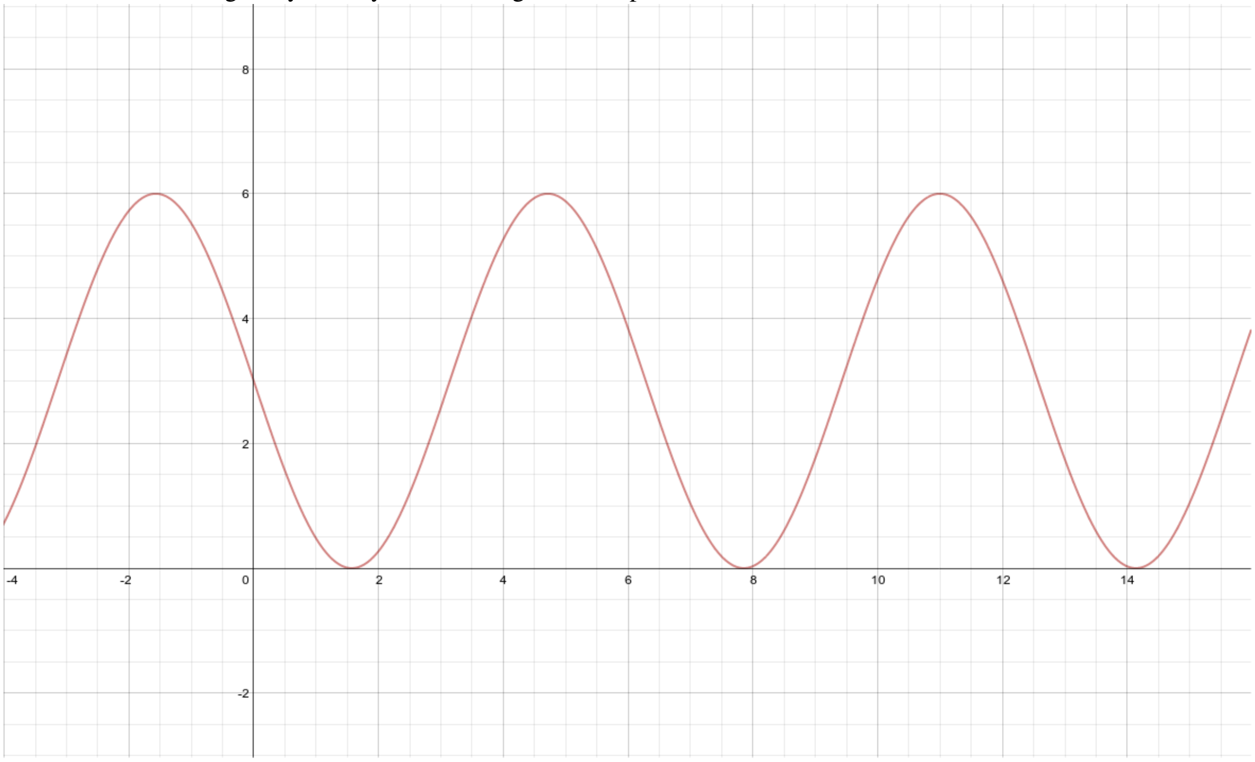
9.4 Graphs of polar equations

Source: Section 6.5 Example 5

Explanation:

This artifact demonstrates graphs of polar equations.

From graphing the polar equation $r = 3 - 3\sin x$ in radian/function mode, I can tell that the maximum r value is 6 because that is the highest y value you can ever get on this particulur sinusoid.



The rest of the information can be gathered visually.

Artifact:

Analyze the graph of $r = 3 - 3\sin\theta$.

