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1 Overview ..... 1
1.1 Who uses NetworkX? ..... 1
1.2 Goals ..... 1
1.3 The Python programming language ..... 1
1.4 Free software ..... 2
1.5 History ..... 2
2 Introduction ..... 3
2.1 NetworkX Basics ..... 3
2.2 Nodes and Edges ..... 4
3 Graph types ..... 9
3.1 Which graph class should I use? ..... 9
3.2 Basic graph types ..... 9
4 Algorithms ..... 119
4.1 Approximation ..... 119
4.2 Assortativity ..... 128
4.3 Bipartite ..... 137
4.4 Boundary ..... 165
4.5 Centrality ..... 166
4.6 Chains ..... 191
4.7 Chordal ..... 192
4.8 Clique ..... 194
4.9 Clustering ..... 199
4.10 Coloring ..... 203
4.11 Communicability ..... 207
4.12 Communities ..... 208
4.13 Components ..... 214
4.14 Connectivity ..... 230
4.15 Cores ..... 249
4.16 Covering ..... 252
4.17 Cycles ..... 254
4.18 Cuts ..... 257
4.19 Directed Acyclic Graphs ..... 261
4.20 Dispersion ..... 266
4.21 Distance Measures ..... 266
4.22 Distance-Regular Graphs ..... 268
4.23 Dominance ..... 270
4.24 Dominating Sets ..... 272
4.25 Efficiency ..... 273
4.26 Eulerian ..... 275
4.27 Flows ..... 276
4.28 Graphical degree sequence ..... 301
4.29 Hierarchy ..... 305
4.30 Hybrid ..... 305
4.31 Isolates ..... 307
4.32 Isomorphism ..... 308
4.33 Link Analysis ..... 322
4.34 Link Prediction ..... 328
4.35 Matching ..... 334
4.36 Minors ..... 336
4.37 Maximal independent set ..... 342
4.38 Operators ..... 342
4.39 Reciprocity ..... 352
4.40 Rich Club ..... 353
4.41 Shortest Paths ..... 354
4.42 Simple Paths ..... 381
4.43 Swap ..... 384
4.44 Tournament ..... 386
4.45 Traversal ..... 389
4.46 Tree ..... 398
4.47 Triads ..... 407
4.48 Vitality ..... 408
4.49 Voronoi cells ..... 408
4.50 Wiener index ..... 409
5 Functions ..... 411
5.1 Graph ..... 411
5.2 Nodes ..... 414
5.3 Edges ..... 415
5.4 Attributes ..... 416
5.5 Freezing graph structure ..... 418
6 Graph generators ..... 421
6.1 Atlas ..... 421
6.2 Classic ..... 422
6.3 Expanders ..... 430
6.4 Small ..... 432
6.5 Random Graphs ..... 436
6.6 Duplication Divergence ..... 445
6.7 Degree Sequence ..... 446
6.8 Random Clustered ..... 452
6.9 Directed ..... 453
6.10 Geometric ..... 456
6.11 Line Graph ..... 461
6.12 Ego Graph ..... 462
6.13 Stochastic ..... 463
6.14 Intersection ..... 463
6.15 Social Networks ..... 465
6.16 Community ..... 466
6.17 Non Isomorphic Trees ..... 471
6.18 Triads ..... 471
6.19 Joint Degree Sequence ..... 472
7 Linear algebra ..... 475
7.1 Graph Matrix ..... 475
7.2 Laplacian Matrix ..... 477
7.3 Spectrum ..... 479
7.4 Algebraic Connectivity ..... 480
7.5 Attribute Matrices ..... 483
8 Converting to and from other data formats ..... 487
8.1 To NetworkX Graph ..... 487
8.2 Dictionaries ..... 488
8.3 Lists ..... 489
8.4 Numpy ..... 490
8.5 Scipy ..... 494
8.6 Pandas ..... 496
9 Reading and writing graphs ..... 501
9.1 Adjacency List ..... 501
9.2 Multiline Adjacency List ..... 504
9.3 Edge List ..... 508
9.4 GEXF ..... 514
9.5 GML ..... 516
9.6 Pickle ..... 520
9.7 GraphML ..... 521
9.8 JSON ..... 523
9.9 LEDA ..... 528
9.10 YAML ..... 529
9.11 SparseGraph6 ..... 532
9.12 Pajek ..... 537
9.13 GIS Shapefile ..... 538
10 Drawing ..... 543
10.1 Matplotlib ..... 543
10.2 Graphviz AGraph (dot) ..... 551
10.3 Graphviz with pydot ..... 553
10.4 Graph Layout ..... 556
11 Exceptions ..... 561
11.1 Exceptions ..... 561
12 Utilities ..... 563
12.1 Helper Functions ..... 563
12.2 Data Structures and Algorithms ..... 564
12.3 Random Sequence Generators ..... 565
12.4 Decorators ..... 567
12.5 Cuthill-Mckee Ordering ..... 568
12.6 Context Managers ..... 570
13 License ..... 571
14 Citing ..... 573
15 Credits ..... 575
15.1 Contributions ..... 575
15.2 Support ..... 577
16 Glossary ..... 579
Bibliography ..... 581
Python Module Index ..... 583

## Overview

NetworkX is a Python language software package for the creation, manipulation, and study of the structure, dynamics, and function of complex networks.
With NetworkX you can load and store networks in standard and nonstandard data formats, generate many types of random and classic networks, analyze network structure, build network models, design new network algorithms, draw networks, and much more.

### 1.1 Who uses NetworkX?

The potential audience for NetworkX includes mathematicians, physicists, biologists, computer scientists, and social scientists. Good reviews of the state-of-the-art in the science of complex networks are presented in Albert and Barabási [BA02], Newman [Newman03], and Dorogovtsev and Mendes [DM03]. See also the classic texts [Bollobas01], [Diestel97] and [West01] for graph theoretic results and terminology. For basic graph algorithms, we recommend the texts of Sedgewick, e.g. [Sedgewick01] and [Sedgewick02] and the survey of Brandes and Erlebach [BE05].

### 1.2 Goals

NetworkX is intended to provide

- tools for the study of the structure and dynamics of social, biological, and infrastructure networks,
- a standard programming interface and graph implementation that is suitable for many applications,
- a rapid development environment for collaborative, multidisciplinary projects,
- an interface to existing numerical algorithms and code written in $\mathrm{C}, \mathrm{C}++$, and FORTRAN,
- the ability to painlessly slurp in large nonstandard data sets.


### 1.3 The Python programming language

Python is a powerful programming language that allows simple and flexible representations of networks, and clear and concise expressions of network algorithms (and other algorithms too). Python has a vibrant and growing ecosystem of packages that NetworkX uses to provide more features such as numerical linear algebra and drawing. In addition Python is also an excellent "glue" language for putting together pieces of software from other languages which allows reuse of legacy code and engineering of high-performance algorithms [Langtangen04].
Equally important, Python is free, well-supported, and a joy to use.

In order to make the most out of NetworkX you will want to know how to write basic programs in Python. Among the many guides to Python, we recommend the documentation at http://www.python.org and the text by Alex Martelli [Martelli03].

### 1.4 Free software

NetworkX is free software; you can redistribute it and/or modify it under the terms of the BSD License. We welcome contributions from the community. Information on NetworkX development is found at the NetworkX Developer Zone at Github https://github.com/networkx/networkx

### 1.5 History

NetworkX was born in May 2002. The original version was designed and written by Aric Hagberg, Dan Schult, and Pieter Swart in 2002 and 2003. The first public release was in April 2005.

Many people have contributed to the success of NetworkX. Some of the contributors are listed in the credits.

### 1.5.1 What Next

- A Brief Tour
- Installing
- Reference
- Examples

The structure of NetworkX can be seen by the organization of its source code. The package provides classes for graph objects, generators to create standard graphs, IO routines for reading in existing datasets, algorithms to analyse the resulting networks and some basic drawing tools.
Most of the NetworkX API is provided by functions which take a graph object as an argument. Methods of the graph object are limited to basic manipulation and reporting. This provides modularity of code and documentation. It also makes it easier for newcomers to learn about the package in stages. The source code for each module is meant to be easy to read and reading this Python code is actually a good way to learn more about network algorithms, but we have put a lot of effort into making the documentation sufficient and friendly. If you have suggestions or questions please contact us by joining the NetworkX Google group.
Classes are named using CamelCase (capital letters at the start of each word). functions, methods and variable names are lower_case_underscore (lowercase with an underscore representing a space between words).

### 2.1 NetworkX Basics

After starting Python, import the networkx module with (the recommended way)

```
>>> import networkx as nx
```

To save repetition, in the documentation we assume that NetworkX has been imported this way.
If importing networkx fails, it means that Python cannot find the installed module. Check your installation and your PYTHONPATH.
The following basic graph types are provided as Python classes:
Graph This class implements an undirected graph. It ignores multiple edges between two nodes. It does allow self-loop edges between a node and itself.

DiGraph Directed graphs, that is, graphs with directed edges. Operations common to directed graphs, (a subclass of Graph).
MultiGraph A flexible graph class that allows multiple undirected edges between pairs of nodes. The additional flexibility leads to some degradation in performance, though usually not significant.
MultiDiGraph A directed version of a MultiGraph.
Empty graph-like objects are created with

```
>>> G=nx.Graph()
>>> G=nx.DiGraph()
```

```
>>> G=nx.MultiGraph()
>>> G=nx.MultiDiGraph()
```

All graph classes allow any hashable object as a node. Hashable objects include strings, tuples, integers, and more. Arbitrary edge attributes such as weights and labels can be associated with an edge.

The graph internal data structures are based on an adjacency list representation and implemented using Python dictionary datastructures. The graph adjaceny structure is implemented as a Python dictionary of dictionaries; the outer dictionary is keyed by nodes to values that are themselves dictionaries keyed by neighboring node to the edge attributes associated with that edge. This "dict-of-dicts" structure allows fast addition, deletion, and lookup of nodes and neighbors in large graphs. The underlying datastructure is accessed directly by methods (the programming interface "API") in the class definitions. All functions, on the other hand, manipulate graph-like objects solely via those API methods and not by acting directly on the datastructure. This design allows for possible replacement of the 'dicts-of-dicts'-based datastructure with an alternative datastructure that implements the same methods.

### 2.1.1 Graphs

The first choice to be made when using NetworkX is what type of graph object to use. A graph (network) is a collection of nodes together with a collection of edges that are pairs of nodes. Attributes are often associated with nodes and/or edges. NetworkX graph objects come in different flavors depending on two main properties of the network:

- Directed: Are the edges directed? Does the order of the edge pairs (u,v) matter? A directed graph is specified by the "Di" prefix in the class name, e.g. DiGraph(). We make this distinction because many classical graph properties are defined differently for directed graphs.
- Multi-edges: Are multiple edges allowed between each pair of nodes? As you might imagine, multiple edges requires a different data structure, though tricky users could design edge data objects to support this functionality. We provide a standard data structure and interface for this type of graph using the prefix "Multi", e.g. MultiGraph().

The basic graph classes are named: Graph, DiGraph, MultiGraph, and MultiDiGraph

### 2.2 Nodes and Edges

The next choice you have to make when specifying a graph is what kinds of nodes and edges to use.
If the topology of the network is all you care about then using integers or strings as the nodes makes sense and you need not worry about edge data. If you have a data structure already in place to describe nodes you can simply use that structure as your nodes provided it is hashable. If it is not hashable you can use a unique identifier to represent the node and assign the data as a node attribute.