Domain: All real numbers

Range: [0, 6]

Symmetric about the y-axis

Maximum r-value = 6

CHAPTER 10

Conic Sections Their Graphs And Applications

Portfolio Entry Reflection Sheet

Name: Conic sections their graphs and applications					
Entry Title: Conic sections their graphs and applications					
Class Objective # (highlight one)					
1 2 3 4 5 6 7 8 9 **10** 11 12					
Class Goals (Highlight all that apply)					
Awareness & Appreciation	Application	Independent Thinking	Appropriate Use of Technology	**Group Work**	Numeric Algebraic Graphical Connection

Explanation:

This artifact demonstrates how to make conic models that fit given conditions.

I'm assuming that the center of this ellipse is (0, 0).

I know b^2 = 4 because the problem tells us what the minor axis length is.

Since the foci are 6 y-units apart, the point between them would be on y = 3, so c = 3.

Knowing b^2 and c, I can solve for a^2 using the pythagorean relation for ellipses $a^2 = b^2 + c^2$

Artifact:

Find an equation of the ellipse with foci $(0, \pm 3)$ whose minor axis has length 4.

$$b^2 = 4$$
$$c = 3$$
$$a^2 = 4 + 3^2 = 13$$
$$\frac{y^2}{13} + \frac{x^2}{4} = 1$$

10.2 How to graph conic sections from equations

Source: Group Quiz 8.2 & 8.3

Group Work

This problem was from a group quiz. We initially did this problem independently, but we got different answers, so we worked together to figure out which one was correct.

Explanation:

The first thing I noticed from the equation was that it had addition, so it had to be ellipsoidal.

I noticed that the value with the highest denominator had y^2 in the numerator, so I knew it was an ellipse opening vertically.

This also told me that $a^2 = 25$, $b^2 = 9$, and that I could rely on the formula $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$.

I knew the center was $(-2, -5)$ by comparing equation with the formula.

I knew that the pythagorean relation for an ellipsoid is $c^2 = a^2 - b^2$, so I used my a and b values solve for the distance from the center to the foci (4)

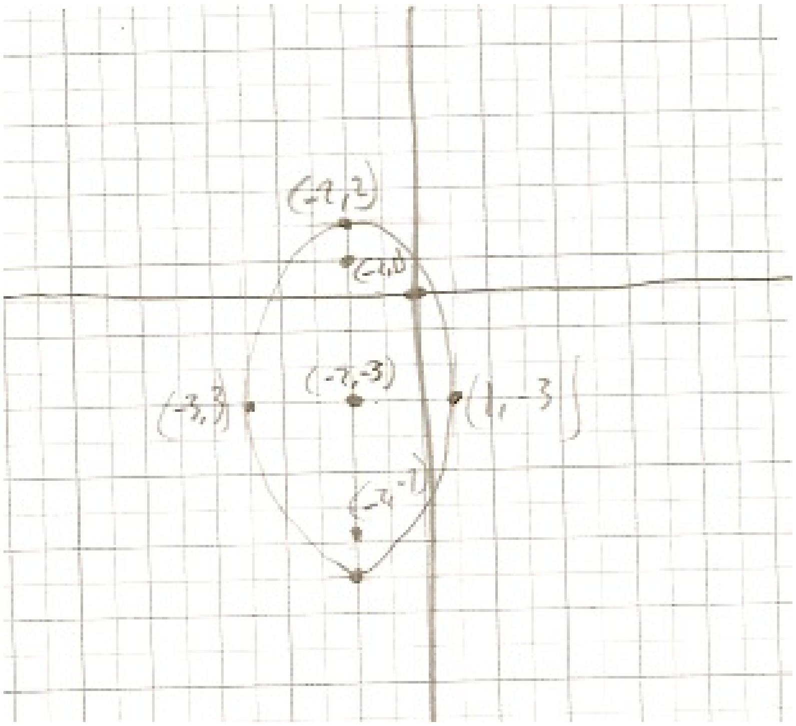
I knew my foci had to be at $(h, k \pm c) = (-2, -3 \pm 4)$.

I also knew that my end point had to be at $(h \pm b, k) = (-2 \pm 3, -3)$ because $b = \sqrt{b^2} = \sqrt{9} = 3$.