Edges often have data associated with them. Arbitrary data can associated with edges as an edge attribute. If the data is numeric and the intent is to represent a weighted graph then use the 'weight' keyword for the attribute. Some of the graph algorithms, such as Dijkstra's shortest path algorithm, use this attribute name to get the weight for each edge.

Other attributes can be assigned to an edge by using keyword/value pairs when adding edges. You can use any keyword except 'weight' to name your attribute and can then easily query the edge data by that attribute keyword.

Once you've decided how to encode the nodes and edges, and whether you have an undirected/directed graph with or without multiedges you are ready to build your network.

### 2.2.1 Graph Creation

NetworkX graph objects can be created in one of three ways:

- Graph generators - standard algorithms to create network topologies.
- Importing data from pre-existing (usually file) sources.
- Adding edges and nodes explicitly.

Explicit addition and removal of nodes/edges is the easiest to describe. Each graph object supplies methods to manipulate the graph. For example,

```
>>> import networkx as nx
>>>G=nx.Graph()
>>> G.add_edge(1,2) # default edge data=1
>>> G.add__edge(2,3,weight=0.9) # specify edge data
```

Edge attributes can be anything:

```
>>> import math
>>> G.add_edge('Y','x', function=math.cos)
>> G.add_node(math.cos) # any hashable can be a node
```

You can add many edges at one time:

```
>>> elist=[('a','b',5.0),('b','c',3.0),('a','c',1.0),('c','d',7.3)]
>>> G.add_weighted_edges_from(elist)
```

See the /tutorial/index for more examples.
Some basic graph operations such as union and intersection are described in the Operators module documentation.
Graph generators such as binomial_graph and powerlaw_graph are provided in the Graph generators subpackage.
For importing network data from formats such as GML, GraphML, edge list text files see the Reading and writing graphs subpackage.

### 2.2.2 Graph Reporting

Class methods are used for the basic reporting functions neighbors, edges and degree. Reporting of lists is often needed only to iterate through that list so we supply iterator versions of many property reporting methods. For example edges() and nodes() have corresponding methods edges_iter() and nodes_iter(). Using these methods when you can will save memory and often time as well.

The basic graph relationship of an edge can be obtained in two basic ways. One can look for neighbors of a node or one can look for edges incident to a node. We jokingly refer to people who focus on nodes/neighbors as node-centric and people who focus on edges as edge-centric. The designers of NetworkX tend to be node-centric and view edges as a relationship between nodes. You can see this by our avoidance of notation like $\mathrm{G}[\mathrm{u}, \mathrm{v}]$ in favor of $\mathrm{G}[\mathrm{u}][\mathrm{v}]$. Most data structures for sparse graphs are essentially adjacency lists and so fit this perspective. In the end, of course, it doesn't really matter which way you examine the graph. G.edges() removes duplicate representations of each edge while G.neighbors( n ) or $\mathrm{G}[\mathrm{n}]$ is slightly faster but doesn't remove duplicates.

Any properties that are more complicated than edges, neighbors and degree are provided by functions. For example nx .triangles( $\mathrm{G}, \mathrm{n}$ ) gives the number of triangles which include node n as a vertex. These functions are grouped in the code and documentation under the term algorithms.

### 2.2.3 Algorithms

A number of graph algorithms are provided with NetworkX. These include shortest path, and breadth first search (see traversal), clustering and isomorphism algorithms and others. There are many that we have not developed yet too. If
you implement a graph algorithm that might be useful for others please let us know through the NetworkX Google group or the Github Developer Zone.

As an example here is code to use Dijkstra's algorithm to find the shortest weighted path:

```
>>> G=nx.Graph()
>>> e=[('a','b',0.3),('b','c',0.9),('a','c',0.5),('c','d',1.2)]
>>> G.add_weighted_edges_from(e)
>>> print(nx.dijkstra_path(G,'a','d'))
['a', 'c', 'd']
```


### 2.2.4 Drawing

While NetworkX is not designed as a network layout tool, we provide a simple interface to drawing packages and some simple layout algorithms. We interface to the excellent Graphviz layout tools like dot and neato with the (suggested) pygraphviz package or the pydot interface. Drawing can be done using external programs or the Matplotlib Python package. Interactive GUI interfaces are possible though not provided. The drawing tools are provided in the module drawing.
The basic drawing functions essentially place the nodes on a scatterplot using the positions in a dictionary or computed with a layout function. The edges are then lines between those dots.

```
>>> G=nx.cubical_graph()
>>> nx.draw(G) # default spring_layout
>>> nx.draw(G,pos=nx.spectral_layout(G), nodecolor='r',edge_color='b')
```

See the examples for more ideas.

### 2.2.5 Data Structure

NetworkX uses a "dictionary of dictionaries of dictionaries" as the basic network data structure. This allows fast lookup with reasonable storage for large sparse networks. The keys are nodes so $\mathrm{G}[\mathrm{u}]$ returns an adjacency dictionary keyed by neighbor to the edge attribute dictionary. The expression $G[u][v]$ returns the edge attribute dictionary itself. A dictionary of lists would have also been possible, but not allowed fast edge detection nor convenient storage of edge data.

Advantages of dict-of-dicts-of-dicts data structure:

- Find edges and remove edges with two dictionary look-ups.
- Prefer to "lists" because of fast lookup with sparse storage.
- Prefer to "sets" since data can be attached to edge.
- $G[u][v]$ returns the edge attribute dictionary.
- n in G tests if node n is in graph G .
- for n in G : iterates through the graph.
- for nbr in $G[n]$ : iterates through neighbors.

As an example, here is a representation of an undirected graph with the edges (' A ',' B '), (' B ',' C ')

```
>>> G=nx.Graph ()
>>> G.add__edge('A','B')
>>> G.add__edge('B','C')
>>> print(G.adj)
{'A': {'B': {}}, 'C': {'B': {}}, '贾': {'A': {}, 'C': {}}}
```

The data structure gets morphed slightly for each base graph class. For DiGraph two dict-of-dicts-of-dicts structures are provided, one for successors and one for predecessors. For MultiGraph/MultiDiGraph we use a dict-of-dicts-of-dicts-of-dicts ${ }^{1}$ where the third dictionary is keyed by an edge key identifier to the fourth dictionary which contains the edge attributes for that edge between the two nodes.
Graphs use a dictionary of attributes for each edge. We use a dict-of-dicts-of-dicts data structure with the inner dictionary storing "name-value" relationships for that edge.

```
>>> G=nx.Graph()
>>> G.add_edge(1,2,color='red',weight=0. 84, size=300)
>>> print(G[1][2]['size'])
300
```

[^0]
## Graph types

NetworkX provides data structures and methods for storing graphs.
All NetworkX graph classes allow (hashable) Python objects as nodes. and any Python object can be assigned as an edge attribute.
The choice of graph class depends on the structure of the graph you want to represent.

### 3.1 Which graph class should I use?

| Graph Type | NetworkX Class |
| :--- | :--- |
| Undirected Simple | Graph |
| Directed Simple | DiGraph |
| With Self-loops | Graph, DiGraph |
| With Parallel edges | MultiGraph, MultiDiGraph |

### 3.2 Basic graph types

### 3.2.1 Graph - Undirected graphs with self loops

## Overview

Graph (data=None, **attr)
Base class for undirected graphs.
A Graph stores nodes and edges with optional data, or attributes.
Graphs hold undirected edges. Self loops are allowed but multiple (parallel) edges are not.
Nodes can be arbitrary (hashable) Python objects with optional key/value attributes.
Edges are represented as links between nodes with optional key/value attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.
- $\operatorname{attr}$ (keyword arguments, optional (default= no attributes)) - Attributes to add to graph as key=value pairs.


## See also:

DiGraph(), MultiGraph(), MultiDiGraph()

## Examples

Create an empty graph structure (a "null graph") with no nodes and no edges.
>>> G $=$ nx.Graph ()
G can be grown in several ways.

## Nodes:

Add one node at a time:

```
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```
>>> G.add_nodes_from([2,3])
>>> G.add_nodes_from(range (100,110))
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```
>>> G.add_node(H)
```


## Edges:

G can also be grown by adding edges.
Add one edge,

```
>>> G.add_edge(1, 2)
```

a list of edges,

```
>>> G.add_edges_from([(1,2),(1,3)])
```

or a collection of edges,

```
>>> G.add_edges_from(H.edges())
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. There are no errors when adding nodes or edges that already exist.

## Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

```
>>> G = nx.Graph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Add node attributes using add_node(), add_nodes_from() or G.node

```
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.node[1]
{'time': '5pm'}
>>> G.node[1]['room'] = 714
>>> del G.node[1]['room'] # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Warning: adding a node to G.node does not add it to the graph.
Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edge.

```
>>> G.add_edge(1, 2, weight=4.7 )
>>> G.add_edges_from([(3,4),(4,5)], color='red')
>>> G.add_edges_from([(1,2,{'color':'blue'}), (2,3,{'weight':8})])
>>> G[1][2]['weight'] = 4.7
>>> G.edge[1][2]['weight'] = 4
```


## Shortcuts:

Many common graph features allow python syntax to speed reporting.

```
>>> 1 in G # check if node in graph
True
>>> [n for n in G if n<3] # iterate through nodes
[1, 2]
>>> len(G) # number of nodes in graph
5
```

The fastest way to traverse all edges of a graph is via adjacency(), but the edges() method is often more convenient.

```
>>> for n,nbrsdict in G.adjacency():
... for nbr,eattr in nbrsdict.items():
... if 'weight' in eattr:
... (n,nbr,eattr['weight'])
(1, 2, 4)
(2, 1, 4)
(2, 3, 8)
(3, 2, 8)
>>> list(G.edges(data='weight'))
[(1, 2, 4), (2, 3, 8), (3, 4, None), (4, 5, None)]
```


## Reporting:

Simple graph information is obtained using methods. Reporting methods usually return iterators instead of containers to reduce memory usage. Methods exist for reporting nodes(), edges(), neighbors() and degree() as well as the number of nodes and edges.

For details on these and other miscellaneous methods, see below.

## Subclasses (Advanced):

The Graph class uses a dict-of-dict-of-dict data structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge data keyed by neighbor. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these three dicts can be replaced in a subclass by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, adjlist_inner_dict_factory, adjlist_outer_dict_factory, and edge_attr_dict_factory.
node_dict_factory [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object
adjlist_outer_dict_factory [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
adjlist_inner_dict_factory [function, (default: dict)] Factory function to be used to create the adjacency list dict which holds edge data keyed by neighbor. It should require no arguments and return a dict-like object
edge_attr_dict_factory [function, (default: dict)] Factory function to be used to create the edge attribute dict which holds attrbute values keyed by attribute name. It should require no arguments and return a dict-like object.

## Examples

Create a graph subclass that tracks the order nodes are added.

```
>>> from collections import OrderedDict
>>> class OrderedNodeGraph(nx.Graph):
... node_dict_factory=OrderedDict
... adjlist_outer_dict_factory=OrderedDict
>>> G=OrderedNodeGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> G.add_edges_from( ((2,2), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 1), (2, 2), (1, 1)]
```

Create a graph object that tracks the order nodes are added and for each node track the order that neighbors are added.

```
>>> class OrderedGraph(nx.Graph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory = OrderedDict
... adjlist_inner_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> G.add_edges_from( ( (2,2), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 2), (2, 1), (1, 1)]
```

Create a low memory graph class that effectively disallows edge attributes by using a single attribute dict for all edges. This reduces the memory used, but you lose edge attributes.

```
>>> class ThinGraph(nx.Graph):
... all_edge_dict = {'weight': 1}
... def single_edge_dict(self):
... return self.all_edge_dict
... edge_attr_dict_factory = single_edge_dict
>>> G = ThinGraph()
>>> G.add_edge (2,1)
>>> list(G.edges(data= True))
[(1, 2, {'weight': 1})]
>>> G.add_edge (2, 2)
>>> G[2][1] is G[2][2]
True
```


### 3.2.2 Methods

## Adding and removing nodes and edges

| Graph.__init___([data]) | Initialize a graph with edges, name, graph attributes. |
| :---: | :---: |
| Graph.add_node(n, \*\*attr) | Add a single node n and update node attributes. |
| Graph.add_nodes_from(nodes, \*\*attr) | Add multiple nodes. |
| Graph.remove_node(n) | Remove node n . |
| Graph.remove_nodes_from(nodes) | Remove multiple nodes. |
| Graph.add_edge(u, v, \*\|*attr) | Add an edge between $u$ and $v$. |
| Graph.add_edges_from(ebunch, ${ }^{*}$ **attr) | Add all the edges in ebunch. |
| Graph.add_weighted_edges_from(ebunch[, weight]) | Add all the edges in ebunch as weighted edges with specified weights. |
| Graph.remove_edge(u, v) | Remove the edge between u and v . |
| Graph.remove_edges_from(ebunch) | Remove all edges specified in ebunch. |
| Graph.clear() | Remove all nodes and edges from the graph. |

$\qquad$
init

Graph.__init__(data=None, **attr)
Initialize a graph with edges, name, graph attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.
- name (string, optional (default=' ')) - An optional name for the graph.
- $\operatorname{attr}$ (keyword arguments, optional (default= no attributes)) - Attributes to add to graph as key=value pairs.


## See also:

convert()

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
>>> e = [(1,2),(2,3),(3,4)] # list of edges
>>> G = nx.Graph(e)
```

Arbitrary graph attribute pairs (key=value) may be assigned

```
>>> G=nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}
```

add_node

Graph.add_node ( $n$, **attr)
Add a single node n and update node attributes.

## Parameters

- n (node) - A node can be any hashable Python object except None.
- $\operatorname{attr}$ (keyword arguments, optional) - Set or change node attributes using key=value.

See also:

```
add_nodes_from()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([ (0,1), (1,2), (2,0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:

```
>>> G.add_node(1,size=10)
>>> G.add_node(3,weight=0.4,UTM=('13S',382871,3972649))
```


## Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn't change on mutables.

```
add_nodes_from
```

Graph.add_nodes_from (nodes, **attr)

Add multiple nodes.

## Parameters

- nodes (iterable container) - A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- attr (keyword arguments, optional (default= no attributes)) - Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.


## See also:

add_node()

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0,1),(1,2), (2,0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(),key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```
>>> G.add_nodes_from([1,2], size=10)
>>> G.add_nodes_from([3,4], weight=0.4)
```

Use (node, attrdict) tuples to update attributes for specific nodes.

```
>>> G.add_nodes_from([(1,dict(size=11)), (2,{'color':'blue'})])
>>> G.node[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.node[1]['size']
11
```

remove_node

Graph. remove_node ( $n$ )
Remove node n .
Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.
Parameters $\mathbf{n}$ (node) - A node in the graph
Raises NetworkXerror - If n is not in the graph.

## See also:

```
remove_nodes_from()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges())
[(0, 1), (1, 2)]
>>> G.remove_node(1)
>>> list(G.edges())
[ ]
```

remove_nodes_from

Graph.remove_nodes_from (nodes)
Remove multiple nodes.
Parameters nodes (iterable container) - A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

## See also:

```
remove node()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes())
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes())
[ ]
```

```
add_edge
```

Graph.add_edge ( $u, v$, **attr)

Add an edge between $u$ and $v$.
The nodes $u$ and $v$ will be automatically added if they are not already in the graph.
Edge attributes can be specified with keywords or by directly accessing the edge's attribute dictionary. See examples below.

## Parameters

- u, v (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## See also:

add_edges_from () add a collection of edges

## Notes

Adding an edge that already exists updates the edge data.
Many NetworkX algorithms designed for weighted graphs use as the edge weight a numerical value assigned to a keyword which by default is 'weight'.

## Examples

The following all add the edge $\mathrm{e}=(1,2)$ to graph G :

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = (1,2)
>>> G.add_edge(1, 2) # explicit two-node form
>>> G.add_edge(*e) # single edge as tuple of two nodes
>>> G.add_edges_from([(1, 2)]) # add edges from iterable container
```

Associate data to edges using keywords:

```
>>> G.add__edge(1, 2, weight=3)
>>> G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string associations, directly access the edge's attribute dictionary.

```
>>> G.add_edge(1, 2)
>>> G[1][2].update({0: 5})
```

add_edges_from

Graph.add_edges_from (ebunch, **attr)
Add all the edges in ebunch.

## Parameters

- ebunch (container of edges) - Each edge given in the container will be added to the graph. The edges must be given as as 2-tuples ( $u, v$ ) or 3-tuples ( $u, v, d$ ) where $d$ is a dictionary containing edge data.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## See also:

add_edge () add a single edge
add_weighted_edges_from () convenient way to add weighted edges

## Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added.
Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0,1),(1,2)]) # using a list of edge tuples
>>> e = zip(range(0,3),range(1,4))
>>> G.add_edges_from(e) # Add the path graph 0-1-2-3
```

Associate data to edges

```
>>> G.add_edges_from([(1,2), (2,3)], weight=3)
>>> G.add_edges_from([(3,4),(1,4)], label='WN2898')
```

add_weighted_edges_from

Graph.add_weighted_edges_from (ebunch, weight='weight', **attr)
Add all the edges in ebunch as weighted edges with specified weights.

## Parameters

- ebunch (container of edges) - Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples ( $u, v, w$ ) where $w$ is a number.
- weight (string, optional (default= 'weight')) - The attribute name for the edge weights to be added.
- attr (keyword arguments, optional (default= no attributes)) - Edge attributes to add/update for all edges.

See also:
add_edge () add a single edge
add_edges_from() add multiple edges

## Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0,1,3.0), (1, 2, 7.5)])
```


## remove_edge

## Graph.remove_edge ( $u, v$ )

Remove the edge between $u$ and $v$.
Parameters $\mathbf{u}, \mathbf{v}$ (nodes) - Remove the edge between nodes $u$ and $v$.
Raises NetworkXError - If there is not an edge between $u$ and $v$.

## See also:

```
remove_edges_from() remove a collection of edges
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, etc
>>> G.remove_edge (0,1)
>>> e = (1,2)
>>> G.remove_edge(*e) # unpacks e from an edge tuple
>>> e = (2,3,{'weight':7}) # an edge with attribute data
>>> G.remove_edge(*e[:2]) # select first part of edge tuple
```

remove_edges_from

Graph.remove_edges_from (ebunch)
Remove all edges specified in ebunch.
Parameters ebunch (list or container of edge tuples) - Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples ( $u, v$ ) edge between $u$ and $v$.
- 3-tuples $(\mathrm{u}, \mathrm{v}, \mathrm{k})$ where k is ignored.

See also:
remove_edge () remove a single edge

## Notes

Will fail silently if an edge in ebunch is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1,2), (2,3)]
>>> G.remove_edges_from(ebunch)
```

clear

Graph.clear()
Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes())
[]
>>> list(G.edges())
[]
```


## Iterating over nodes and edges

| Graph.nodes([data, default]) | Returns an iterator over the nodes. |
| :---: | :---: |
| Graph._-iter_() | Iterate over the nodes. |
| Graph.edges([nbunch, data, default]) | Return an iterator over the edges. |
| Graph.get_edge_data(u, v[, default]) | Return the attribute dictionary associated with edge (u,v). |
| Graph.neighbors(n) | Return an iterator over all neighbors of node n . |
| Graph.__getitem__( n ) | Return a dict of neighbors of node n . |
| Graph.adjacency() | Return an iterator over (node, adjacency dict) tuples for all nodes. |
| Graph.nbunch_iter([nbunch]) | Return an iterator over nodes contained in nbunch that are also in the graph. |

nodes

Graph. nodes (data=False, default=None)
Returns an iterator over the nodes.

## Parameters

- data (string or bool, optional (default=False)) - The node attribute returned in 2-tuple
 the nodes $n$.
- default (value, optional (default=None)) - Value used for nodes that dont have the requested attribute. Only relevant if data is not True or False.

Returns An iterator over nodes, or ( $\mathrm{n}, \mathrm{d}$ ) tuples of node with data. If data is False, an iterator over nodes. Otherwise an iterator of 2 -tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

Return type iterator

## Notes

If the node data is not required, it is simpler and equivalent to use the expression for $n$ in $G$, or list ( $G$ ).

## Examples

There are two simple ways of getting a list of all nodes in the graph:

```
>>> G = nx.path_graph(3)
>>> list(G.nodes())
[0, 1, 2]
```

```
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```
>>> G.add_node(1, time='5pm')
>>> G.node[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```
>>> G = nx.Graph()
>>> G.add_node (0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

$\qquad$

Graph.__iter_()
Iterate over the nodes. Use the expression 'for n in G '.
Returns niter-An iterator over all nodes in the graph.
Return type iterator

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for }n\mathrm{ in G]
[0, 1, 2, 3]
```

edges

Graph.edges (nbunch=None, data=False, default=None)
Return an iterator over the edges.
Edges are returned as tuples with optional data in the order (node, neighbor, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns edges - An iterator over ( $u, v$ ) or ( $u, v, d$ ) tuples of edges.
Return type iterator

## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.path_graph(3) # or MultiGraph, etc
>>> G.add_edge (2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges([0,3]))
[(0, 1), (3, 2)]
>>> list(G.edges(0))
[(0, 1)]
```

get_edge_data

Graph.get_edge_data ( $u, v$, default=None)
Return the attribute dictionary associated with edge (u,v).

## Parameters

- u, $\mathbf{v}$ (nodes)
- default (any Python object (default=None)) - Value to return if the edge (u,v) is not found.

Returns edge_dict - The edge attribute dictionary.
Return type dictionary

## Notes

It is faster to use $\mathrm{G}[\mathrm{u}][\mathrm{v}]$.

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0][1]
{ }
```

Warning: Assigning $G[u][v]$ corrupts the graph data structure. But it is safe to assign attributes to that dictionary,

```
>>> G[0][1]['weight'] = 7
>>> G[0][1]['weight']
7
>>> G[1][0]['weight']
7
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.get_edge_data(0, 1) # default edge data is {}
{ }
>>> e = (0,1)
>>> G.get_edge_data(*e) # tuple form
{ }
>>> G.get_edge_data('a','b',default=0) # edge not in graph, return 0
0
```