From there, all I had to do was graph.

Artifact:

Graph $\frac{(x+2)^2}{9} + \frac{(y+3)^2}{25} = 1$



10.3 The applications of conic sections

Source: Section 8.1 Example 6/my own little twist

Explanation:

This artifact demonstrates the applications of conic sections.

This artifact demonstrates how to graph conic sections from equations.

To solve this problem, I went off the assumption that the vertex of the parabolic reflector was at (0,0).

This assumption allowed me to use the formula $x^2 = 4py$

Since the vertex was at (0,0), the distance accross the reflector (6 feet) must divided symmetrically by the y-axis.

Since parabolas are always perfectly symmetric, the distance from the center to the endpoints of the parabolic reflector must be $\frac{1}{2}$ of the distance of the width of the parabolic reflector.

I also knew from the problem that the vertical distance from the center is 2.

Knowing these things, I was able to deduce that the endpoints of the parabolic reflector had to be at $(\pm 3, 2)$.

With that coordinate, all I had to do was plug into the parabolic formula, and solve for p (because p is the distance from the center to the foci).

Artifact:

At his weekend job as a secret agent in the CIA, Luis uses a parabolic reflector that he wears on his head with a microphone at the reflector's focus to capture the conversations of russian spies.

If the parabolic reflector is 6 feet across and 2 feet deep, where should the microphone be placed?

Center = $(0, 0)$

$$(\pm 3)^2 = 4p(2)$$
$$p = \frac{9}{8}$$

The microphone should be placed ≈ 1.125 feet from the vertex of the parabolic reflector.

10.4 How to algebraically manipulate conic equations into standard form

Source: Group Quiz 8.2 & 8.3

Group Work

This problem was from a group quiz. The three of us worked on this problem together.

Explanation:

This artifact demonstrates how to algebraically manipulate conic equations into standard form.

The first thing I did was organizational. I put the x values next to the other x values and the y values next to the other y values.

Then I pulled a common factor from the x's (25) and a common factor from the y's (16).

The result of this was two quadratic equations which I then factored using completing the square.

Once I had completed the square, I divided both sides by 400 to make the equation equal to 1 (because I knew it is a conic).

Then I divided the remaining common factors (25 and 16) by both sides to complete the hyperbolic equation.

Artifact:

Use completing the square and find the equation for $25y^2 - 16x^2 - 100y - 224x - 1084 = 0$

$$25y^2 - 100y - 16x - 224x = 1084$$
$$25(y^2 - 4y + 4) - 15(x^2 + 14x + 49) = 1084 + 100 - 784$$
$$25(y-2)^2 - 16(x+7)^2 = 400$$
$$\frac{25(y-2)^2}{400} - \frac{16(x+7)^2}{400} = \frac{400}{400}$$
$$\frac{(y-2)^2}{16} - \frac{(x+7)^2}{25} = 1$$

10.5 Parabolas, ellipses and hyperbolas

Source: Group Quiz 8.2 & 8.3

Explanation:

This artifact demonstrates parabolas, ellipses and hyperbolas.

The transverse axis is $2a$, so $2a = 6$.

The distance between the center and the foci (c) is the same as the distance between the two foci (in the x) divided by 2, which is 4.

I used the pythagorean relation for a hyperbola, which is $a^2 + b^2 = c^2$, to solve for b^2.

I took half the distance between the two foci to get the x value of my center (-1).

I knew the y value of my center was 4, because the center is on the same level as the two given foci.

Now I could just plug into the hyperbolic formula to get the standard form of this hyperbola.