## neighbors

Graph.neighbors ( $n$ )
Return an iterator over all neighbors of node $n$.
Parameters $\mathbf{n}$ (node) - A node in the graph
Returns neighbors - An iterator over all neighbors of node n
Return type iterator
Raises NetworkXError - If the node n is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G.neighbors(0)]
[1]
```


## Notes

It is usually more convenient (and faster) to access the adjacency dictionary as $G[n]$ :

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=7)
>>> G['a']
{'b': {'weight': 7} }
>>> G = nx.path_graph(4)
>>> [n for n in G[0]]
[1]
```

__getitem
Graph.__getitem_( $n$ )
Return a dict of neighbors of node n . Use the expression ' $\mathrm{G}[\mathrm{n}]$ '.
Parameters $\mathbf{n}$ (node) - A node in the graph.
Returns adj_dict - The adjacency dictionary for nodes connected to n .
Return type dictionary

## Notes

$\mathrm{G}[\mathrm{n}]$ is similar to G. neighbors( n ) but the internal data dictionary is returned instead of an iterator.
Assigning $\mathrm{G}[\mathrm{n}]$ will corrupt the internal graph data structure. Use $\mathrm{G}[\mathrm{n}]$ for reading data only.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
{1: {}}
```


## adjacency

## Graph.adjacency ()

Return an iterator over (node, adjacency dict) tuples for all nodes.
This is the fastest way to look at every edge. For directed graphs, only outgoing adjacencies are included.
Returns adj_iter - An iterator over (node, adjacency dictionary) for all nodes in the graph.
Return type iterator

Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n,nbrdict) for n, nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

nbunch_iter

Graph.nbunch_iter (nbunch=None)
Return an iterator over nodes contained in nbunch that are also in the graph.
The nodes in nbunch are checked for membership in the graph and if not are silently ignored.
Parameters nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
Returns niter - An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.

## Return type iterator

Raises NetworkXError - If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

## See also:

Graph.__iter__()

## Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use "if nbunch in self:", even after processing with this routine.
If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.

## Information about graph structure

| Graph.has_node(n) | Return True if the graph contains the node n. |
| :--- | :--- |
| Graph.__contains_(n) | Return True if n is a node, False otherwise. |
| Graph.has_edge(u, v) | Return True if the edge (u,v) is in the graph. |
| Graph.order() | Return the number of nodes in the graph. |
| Graph.number_of_nodes () | Return the number of nodes in the graph. |
| Graph.__len_( $)$ | Return the number of nodes. |
| Graph.degree([nbunch, weight]) | Return an iterator for (node, degree) or degree for single <br> node. |
| Graph.size([weight]) | Return the number of edges or total of all edge weights. |
| Graph. number_of_edges([u, v]) | Return the number of edges between two nodes. |
| Graph.nodes_with_selfloops() | Returns an iterator over nodes with self loops. |
| Graph.selfloop_edges([data, default]) | Returns an iterator over selfloop edges. |
| Graph.number_of_selfloops () | Return the number of selfloop edges. |

has_node

Graph.has_node ( $n$ )
Return True if the graph contains the node $n$.

## Parameters n (node)

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```
>>> 0 in G
True
```

contains

Graph.__contains__( $n$ )
Return True if n is a node, False otherwise. Use the expression ' n in G '.

Examples
>>> G = nx.path_graph(4) \# or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in $G$
True
has_edge

Graph.has_edge ( $u, v$ )
Return True if the edge ( $u, v$ ) is in the graph.
Parameters u, $\mathbf{v}$ (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

Returns edge_ind - True if edge is in the graph, False otherwise.
Return type bool

## Examples

Can be called either using two nodes $u, v$ or edge tuple ( $u, v$ )

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_edge(0,1) # using two nodes
True
>>> e = (0,1)
>>> G.has_edge(*e) # e is a 2-tuple (u,v)
True
>>> e = (0,1,{'weight':7})
>>> G.has_edge(*e[:2]) # e is a 3-tuple (u,v,data_dictionary)
True
```

The following syntax are all equivalent:

```
>>> G.has_edge(0,1)
True
>>> 1 in G[0] # though this gives KeyError if 0 not in G
True
```

order

Graph.order()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int

## See also:

```
number_of_nodes(),___len___()
```

number_of_nodes

Graph.number_of_nodes ()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
order(),__len_()

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
3
```

_len

Graph.__len ()

Return the number of nodes. Use the expression 'len(G)'.
Returns nnodes - The number of nodes in the graph.
Return type int

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

degree

Graph.degree (nbunch=None, weight=None)
Return an iterator for (node, degree) or degree for single node.
The node degree is the number of edges adjacent to the node. This function returns the degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - Degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, degree).


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.degree(0) # node 0 with degree I
1
>>> list(G.degree([0,1]))
[(0, 1), (1, 2)]
```

size

Graph.size (weight=None)
Return the number of edges or total of all edge weights.
Parameters weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

## Returns

size - The number of edges or (if weight keyword is provided) the total weight sum.
If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).

Return type numeric
See also:

```
number_of_edges()
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3
```

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a','b',weight=2)
>>> G.add_edge('b','c',weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```


## number_of_edges

Graph.number_of_edges ( $u=$ None, $v=$ None)
Return the number of edges between two nodes.
Parameters u, v (nodes, optional (default=all edges)) - If $u$ and $v$ are specified, return the number of edges between $u$ and $v$. Otherwise return the total number of all edges.

Returns nedges - The number of edges in the graph. If nodes $u$ and $v$ are specified return the number of edges between those nodes.

Return type int

## See also:

size()

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_edges()
3
>>> G.number_of_edges (0,1)
1
>>> e = (0,1)
>>> G.number_of_edges(*e)
1
```


## nodes_with_selfloops

```
Graph.nodes_with_selfloops()
```

Returns an iterator over nodes with self loops.
A node with a self loop has an edge with both ends adjacent to that node.
Returns nodelist - A iterator over nodes with self loops.
Return type iterator
See also:

```
selfloop_edges(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge(1, 1)
>>> G.add_edge(1, 2)
>>> list(G.nodes_with_selfloops())
[1]
```


## selfloop_edges

Graph.selfloop_edges (data=False, default=None)
Returns an iterator over selfloop edges.
A selfloop edge has the same node at both ends.

## Parameters

- data (string or bool, optional (default=False)) - Return selfloop edges as two tuples (u,v) (data=False) or three-tuples (u,v,datadict) (data=True) or three-tuples (u,v,datavalue) (data='attrname')
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns edgeiter - An iterator over all selfloop edges.
Return type iterator over edge tuples

## See also:

```
nodes_with_selfloops(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1,1)
>>> G.add_edge (1,2)
>>> list(G.selfloop_edges())
[(1, 1)]
>>> list(G.selfloop_edges(data=True))
[(1, 1, {})]
```

number_of_selfloops

Graph.number_of_selfloops()
Return the number of selfloop edges.
A selfloop edge has the same node at both ends.
Returns nloops - The number of selfloops.

## Return type int

See also:
nodes_with_selfloops(), selfloop_edges()

## Examples

```
>>> G=nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add__edge (1,1)
>>> G.add_edge (1,2)
>>> G.number_of_selfloops()
1
```


## Making copies and subgraphs

| Graph.copy([with_data]) | Return a copy of the graph. |
| :--- | :--- |
| Graph.to_undirected() | Return an undirected copy of the graph. |
| Graph.to_directed() | Return a directed representation of the graph. |
| Graph.subgraph(nbunch) | Return the subgraph induced on nodes in nbunch. |
| Graph.edge_subgraph(edges) | Returns the subgraph induced by the specified edges. |

## copy

Graph.copy (with_data=True)
Return a copy of the graph.
All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy - The default behavior is a "deepcopy" where the graph structure as well as all data attributes and any objects they might contain are copied. The entire graph object is new so that changes in the copy do not affect the original object.

Data Reference (Shallow) - For a shallow copy (with_data=False) the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other.
Independent Shallow - This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This type of copy is not enabled. Instead use:

```
>>> G = nx.path_graph(5)
>>>H = G.__class__(G)
```

Fresh Data- For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges())
```

See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Parameters with_data (bool, optional (default=True)) - If True, the returned graph will have a deep copy of the graph, node, and edge attributes of this object. Otherwise, the returned graph will be a shallow copy.

Returns G-A copy of the graph.
Return type Graph

## See also:

to_directed () return a directed copy of the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```


## to_undirected

## Graph.to_undirected()

Return an undirected copy of the graph.
Returns G-A deepcopy of the graph.
Return type Graph/MultiGraph

## See also:

copy (), add_edge(), add_edges_from()

## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{G}=\mathrm{DiGraph}(\mathrm{D})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

## Examples

```
>>> G = nx.path_graph(2) # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges())
[(0, 1)]
```


## to_directed

## Graph.to_directed()

Return a directed representation of the graph.
Returns G - A directed graph with the same name, same nodes, and with each edge (u,v,data) replaced by two directed edges (u,v,data) and (v,u,data).

## Return type DiGraph

## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{D}=\mathrm{DiGraph}(\mathrm{G})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Warning: If you have subclassed Graph to use dict-like objects in the data structure, those changes do not transfer to the DiGraph created by this method.

## Examples

```
>>> G = nx.Graph() # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```
>>> G = nx.DiGraph() # or MultiDiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1)]
```


## subgraph

## Graph.subgraph (nbunch)

Return the subgraph induced on nodes in nbunch.
The induced subgraph of the graph contains the nodes in nbunch and the edges between those nodes.
Parameters nbunch (list, iterable) - A container of nodes which will be iterated through once.
Returns $\mathbf{G}$ - A subgraph of the graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge or node attributes just point to the original graph. So changes to the node or edge structure will not be reflected in the original graph while changes to the attributes will.

To create a subgraph with its own copy of the edge/node attributes use: nx.Graph(G.subgraph(nbunch))
If edge attributes are containers, a deep copy can be obtained using: G.subgraph(nbunch).copy()
For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from([ n in G if n not in set(nbunch)])

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0,1,2])
>>> list(H.edges())
[(0, 1), (1, 2)]
```


## edge_subgraph

Graph.edge_subgraph (edges)
Returns the subgraph induced by the specified edges.
The induced subgraph contains each edge in edges and each node incident to any one of those edges.
Parameters edges (iterable) - An iterable of edges in this graph.
Returns $\mathbf{G}$ - An edge-induced subgraph of this graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge, and node attributes in the returned subgraph are references to the corresponding attributes in the original graph. Thus changes to the node or edge structure of the returned graph will not be reflected in the original graph, but changes to the attributes will.

To create a subgraph with its own copy of the edge or node attributes, use:

```
>>> nx.Graph(G.edge_subgraph(edges))
```

If edge attributes are containers, a deep copy of the attributes can be obtained using:

```
>>> G.edge_subgraph(edges) .copy()
```


## Examples

```
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes())
[0, 1, 3, 4]
>>> list(H.edges())
[(0, 1), (3, 4)]
```


### 3.2.3 DiGraph - Directed graphs with self loops

## Overview

DiGraph (data=None, **attr)
Base class for directed graphs.
A DiGraph stores nodes and edges with optional data, or attributes.
DiGraphs hold directed edges. Self loops are allowed but multiple (parallel) edges are not.
Nodes can be arbitrary (hashable) Python objects with optional key/value attributes.
Edges are represented as links between nodes with optional key/value attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function,
currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.
- attr (keyword arguments, optional (default= no attributes)) - Attributes to add to graph as key=value pairs.