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Artifact:

Find the equation for a hyperbola with foci at $(3, 4)$ and at $(-5, 4)$ and a transverse axis length of 6.

$$2a = 6$$
$$a = 3$$
$$c = 4$$
$$3^2 + b^2 = 4^2$$
$$b^2 = 7$$
$$\frac{(x+1)^2}{9} - \frac{(y-4)^2}{7} = 1$$

CHAPTER 11

Sequences And Series

Portfolio Entry Reflection Sheet

Name: Sequences and series					
Entry Title: Sequences and series					
Class Objective # (highlight one)					
1 2 3 4 5 6 7 8 9 10 **11** 12					
Class Goals (Highlight all that apply)					
Awareness & Appreciation	Application	Independent Thinking	Appropriate Use of Technology	**Group Work**	Numeric Algebraic Graphical Connection

Explanation:

This artifact demonstrates the usage of both geometric and arithmetic sequences.

The first example is geometric, and the second is arithmetic.

Artifact:

1. Find the first 3 terms and the 50th term of the sequence $\{a_k\}$ in which $a_k = k^2 - k$.

$$a_1 = 1^2 - 1 = 0$$
$$a_2 = 2^2 - 2 = 2$$
$$a_3 = 3^2 - 3 = 6$$
$$a_50 = 50^2 - 50 = 2450$$

1. Find the first 3 terms and the 100th term of the sequence $\{a_k\}$ in which $a_k = k + 4$.

$$a_1 = 1 + 4 = 5$$
$$a_2 = 2 + 4 = 6$$
$$a_3 = 3 + 4 = 7$$
$$a_100 = 100 + 4 = 104$$

11.2 Defining sequences explicitly and recursively

Source: Group Quiz 9.4 & 9.5

Group Work:

Matthew and I disagreed on whether to start the index numbers at 0 or 1 for the explicit functions.

He thought we should start at 1, and I thought we should start at 0 because it was easier.

He was right, but we worked together to compromise.

We had one of them start at 0, and the other start at 1.

Explanation:

This artifact demonstrates defining sequences explicitly and recursively.

The recursive formula works by multiplying the previous number in the series by 3, starting with -2.

The explicit formula works by using indices (n) starting at 1.

The exponent on the formula is n-1. Since the first index is 1, the -3 will be nulled because anything to the 0 power is 1.

The explicit formula is a bit nicer for humans because you don't have to calculate the values recursively.

Artifact:

Find the **explicit** and **recursive** formulas that model -2, 6, -18, 54, -162...

Recursive:

$$A_1 = -2$$
$$A_n = A_{n-1} * 3$$

Explicit:

$$A_n = -2(-3)^{n-1}$$

11.3 Summations notation

Source: Notes June 06, 2012

Explanation:

This artifact demonstrates summations notation.

The slope of the explicit function is 7 because that is the rate of change in the series that is suggested by the data.

I plugged in (1, 2) because 2 is the first item in the series (1 is the first index value).

I set it equal to the last term of the series to solve for the last index number, because that goes ontop of the sigma.

I used the gaussian method to find the sum of the finite arithmetic series.

Artifact:

Express the [2, 9, 16, 23, ..., 107] in summation notation.

$$
\begin{gathered}
y = mx + b \\
y = 7x + b \\
2 = 7(1) + b \\
b = -5 \\
y = 7x - 5 \\
107 = 7x - 5 \\
x = 16 \\
(\frac{(2+107)}{2}) * 16 = 872 \\
\sum_{x=1}^{16} 7x - 5 = 872
\end{gathered}
$$

11.4 Summing finite arithmetic and geometric sequences

Source: Section 9.5 Example 2 and Group Quiz 9.4 & 9.5

Group Work:

The second example in this artifact comes our most recent group quiz.

I remember that we were very systematic and efficient in this quiz.

I calculated the first value in the series while he calculated the last one.

We were like Batman and Robin.

Matthew was Robin.

Explanation:

This artifact demonstrates summing finite arithmetic and geometric sequences.

1.

I used the formula $\frac{a_1(1-r^n)}{1-r}$ to find the sum of this geometric series.

I was given a_1 and a_n. I just needed to find the r value by using basic algebra.

2.

I found the average of the first and last values in the series and then multiplied that by the number of values in the sequence to get the sum.