## See also:

Graph(), MuItiGraph(), MuItiDiGraph()

## Examples

Create an empty graph structure (a "null graph") with no nodes and no edges.

```
>>> G = nx.DiGraph()
```

G can be grown in several ways.

## Nodes:

Add one node at a time:

```
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```
>>> G.add_nodes_from([2,3])
>>> G.add_nodes_from(range (100,110))
>>> H=nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.
>>> G.add_node (H)

## Edges:

G can also be grown by adding edges.
Add one edge,

```
>>> G.add_edge(1, 2)
```

a list of edges,

```
>>> G.add_edges_from([(1,2),(1,3)])
```

or a collection of edges,

```
>>> G.add_edges_from(H.edges())
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. There are no errors when adding nodes or edges that already exist.

## Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

```
>>> G = nx.DiGraph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Add node attributes using add_node(), add_nodes_from() or G.node

```
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.node[1]
{'time': '5pm'}
>>> G.node[1]['room'] = 714
>>> del G.node[1]['room'] # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Warning: adding a node to G.node does not add it to the graph.
Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edge.

```
>>> G.add_edge(1, 2, weight=4.7 )
>>> G.add_edges_from([(3,4),(4,5)], color='red')
>>> G.add_edges_from([(1,2,{'color':'blue'}), (2,3,{'weight':8})])
>>> G[1][2]['weight'] = 4.7
>>> G.edge[1][2]['weight'] = 4
```


## Shortcuts:

Many common graph features allow python syntax to speed reporting.

```
>>> 1 in G # check if node in graph
True
>>> [n for n in G if n<3] # iterate through nodes
[1, 2]
>>> len(G) # number of nodes in graph
5
```

The fastest way to traverse all edges of a graph is via adjacency(), but the edges() method is often more convenient.

```
>>> for n,nbrsdict in G.adjacency():
... for nbr,eattr in nbrsdict.items():
... if 'weight' in eattr:
... (n,nbr,eattr['weight'])
(1, 2, 4)
(2, 3, 8)
>>> list(G.edges(data='weight'))
[(1, 2, 4), (2, 3, 8), (3, 4, None), (4, 5, None)]
```


## Reporting:

Simple graph information is obtained using methods. Reporting methods usually return iterators instead of containers to reduce memory usage. Methods exist for reporting nodes(), edges(), neighbors() and degree() as well as the number of nodes and edges.
For details on these and other miscellaneous methods, see below.

## Subclasses (Advanced):

The Graph class uses a dict-of-dict-of-dict data structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge data keyed by
neighbor. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these three dicts can be replaced in a subclass by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, adjlist_inner_dict_factory, adjlist_outer_dict_factory, and edge_attr_dict_factory.
node_dict_factory [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object
adjlist_outer_dict_factory [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
adjlist_inner_dict_factory [function, optional (default: dict)] Factory function to be used to create the adjacency list dict which holds edge data keyed by neighbor. It should require no arguments and return a dict-like object
edge_attr_dict_factory [function, optional (default: dict)] Factory function to be used to create the edge attribute dict which holds attrbute values keyed by attribute name. It should require no arguments and return a dict-like object.

## Examples

Create a graph subclass that tracks the order nodes are added.

```
>>> from collections import OrderedDict
>>> class OrderedNodeGraph(nx.Graph):
... node_dict_factory=OrderedDict
... adjlist_outer_dict_factory=OrderedDict
>>> G=OrderedNodeGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> G.add_edges_from( ( (2,2), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 1), (2, 2), (1, 1)]
```

Create a graph object that tracks the order nodes are added and for each node track the order that neighbors are added.

```
>>> class OrderedGraph(nx.Graph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory=OrderedDict
... adjlist_inner_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> G.add_edges_from( ( (2,2), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 2), (2, 1), (1, 1)]
```

Create a low memory graph class that effectively disallows edge attributes by using a single attribute dict for all edges. This reduces the memory used, but you lose edge attributes.

```
>>> class ThinGraph(nx.Graph):
... all_edge_dict = {'weight': 1}
... def single_edge_dict(self):
... return self.all_edge_dict
... edge_attr_dict_factory = single_edge_dict
>>> G = ThinGraph()
>>> G.add_edge (2,1)
>>> list(G.edges(data= True))
[(1, 2, {'weight': 1})]
>>> G.add_edge (2,2)
>>> G[2][1] is G[2][2]
True
```


### 3.2.4 Methods

## Adding and removing nodes and edges

| DiGraph.__init__([data]) | Initialize a graph with edges, name, graph attributes. |
| :---: | :---: |
| DiGraph.add_node(n, \***attr) | Add a single node n and update node attributes. |
| DiGraph.add_nodes_from(nodes, \*1*attr) | Add multiple nodes. |
| DiGraph.remove_node(n) | Remove node n . |
| DiGraph.remove_nodes_from(nbunch) | Remove multiple nodes. |
| DiGraph.add_edge(u, v, \*\*attr) | Add an edge between $u$ and $v$. |
| DiGraph.add_edges_from(ebunch, \***attr) | Add all the edges in ebunch. |
| ```DiGraph.add_weighted_edges_from(ebunch[, weight])``` | Add all the edges in ebunch as weighted edges with specified weights. |
| DiGraph.remove_edge(u, v) | Remove the edge between u and v . |
| DiGraph.remove_edges_from(ebunch) | Remove all edges specified in ebunch. |
| DiGraph.clear() | Remove all nodes and edges from the graph. |

$\qquad$

DiGraph.__init__(data=None, **attr)
Initialize a graph with edges, name, graph attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.
- name (string, optional (default=' ')) - An optional name for the graph.
- attr (keyword arguments, optional (default= no attributes)) - Attributes to add to graph as key=value pairs.


## See also:

convert()

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
>>> e = [(1,2),(2,3), (3,4)] # list of edges
>>> G = nx.Graph(e)
```

Arbitrary graph attribute pairs (key=value) may be assigned

```
>>> G=nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}
```

add_node

DiGraph.add_node (n, **attr)
Add a single node n and update node attributes.

## Parameters

- n (node) - A node can be any hashable Python object except None.
- $\operatorname{attr}$ (keyword arguments, optional) - Set or change node attributes using key=value.

See also:

```
add_nodes_from()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0,1), (1,2), (2,0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:
$\ggg$ G.add_node $(1$, size $=10)$
$\ggg$ G.add_node(3,weight=0.4,UTM=('13S',382871,3972649))

## Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn't change on mutables.

```
add_nodes_from
```

DiGraph.add_nodes_from (nodes, **attr)

Add multiple nodes.

## Parameters

- nodes (iterable container) - A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- attr (keyword arguments, optional (default= no attributes)) - Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.


## See also:

add_node()

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MuItiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0,1), (1,2), (2,0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(),key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```
>>> G.add_nodes_from([1,2], size=10)
>>> G.add_nodes_from([3,4], weight=0.4)
```

Use (node, attrdict) tuples to update attributes for specific nodes.

```
>>> G.add_nodes_from([(1,dict(size=11)), (2,{'color':'blue'})])
>>> G.node[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.node[1]['size']
11
```

remove_node

DiGraph.remove_node ( $n$ )
Remove node n .
Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.
Parameters $\mathbf{n}$ (node) - A node in the graph
Raises NetworkXerror - If n is not in the graph.

## See also:

```
remove_nodes_from()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges())
[(0, 1), (1, 2)]
>>> G.remove_node (1)
>>> list(G.edges())
[]
```

remove_nodes_from

DiGraph.remove_nodes_from (nbunch)
Remove multiple nodes.
Parameters nodes (iterable container) - A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

## See also:

```
remove_node()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes())
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes())
[ ]
```

add_edge

DiGraph.add_edge ( $u, v, * * a t t r$ )
Add an edge between $u$ and $v$.
The nodes u and v will be automatically added if they are not already in the graph.
Edge attributes can be specified with keywords or by directly accessing the edge's attribute dictionary. See examples below.

## Parameters

- u, $\mathbf{v}$ (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## See also:

add_edges_from () add a collection of edges

## Notes

Adding an edge that already exists updates the edge data.
Many NetworkX algorithms designed for weighted graphs use as the edge weight a numerical value assigned to a keyword which by default is 'weight'.

## Examples

The following all add the edge $\mathrm{e}=(1,2)$ to graph G :

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = (1,2)
>>> G.add_edge(1, 2) # explicit two-node form
>>> G.add_edge(*e) # single edge as tuple of two nodes
>>> G.add_edges_from( [(1,2)] ) # add edges from iterable container
```

Associate data to edges using keywords:

```
>>> G.add_edge(1, 2, weight=3)
>>> G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string associations, directly access the edge's attribute dictionary.

```
>>> G.add_edge(1, 2)
>>> G[1][2].update({0: 5})
```

add_edges_from

DiGraph.add_edges_from (ebunch, **attr)
Add all the edges in ebunch.

## Parameters

- ebunch (container of edges) - Each edge given in the container will be added to the graph. The edges must be given as 2-tuples ( $u, v$ ) or 3-tuples ( $u, v, d$ ) where $d$ is a dictionary containing edge data.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## See also:

add_edge () add a single edge
add_weighted_edges_from () convenient way to add weighted edges

## Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added.
Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0,1),(1,2)]) # using a list of edge tuples
>>> e = zip(range(0,3),range(1,4))
>>> G.add_edges_from(e) # Add the path graph 0-1-2-3
```

Associate data to edges

```
>>> G.add_edges_from([(1,2), (2,3)], weight=3)
>>> G.add_edges_from([(3,4), (1,4)], label='WN2898')
```

add_weighted_edges_from

DiGraph.add_weighted_edges_from (ebunch, weight='weight', **attr)
Add all the edges in ebunch as weighted edges with specified weights.

## Parameters

- ebunch (container of edges) - Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples ( $u, v, w$ ) where $w$ is a number.
- weight (string, optional (default= 'weight')) - The attribute name for the edge weights to be added.
- attr (keyword arguments, optional (default= no attributes)) - Edge attributes to add/update for all edges.

See also:
add_edge () add a single edge
add_edges_from() add multiple edges

## Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0,1,3.0), (1, 2, 7.5)])
```

remove_edge

DiGraph.remove_edge ( $u, v$ )
Remove the edge between $u$ and $v$.
Parameters u, $\mathbf{v}$ (nodes) - Remove the edge between nodes $u$ and $v$.
Raises NetworkXError - If there is not an edge between $\mathbf{u}$ and v .

## See also:

remove_edges_from () remove a collection of edges

## Examples

```
>>> G = nx.Graph() # or DiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge (0,1)
>>> e = (1,2)
>>> G.remove_edge(*e) # unpacks e from an edge tuple
>>> e = (2,3,{'weight':7}) # an edge with attribute data
>>> G.remove_edge(*e[:2]) # select first part of edge tuple
```

remove_edges_from

DiGraph.remove_edges_from (ebunch)
Remove all edges specified in ebunch.
Parameters ebunch (list or container of edge tuples) - Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples ( $u, v$ ) edge between $u$ and $v$.
- 3-tuples $(\mathrm{u}, \mathrm{v}, \mathrm{k})$ where k is ignored.


## See also:

remove_edge () remove a single edge

## Notes

Will fail silently if an edge in ebunch is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1,2), (2,3)]
>>> G.remove_edges_from(ebunch)
```

clear

DiGraph.clear()
Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes())
[]
>>> list(G.edges())
[]
```


## Iterating over nodes and edges

| DiGraph.nodes([data, default]) | Returns an iterator over the nodes. |
| :---: | :---: |
| DiGraph.__iter_() | Iterate over the nodes. |
| DiGraph.edges([nbunch, data, default]) | Return an iterator over the edges. |
| DiGraph.out_edges([nbunch, data, default]) | Return an iterator over the edges. |
| DiGraph.in_edges([nbunch, data, default]) | Return an iterator over the incoming edges. |
| DiGraph.get_edge_data(u, v[, default]) | Return the attribute dictionary associated with edge (u,v). |
| DiGraph.neighbors(n) | Return an iterator over successor nodes of n . |
| DiGraph.__getitem__(n) | Return a dict of neighbors of node n . |
| DiGraph.successors(n) | Return an iterator over successor nodes of n . |
| DiGraph.predecessors(n) | Return an iterator over predecessor nodes of n . |
| DiGraph.adjacency() | Return an iterator over (node, adjacency dict) tuples for all nodes. |
| DiGraph.nbunch_iter([nbunch]) | Return an iterator over nodes contained in nbunch that are also in the graph. |

nodes

DiGraph.nodes (data=False, default=None)
Returns an iterator over the nodes.

## Parameters

- data (string or bool, optional (default=False)) - The node attribute returned in 2-tuple ( $n$,ddict[data]). If True, return entire node attribute dict as ( $n$, ddict). If False, return just the nodes $n$.
- default (value, optional (default=None)) - Value used for nodes that dont have the requested attribute. Only relevant if data is not True or False.

Returns An iterator over nodes, or ( $\mathrm{n}, \mathrm{d}$ ) tuples of node with data. If data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

## Return type iterator

## Notes

If the node data is not required, it is simpler and equivalent to use the expression for $n$ in $G$, or list ( $G$ ).

## Examples

There are two simple ways of getting a list of all nodes in the graph:

```
>>> G = nx.path_graph(3)
>>> list(G.nodes())
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```
>>> G.add_node(1, time='5pm')
>>> G.node[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

$\qquad$

DiGraph.__iter__()
Iterate over the nodes. Use the expression 'for n in G '.
Returns niter - An iterator over all nodes in the graph.
Return type iterator

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for }n\mathrm{ in G]
[0, 1, 2, 3]
```


## edges

DiGraph.edges (nbunch=None, data=False, default=None)
Return an iterator over the edges.

Edges are returned as tuples with optional data in the order (node, neighbor, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns edge - An iterator over ( $u, v$ ) or ( $u, v, d$ ) tuples of edges.
Return type iterator

## See also:

```
in_edges(),out_edges()
```


## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.DiGraph() # or MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2])
>>> G.add_edge(2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges([0,2]))
[(0, 1), (2, 3)]
>>> list(G.edges(0))
[(0, 1)]
```


## out_edges

DiGraph.out_edges (nbunch=None, data=False, default=None)
Return an iterator over the edges.
Edges are returned as tuples with optional data in the order (node, neighbor, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.
Returns edge - An iterator over (u,v) or (u,v,d) tuples of edges.
Return type iterator


## See also:

```
in_edges(),out_edges()
```


## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.DiGraph() # or MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2])
>>> G.add_edge(2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges([0,2]))
[(0, 1), (2, 3)]
>>> list(G.edges(0))
[(0, 1)]
```

in_edges

DiGraph.in_edges (nbunch=None, data=False, default=None)
Return an iterator over the incoming edges.

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns in_edge - An iterator over (u,v) or (u,v,d) tuples of incoming edges.
Return type iterator

## See also:

edges () return an iterator over edges
get_edge_data
DiGraph.get_edge_data ( $u, v$, default=None)
Return the attribute dictionary associated with edge ( $u, v$ ).

## Parameters

- u, $\mathbf{v}$ (nodes)
- default (any Python object (default=None)) - Value to return if the edge (u,v) is not found.

Returns edge_dict - The edge attribute dictionary.
Return type dictionary

## Notes

It is faster to use G[u][v].

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0][1]
{ }
```

Warning: Assigning $G[u][v]$ corrupts the graph data structure. But it is safe to assign attributes to that dictionary,

```
>>> G[0][1]['weight'] = 7
>>> G[0][1]['weight']
7
>>> G[1][0]['weight']
7
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.get_edge_data(0, 1) # default edge data is {}
{ }
>>> e = (0,1)
>>> G.get_edge_data(*e) # tuple form
{}
>>> G.get_edge_data('a','b',default=0) # edge not in graph, return 0
0
```

neighbors

DiGraph.neighbors ( $n$ )
Return an iterator over successor nodes of $n$.
neighbors() and successors() are the same.
__getitem

DiGraph.___getitem_(n)
Return a dict of neighbors of node $n$. Use the expression ' $\mathrm{G}[\mathrm{n}]$ '.

Parameters $\mathbf{n}$ (node) - A node in the graph.
Returns adj_dict - The adjacency dictionary for nodes connected to n .
Return type dictionary

## Notes

$\mathrm{G}[\mathrm{n}]$ is similar to G.neighbors( n ) but the internal data dictionary is returned instead of an iterator.
Assigning $\mathrm{G}[\mathrm{n}]$ will corrupt the internal graph data structure. Use $\mathrm{G}[\mathrm{n}]$ for reading data only.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
{1: {}}
```


## successors

## DiGraph.successors (n)

Return an iterator over successor nodes of $n$.
neighbors() and successors() are the same.

## predecessors

DiGraph.predecessors ( $n$ )
Return an iterator over predecessor nodes of $n$.
adjacency

DiGraph.adjacency()
Return an iterator over (node, adjacency dict) tuples for all nodes.
This is the fastest way to look at every edge. For directed graphs, only outgoing adjacencies are included.
Returns adj_iter - An iterator over (node, adjacency dictionary) for all nodes in the graph.
Return type iterator

Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n,nbrdict) for n,nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

nbunch_iter

DiGraph.nbunch_iter (nbunch=None)
Return an iterator over nodes contained in nbunch that are also in the graph.
The nodes in nbunch are checked for membership in the graph and if not are silently ignored.
Parameters nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
Returns niter - An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.
Return type iterator
Raises NetworkXError - If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

## See also:

$\qquad$
Graph.__iter_()

## Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use "if nbunch in self:", even after processing with this routine.
If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.

## Information about graph structure

| DiGraph.has_node(n) | Return True if the graph contains the node n. |
| :--- | :--- |
| DiGraph.__Contains__(n) | Return True if n is a node, False otherwise. |
| DiGraph.has_edge(u, v) | Return True if the edge (u,v) is in the graph. |
| DiGraph.order() | Return the number of nodes in the graph. |
| DiGraph.number_of_nodes() | Return the number of nodes in the graph. |
| DiGraph.__len_() | Return the number of nodes. |
| DiGraph.degree([nbunch, weight]) | Return an iterator for (node, degree) or degree for single <br> node. |
| DiGraph.in_degree([nbunch, weight]) | Return an iterator for (node, in-degree) or in-degree for sin- <br> gle node. |
| DiGraph.out_degree([nbunch, weight]) | Return an iterator for (node, out-degree) or out-degree for <br> single node. |
| DiGraph.size([weight]) | Return the number of edges or total of all edge weights. |
| DiGraph.number_of_edges([u, v]) | Return the number of edges between two nodes. |
| DiGraph.nodes_with_selfloops() | Returns an iterator over nodes with self loops. |
| DiGraph.selfloop_edges([data, default]) | Returns an iterator over selfloop edges. |
| DiGraph. number_of_selfloops() | Return the number of selfloop edges. |

has_node
DiGraph.has_node ( $n$ )
Return True if the graph contains the node n .
Parameters $\mathbf{n}$ (node)

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```
>>> 0 in G
True
```

__contains

DiGraph.__contains (n)

Return True if n is a node, False otherwise. Use the expression ' n in G '.

Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

has_edge

DiGraph.has_edge $(u, v)$
Return True if the edge ( $u, v$ ) is in the graph.
Parameters u,v (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
Returns edge_ind - True if edge is in the graph, False otherwise.
Return type bool

## Examples

Can be called either using two nodes $u, v$ or edge tuple (u,v)

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
```

>>> G.has_edge $(0,1)$ \# using two nodes
True
$\ggg e=(0,1)$
$\ggg$ G.has_edge $(* e) \quad \#$ e is a 2 -tuple $(u, v)$

```
True
>>> e = (0,1,{'weight':7})
>>> G.has_edge(*e[:2]) # e is a 3-tuple (u,v,data_dictionary)
True
```

The following syntax are all equivalent:

```
>>> G.has_edge(0,1)
True
>>> 1 in G[0] # though this gives KeyError if 0 not in G
True
```

order

DiGraph.order()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int

## See also:

number_of_nodes(),__len__()
number_of_nodes

DiGraph.number_of_nodes()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
order(),__Ien_()

Examples
$\ggg$ G $=$ nx.path_graph(3) \# or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len (G)
3
_len

DiGraph.__len_()
Return the number of nodes. Use the expression 'len(G)'.
Returns nnodes - The number of nodes in the graph.
Return type int

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

degree

DiGraph. degree (nbunch=None, weight=None)
Return an iterator for (node, degree) or degree for single node.
The node degree is the number of edges adjacent to the node. This function returns the degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - Degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, degree).

See also:

```
in_degree(),out_degree()
```


## Examples

```
>>> G = nx.DiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.degree(0) # node 0 with degree 1
1
>>> list(G.degree([0,1]))
[(0, 1), (1, 2)]
```


## in_degree

## DiGraph.in_degree (nbunch=None, weight=None)

Return an iterator for (node, in-degree) or in-degree for single node.
The node in-degree is the number of edges pointing in to the node. This function returns the in-degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - In-degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, in-degree).


## See also:

```
degree(),out_degree()
```


## Examples

```
>>> G = nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.in_degree(0) # node 0 with degree 0
0
>>> list(G.in_degree([0,1]))
[(0, 0), (1, 1)]
```


## out_degree

DiGraph.out_degree (nbunch=None, weight=None)
Return an iterator for (node, out-degree) or out-degree for single node.
The node out-degree is the number of edges pointing out of the node. This function returns the out-degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - Out-degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, out-degree).


## See also:

```
degree(),in_degree()
```


## Examples

```
>>> G = nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.out_degree(0) # node 0 with degree 1
1
>>> list(G.out_degree([0,1]))
[(0, 1), (1, 1)]
```

size

DiGraph.size (weight=None)
Return the number of edges or total of all edge weights.
Parameters weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

## Returns

size - The number of edges or (if weight keyword is provided) the total weight sum.
If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).

Return type numeric

## See also:

```
number_of_edges()
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3
```

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a','b',weight=2)
>>> G.add_edge('b','c',weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```


## number_of_edges

DiGraph.number_of_edges ( $u=$ None, $v=$ None)
Return the number of edges between two nodes.
Parameters u, v (nodes, optional (default=all edges)) - If $u$ and $v$ are specified, return the number of edges between $u$ and $v$. Otherwise return the total number of all edges.

Returns nedges - The number of edges in the graph. If nodes $u$ and $v$ are specified return the number of edges between those nodes.

Return type int
See also:
size()

Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_edges()
3
>>> G.number_of_edges (0,1)
1
>>> e = (0,1)
>>> G.number_of_edges(*e)
1
```

nodes_with_selfloops

DiGraph.nodes_with_selfloops()
Returns an iterator over nodes with self loops.
A node with a self loop has an edge with both ends adjacent to that node.
Returns nodelist - A iterator over nodes with self loops.
Return type iterator

## See also:

```
selfloop_edges(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge(1, 1)
>>> G.add_edge (1, 2)
>>> list(G.nodes_with_selfloops())
[1]
```

selfloop_edges

DiGraph.selfloop_edges (data=False, default=None)
Returns an iterator over selfloop edges.
A selfloop edge has the same node at both ends.