Artifact:

1. Find the sum of the geometric series $4, -\frac{4}{3}, \frac{4}{9}, -\frac{4}{27}, ..., 4(-\frac{1}{3})^{10}$

$$A_1 = 4$$
$$4r = -\frac{4}{3}$$
$$r = -\frac{1}{3}$$
$$A_n = 4(-\frac{1}{3})^{10}$$
$$n = 11$$
$$\frac{4(1 + \frac{1}{3}^{11})}{1 + \frac{1}{3}} \approx 3.000016935$$

2. Find the sum of the arithmetic series $\sum_{k=1}^{4} -6 + k$.

$$A_1 = -6 + 1 = -5$$
$$A_4 = -6 + 4 = -2$$
$$(\frac{(-5 + (-2))}{2}) * 4 = -14$$

11.5 Summing infinite geometric sequences

Source: Notes

Explanation:

This artifact demonstrates summing infinite geometric sequences.

To find the sum, I used the formula for the sum of an infinite geometric series ($\frac{A_1}{1-r}$).

This formula only works for geometric series that converge (eventually reaches a limit, usually 0).

Artifact:

Find the sum of the infinite geometric sequence [32, 16, 8, 4, 2, 1...]:

$$\text{sum } = \frac{32}{1 - \frac{1}{2}} = 64$$
$$\sum_{k=1}^{\infty} 32(\frac{1}{2})^{k-1} = 64$$

CHAPTER 12

Limits

Portfolio Entry Reflection Sheet

<table>
<tr><td colspan="12">Name: Limits</td></tr>
<tr><td colspan="12">Entry Title: Limits</td></tr>
<tr><td colspan="12" align="center">Class Objective # (highlight one)</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td colspan="12" align="center">Class Goals
(Highlight all that apply)</td></tr>
<tr><td colspan="2">Awareness & Appreciation</td><td colspan="2">Application</td><td colspan="2">Independent Thinking</td><td colspan="2">Appropriate Use of Technology</td><td colspan="2">Group Work</td><td colspan="2">Numeric Algebraic Graphical Connection</td></tr>
</table>

Explanation:

This artifact demonstrates how to write asymptotes in limit notation.

Example 1)

As x becomes extremely large, the value of f(x) approaches 2, and the value of f(x) can be made as close to 2 as one could wish just by picking x sufficiently large.

The limit of f(x) as x approaches infinity is 2.

Example 2)

As x approaches 0, $f(x)$ approaches positive or negative infinity, depending on which direction x is approaching from. This is because the closer the number gets to 0, the smaller it has got to be. For x-values like 0.0001, $f(x)$ will actually be a big number because $f(x) = \frac{1}{x}$

Inversely, as x approaches positive or negative 0, $f(x)$ approaches 0 because $f(x)$ is always a fraction. As the demominator of the fraction increases, it's value decreases and is getting closer and closer to 0.