## Parameters

- data (string or bool, optional (default=False)) - Return selfloop edges as two tuples $(u, v)($ data=False) or three-tuples ( $u, v$, datadict) $($ data=True) or three-tuples (u,v,datavalue) (data='attrname')
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns edgeiter - An iterator over all selfloop edges.
Return type iterator over edge tuples

## See also:

```
nodes_with_selfloops(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1,1)
>>> G.add_edge (1, 2)
>>> list(G.selfloop_edges())
[(1, 1)]
>>> list(G.selfloop_edges(data=True))
[(1, 1, {})]
```

number_of_selfloops

DiGraph.number_of_selfloops()
Return the number of selfloop edges.
A selfloop edge has the same node at both ends.
Returns nloops - The number of selfloops.
Return type int
See also:

```
nodes_with_selfloops(),selfloop_edges()
```


## Examples

```
>>> G=nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1,1)
>>> G.add_edge (1, 2)
>>> G.number_of_selfloops()
1
```

Making copies and subgraphs

| DiGraph.copy([with_data]) | Return a copy of the graph. |
| :--- | :--- |
| DiGraph.to_undirected([reciprocal]) | Return an undirected representation of the digraph. |
| DiGraph.to_directed() | Return a directed copy of the graph. |
| DiGraph.subgraph(nbunch) | Return the subgraph induced on nodes in nbunch. |
| DiGraph.edge_subgraph(edges) | Returns the subgraph induced by the specified edges. |
| DiGraph.reverse([copy]) | Return the reverse of the graph. |

## copy

DiGraph.copy (with_data=True)
Return a copy of the graph.
All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy - The default behavior is a "deepcopy" where the graph structure as well as all data attributes and any objects they might contain are copied. The entire graph object is new so that changes in the copy do not affect the original object.

Data Reference (Shallow) - For a shallow copy (with_data=False) the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other.

Independent Shallow - This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This type of copy is not enabled. Instead use:

```
>>> G = nx.path_graph(5)
>>> H = G.__class__(G)
```

Fresh Data- For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```
>>>H = G.__class___()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges())
```

See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Parameters with_data (bool, optional (default=True)) - If True, the returned graph will have a deep copy of the graph, node, and edge attributes of this object. Otherwise, the returned graph will be a shallow copy.

Returns G-A copy of the graph.
Return type Graph

## See also:

to_directed() return a directed copy of the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```


## to_undirected

DiGraph.to_undirected (reciprocal=False)
Return an undirected representation of the digraph.

Parameters reciprocal (bool (optional)) - If True only keep edges that appear in both directions in the original digraph.
Returns G - An undirected graph with the same name and nodes and with edge (u,v,data) if either ( $u, v$, data) or ( $v, u, d a t a)$ is in the digraph. If both edges exist in digraph and their edge data is different, only one edge is created with an arbitrary choice of which edge data to use. You must check and correct for this manually if desired.

## Return type Graph

## Notes

If edges in both directions ( $u, v$ ) and ( $\mathrm{v}, \mathrm{u}$ ) exist in the graph, attributes for the new undirected edge will be a combination of the attributes of the directed edges. The edge data is updated in the (arbitrary) order that the edges are encountered. For more customized control of the edge attributes use add_edge().
This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{G}=\mathrm{DiGraph}(\mathrm{D})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Warning: If you have subclassed DiGraph to use dict-like objects in the data structure, those changes do not transfer to the Graph created by this method.

## to_directed

DiGraph.to_directed()
Return a directed copy of the graph.
Returns G - A deepcopy of the graph.
Return type DiGraph

## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{D}=\mathrm{DiGraph}(\mathrm{G})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

## Examples

```
>>> G = nx.Graph() # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```
>>> G = nx.DiGraph() # or MultiDiGraph, etc
>>> G.add_edge (0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1)]
```


## subgraph

DiGraph.subgraph (nbunch)
Return the subgraph induced on nodes in nbunch.
The induced subgraph of the graph contains the nodes in nbunch and the edges between those nodes.
Parameters nbunch (list, iterable) - A container of nodes which will be iterated through once.
Returns $\mathbf{G}$ - A subgraph of the graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge or node attributes just point to the original graph. So changes to the node or edge structure will not be reflected in the original graph while changes to the attributes will.

To create a subgraph with its own copy of the edge/node attributes use: nx.Graph(G.subgraph(nbunch))
If edge attributes are containers, a deep copy can be obtained using: G.subgraph(nbunch).copy()
For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from([ $n$ in $G$ if $n$ not in $\operatorname{set}($ nbunch $)]$ )

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0,1,2])
>>> list(H.edges())
[(0, 1), (1, 2)]
```

edge_subgraph

DiGraph.edge_subgraph (edges)
Returns the subgraph induced by the specified edges.
The induced subgraph contains each edge in edges and each node incident to any one of those edges.
Parameters edges (iterable) - An iterable of edges in this graph.
Returns $\mathbf{G}$ - An edge-induced subgraph of this graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge, and node attributes in the returned subgraph are references to the corresponding attributes in the original graph. Thus changes to the node or edge structure of the returned graph will not be reflected in the original graph, but changes to the attributes will.
To create a subgraph with its own copy of the edge or node attributes, use:

```
>>> nx.DiGraph(G.edge_subgraph(edges))
```

If edge attributes are containers, a deep copy of the attributes can be obtained using:

```
>>> G.edge_subgraph(edges).copy()
```


## Examples

```
>>> G = nx.DiGraph(nx.path_graph(5))
>>>H=G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes())
[0, 1, 3, 4]
>>> list(H.edges())
[(0, 1), (3, 4)]
```


## reverse

DiGraph.reverse (copy=True)
Return the reverse of the graph.
The reverse is a graph with the same nodes and edges but with the directions of the edges reversed.
Parameters copy (bool optional (default=True)) - If True, return a new DiGraph holding the reversed edges. If False, reverse the reverse graph is created using the original graph (this changes the original graph).

### 3.2.5 MultiGraph - Undirected graphs with self loops and parallel edges

## Overview

MultiGraph (data=None, **attr)
An undirected graph class that can store multiedges.
Multiedges are multiple edges between two nodes. Each edge can hold optional data or attributes.
A MultiGraph holds undirected edges. Self loops are allowed.
Nodes can be arbitrary (hashable) Python objects with optional key/value attributes.
Edges are represented as links between nodes with optional key/value attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.
- attr (keyword arguments, optional (default= no attributes) - Attributes to add to graph as key=value pairs.


## See also:

Graph(), DiGraph(), MultiDiGraph()

## Examples

Create an empty graph structure (a "null graph") with no nodes and no edges.

```
>>> G = nx.MultiGraph()
```

G can be grown in several ways.

## Nodes:

Add one node at a time:

```
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```
>>> G.add_nodes_from([2,3])
>>> G.add_nodes_from(range (100,110))
>>> H=nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```
>>> G.add node (H)
```


## Edges:

G can also be grown by adding edges.
Add one edge,

```
>>> key = G.add_edge(1, 2)
```

a list of edges,

```
>>> keys = G.add_edges_from([(1,2),(1,3)])
```

or a collection of edges,

```
>>> keys = G.add_edges_from(list(H.edges()))
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. If an edge already exists, an additional edge is created and stored using a key to identify the edge. By default the key is the lowest unused integer.

```
>>> keys = G.add_edges_from([(4,5,dict(route=282)), (4,5,dict(route=37))])
>>> G[4]
{3: {0: {}}, 5: {0: {}, 1: {'route': 282}, 2: {'route': 37}}}
```


## Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

```
>>> G = nx.MultiGraph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Add node attributes using add_node(), add_nodes_from() or G.node

```
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.node[1]
{'time': '5pm'}
>>> G.node[1]['room'] = 714
>>> del G.node[1]['room'] # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Warning: adding a node to G.node does not add it to the graph.
Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edge.

```
>>> key = G.add_edge(1, 2, weight=4.7 )
>>> keys = G.add_edges_from([(3,4), (4,5)], color='red')
>>> keys = G.add_edges_from([(1,2,{'color':'blue'}), (2,3,{'weight':8})])
>>> G[1][2][0]['weight'] = 4.7
>>> G.edge[1][2][0]['weight'] = 4
```


## Shortcuts:

Many common graph features allow python syntax to speed reporting.

```
>>> 1 in G # check if node in graph
True
>> [n for n in G if n<3] # iterate through nodes
[1, 2]
>>> len(G) # number of nodes in graph
5
>>> G[1] # adjacency dict keyed by neighbor to edge attributes
... # Note: you should not change this dict manually!
{2: {0: {'weight': 4}, 1: {'color': 'blue'}}}
```

The fastest way to traverse all edges of a graph is via adjacency(), but the edges() method is often more convenient.

```
>>> for n,nbrsdict in G.adjacency():
... for nbr,keydict in nbrsdict.items():
... for key,eattr in keydict.items():
... if 'weight' in eattr:
... (n,nbr,key,eattr['weight'])
(1, 2, 0, 4)
(2, 1, 0, 4)
(2, 3, 0, 8)
(3, 2, 0, 8)
>>> list(G.edges(data='weight', keys=True))
[(1, 2, 0, 4), (1, 2, 1, None), (2, 3, 0, 8), (3, 4, 0, None), (4, 5, 0, None)]
```


## Reporting:

Simple graph information is obtained using methods. Reporting methods usually return iterators instead of containers to reduce memory usage. Methods exist for reporting nodes(), edges(), neighbors() and degree() as well as the number of nodes and edges.

For details on these and other miscellaneous methods, see below.

## Subclasses (Advanced):

The MultiGraph class uses a dict-of-dict-of-dict-of-dict data structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge_key dicts keyed by neighbor. The edge_key dict holds each edge_attr dict keyed by edge key. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these four dicts in the dict-of-dict-of-dict-of-dict structure can be replaced by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, adjlist_inner_dict_factory, adjlist_outer_dict_factory, and edge_attr_dict_factory.
node_dict_factory [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object
adjlist_outer_dict_factory [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
adjlist_inner_dict_factory [function, (default: dict)] Factory function to be used to create the adjacency list dict which holds multiedge key dicts keyed by neighbor. It should require no arguments and return a dict-like object.
edge_key_dict_factory [function, (default: dict)] Factory function to be used to create the edge key dict which holds edge data keyed by edge key. It should require no arguments and return a dict-like object.
edge_attr_dict_factory [function, (default: dict)] Factory function to be used to create the edge attribute dict which holds attrbute values keyed by attribute name. It should require no arguments and return a dict-like object.

## Examples

Create a multigraph subclass that tracks the order nodes are added.