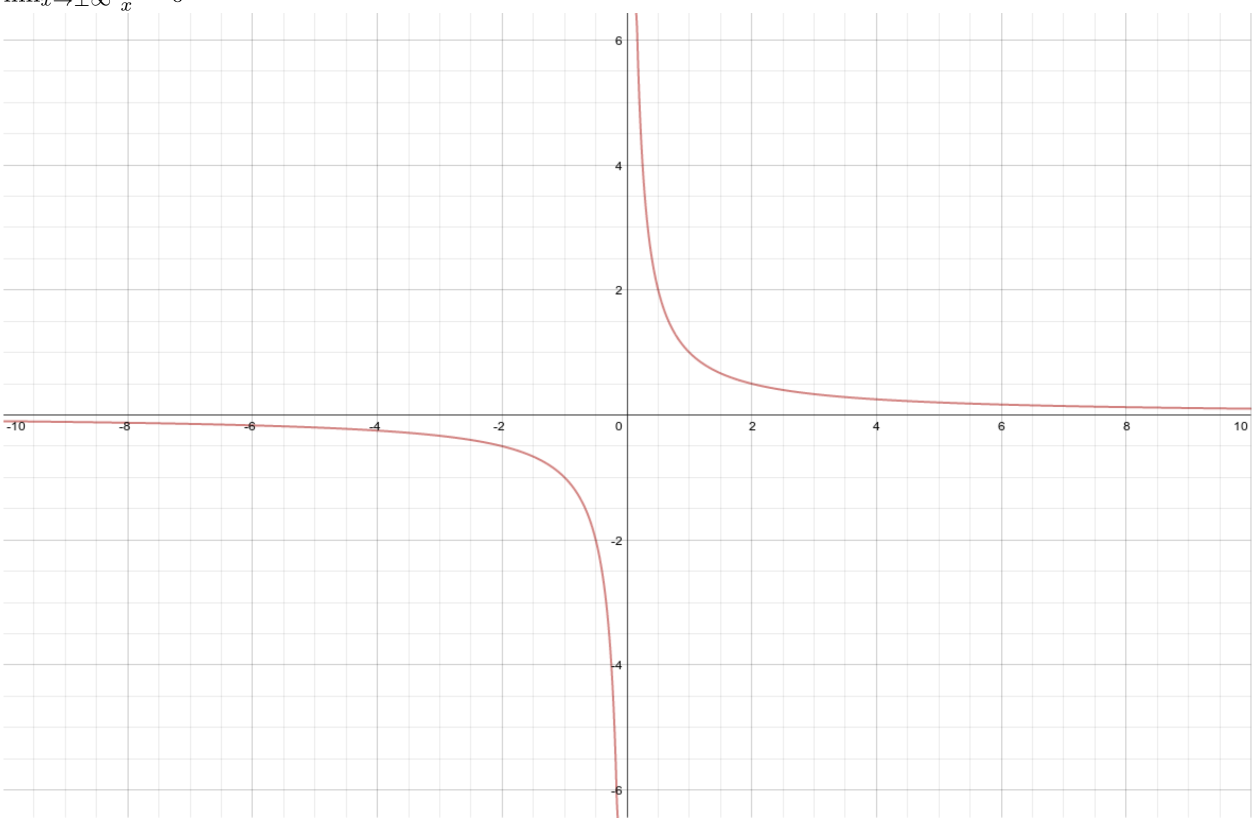
Artifact:

Example 1)

$f(x) = \frac{2x-1}{x}$
$\lim_{x\to\infty} \frac{2x-1}{x} = 2$

Example 2)

$f(x) = \frac{1}{x}$
$\lim_{x\to 0} \frac{1}{x} = \pm\infty$
$\lim_{x\to\pm\infty} \frac{1}{x} = 0$



12.2 Removable discontinuity

Source: Online Tutorial

Awareness and Appreciation:

I didn't know how to do this, but Igoogled it succesfully.

This demonstrates that I am aware of the magnificient resources that are available to me, and that I appreciate them because I use them.

Independent Thinking

I didn't know what to show about removable discontinuity, I just knew what it was.

So I went online and read about it to figure out how it works.

Explanation:

This artifact demonstrates removable discontinuity.

Discontinuity is removable if you can easily plug in the holes in its graph by redefining the function.

In the original function, for -2 and 2 f(x) is undefined. But a little bit of algebraic magic reveals that the function can be "patched" for f(2) to be successful.

Artifact:

$$f(x) = \frac{x-2}{x^2-4}$$

$$\frac{x-2}{(x-2)(x+2)}$$

$$\frac{1}{x+2} \text{ The (x-2)s cancel out}$$

$$f(2) = \frac{1}{4}$$

$$f(x) = \frac{1}{4} \text{ if x = 2}$$

$$x = -2 \text{ Vertical Asymptote}$$

$$x = 2 \text{ Removable Discontinuity}$$

CHAPTER 13

Indices and tables

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- *modindex*
- *search*