```
>>> from collections import OrderedDict
>>> class OrderedGraph(nx.MultiGraph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> keys = G.add_edges_from( ((2,2), (2,1), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 1), (2, 1), (2, 2), (1, 1)]
```

Create a multgraph object that tracks the order nodes are added and for each node track the order that neighbors are added and for each neighbor tracks the order that multiedges are added.

```
>>> class OrderedGraph(nx.MultiGraph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory = OrderedDict
... adjlist_inner_dict_factory = OrderedDict
... edge_key_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> elist = ( (2, 2), (2,1,2,{'weight':0.1}), (2,1,1,{'weight':0.2}), (1,1))
>>> keys = G.add_edges_from(elist)
>>> list (G.edges(keys=True))
[(2, 2, 0), (2, 1, 2), (2, 1, 1), (1, 1, 0)]
```


### 3.2.6 Methods

## Adding and removing nodes and edges

| MultiGraph.__init__([data]) |  |
| :---: | :---: |
| MultiGraph.add_node(n, \**attr) | Add a single node n and update node attributes. |
| MultiGraph.add_nodes_from(nodes, \***attr) | Add multiple nodes. |
| MultiGraph.remove_node(n) | Remove node n . |
| MultiGraph.remove_nodes_from(nodes) | Remove multiple nodes. |
| MultiGraph.add_edge(u, v[, key]) | Add an edge between $u$ and v . |
| MultiGraph.add_edges_from(ebunch, \***attr) | Add all the edges in ebunch. |
| MultiGraph.add_weighted_edges_from(ebunch[, ...]) | Add all the edges in ebunch as weighted edges with specified weights. |
| MultiGraph.new_edge_key(u, v) | Return an unused key for edges between nodes $u$ and $v$. |
| MultiGraph.remove_edge(u, v[, key]) | Remove an edge between $u$ and v. |
| MultiGraph.remove_edges_from(ebunch) | Remove all edges specified in ebunch. |
| MultiGraph.clear() | Remove all nodes and edges from the graph. |

$\qquad$

MultiGraph.__init__(data=None, **attr)
add_node

MultiGraph.add_node ( $n$, **attr)
Add a single node n and update node attributes.

## Parameters

- n (node) - A node can be any hashable Python object except None.
- $\boldsymbol{a t t r}$ (keyword arguments, optional) - Set or change node attributes using key=value.


## See also:

```
add_nodes_from()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0,1),(1,2),(2,0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:

```
>>> G.add_node(1,size=10)
>>> G.add_node(3,weight=0.4,UTM=('13S',382871,3972649))
```


## Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn't change on mutables.
add_nodes_from

MultiGraph.add_nodes_from (nodes, **attr)
Add multiple nodes.

## Parameters

- nodes (iterable container) - A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- attr (keyword arguments, optional (default= no attributes)) - Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.


## See also:

```
add_node()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0,1), (1,2), (2,0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(),key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```
>>> G.add_nodes_from([1,2], size=10)
>>> G.add_nodes_from([3,4], weight=0.4)
```

Use (node, attrdict) tuples to update attributes for specific nodes.

```
>>> G.add__nodes_from([(1,dict(size=11)), (2,{'color':'blue'})])
>>> G.node[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.node[1]['size']
11
```

remove_node

MultiGraph. remove_node ( $n$ )
Remove node n .
Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.
Parameters $\mathbf{n}$ (node) - A node in the graph
Raises NetworkXError - If n is not in the graph.

## See also:

```
remove_nodes_from()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges())
[(0, 1), (1, 2)]
>>> G.remove_node(1)
>>> list(G.edges())
[]
```

```
remove_nodes_from
```

MultiGraph.remove_nodes_from (nodes)

Remove multiple nodes.
Parameters nodes (iterable container) - A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

## See also:

remove_node()

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes())
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
```

```
>>> list(G.nodes())
[ ]
```

add_edge

MultiGraph.add_edge ( $u, v$, key=None, **attr)
Add an edge between $u$ and $v$.
The nodes $u$ and $v$ will be automatically added if they are not already in the graph.
Edge attributes can be specified with keywords or by directly accessing the edge's attribute dictionary. See examples below.

## Parameters

- u, $\mathbf{v}$ (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
- key (hashable identifier, optional (default=lowest unused integer)) - Used to distinguish multiedges between a pair of nodes.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## Returns

Return type The edge key assigned to the edge.

## See also:

add_edges_from () add a collection of edges

## Notes

To replace/update edge data, use the optional key argument to identify a unique edge. Otherwise a new edge will be created.

NetworkX algorithms designed for weighted graphs cannot use multigraphs directly because it is not clear how to handle multiedge weights. Convert to Graph using edge attribute 'weight' to enable weighted graph algorithms.
Default keys are generated using the method new_edge_key (). This method can be overridden by subclassing the base class and providing a custom new_edge_key () method.

## Examples

The following all add the edge $\mathrm{e}=(1,2)$ to graph G :

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = (1,2)
>>> G.add_edge(1, 2) # explicit two-node form
>>> G.add_edge(*e) # single edge as tuple of two nodes
>>> G.add_edges_from( [(1,2)] ) # add edges from iterable container
```

Associate data to edges using keywords:

```
>>> G.add_edge(1, 2, weight=3)
>>> G.add_edge(1, 2, key=0, weight=4) # update data for key=0
>>> G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```


## add_edges_from

MultiGraph.add_edges_from (ebunch, **attr)
Add all the edges in ebunch.

## Parameters

- ebunch (container of edges) - Each edge given in the container will be added to the graph. The edges can be:
- 2-tuples (u,v) or
- 3-tuples ( $u, v, d$ ) for an edge attribute dict $d$, or
- 4-tuples ( $u, v, k, d$ ) for an edge identified by key $k$
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## Returns

Return type A list of edge keys assigned to the edges in ebunch.

## See also:

add_edge () add a single edge
add_weighted_edges_from() convenient way to add weighted edges

## Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added.
Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments.
Default keys are generated using the method new_edge_key (). This method can be overridden by subclassing the base class and providing a custom new_edge_key () method.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0,1),(1,2)]) # using a list of edge tuples
>>> e = zip(range(0,3),range(1,4))
>>> G.add_edges_from(e) # Add the path graph 0-1-2-3
```

Associate data to edges

```
>>> G.add_edges_from([(1,2), (2,3)], weight=3)
>>> G.add_edges_from([(3,4),(1,4)], label='WN2898')
```

```
add_weighted_edges_from
```

MultiGraph.add_weighted_edges_from(ebunch, weight='weight', **attr)
Add all the edges in ebunch as weighted edges with specified weights.

## Parameters

- ebunch (container of edges) - Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples ( $u, v, w$ ) where $w$ is a number.
- weight (string, optional (default= 'weight')) - The attribute name for the edge weights to be added.
- attr (keyword arguments, optional (default= no attributes)) - Edge attributes to add/update for all edges.


## See also:

add_edge () add a single edge
add_edges_from() add multiple edges

## Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0,1,3.0),(1,2,7.5)])
```

new_edge_key

MultiGraph.new_edge_key ( $u, v$ )
Return an unused key for edges between nodes $u$ and $v$.
The nodes $u$ and $v$ do not need to be already in the graph.

## Notes

In the standard MultiGraph class the new key is the number of existing edges between $u$ and $v$ (increased if necessary to ensure unused). The first edge will have key 0 , then 1 , etc. If an edge is removed further new_edge_keys may not be in this order.

Parameters u, v (nodes)
Returns key
Return type int

## remove_edge

MultiGraph.remove_edge ( $u, v$, key=None)
Remove an edge between $u$ and $v$.

## Parameters

- u, $\mathbf{v}$ (nodes) - Remove an edge between nodes $u$ and $v$.
- key (hashable identifier, optional (default=None)) - Used to distinguish multiple edges between a pair of nodes. If None remove a single (arbitrary) edge between $u$ and $v$.
Raises NetworkXError - If there is not an edge between $u$ and $v$, or if there is no edge with the specified key.
See also:
remove_edges_from () remove a collection of edges


## Examples

```
>>> G = nx.MultiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge (0,1)
>>> e = (1,2)
>>> G.remove_edge(*e) # unpacks e from an edge tuple
```

For multiple edges

```
>>> G = nx.MultiGraph() # or MultiDiGraph, etc
>>> G.add_edges_from([(1,2),(1,2),(1,2)]) # key_list returned
[0, 1, 2]
>>> G.remove_edge(1,2) # remove a single (arbitrary) edge
```

For edges with keys

```
>>> G = nx.MultiGraph() # or MultiDiGraph, etc
>>> G.add_edge(1, 2,key='first')
'first'
>>> G.add_edge(1,2,key='second')
'second'
>>> G.remove_edge(1, 2,key='second')
```


## remove_edges_from

## MultiGraph.remove_edges_from (ebunch)

Remove all edges specified in ebunch.
Parameters ebunch (list or container of edge tuples) - Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples ( $u, v$ ) All edges between $u$ and $v$ are removed.
- 3-tuples (u,v,key) The edge identified by key is removed.
- 4-tuples (u,v,key,data) where data is ignored.


## See also:

```
remove_edge() remove a single edge
```


## Notes

Will fail silently if an edge in ebunch is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1,2), (2,3)]
>>> G.remove_edges_from(ebunch)
```

Removing multiple copies of edges

```
>>> G = nx.MultiGraph()
>>> keys = G.add_edges_from([(1,2),(1,2),(1,2)])
>>> G.remove_edges_from([(1,2), (1,2)])
>>> list(G.edges())
[(1, 2)]
>>> G.remove_edges_from([(1,2), (1,2)]) # silently ignore extra copy
>>> list(G.edges()) # now empty graph
[ ]
```

clear

MultiGraph.clear()
Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes())
[]
>>> list(G.edges())
[]
```

Iterating over nodes and edges

| MultiGraph. nodes $([$ data, default $])$ | Returns an iterator over the nodes. |
| :--- | :--- |
| MultiGraph.__iter__() | Iterate over the nodes. |
| MultiGraph.edges $([$ nbunch, data, keys, default $])$ | Return an iterator over the edges. |
| MultiGraph.get_edge_data(u, v[, key, default $])$ | Return the attribute dictionary associated with edge (u,v). |
| MultiGraph.neighbors(n) | Return an iterator over all neighbors of node n. |
|  |  |

Table 3.10 - continued from previous page

| MultiGraph._Getitem_(n) | Return a dict of neighbors of node n. |
| :--- | :--- |
| MultiGraph.adjacency() | Return an iterator over (node, adjacency dict) tuples for all <br> nodes. |
| MultiGraph. nbunch_iter([nbunch]) | Return an iterator over nodes contained in nbunch that are <br> also in the graph. |

## nodes

MultiGraph.nodes (data=False, default=None)
Returns an iterator over the nodes.

## Parameters

- data (string or bool, optional (default=False)) - The node attribute returned in 2-tuple ( n , ddict[data]). If True, return entire node attribute dict as (n,ddict). If False, return just the nodes n .
- default (value, optional (default=None)) - Value used for nodes that dont have the requested attribute. Only relevant if data is not True or False.

Returns An iterator over nodes, or ( $\mathrm{n}, \mathrm{d}$ ) tuples of node with data. If data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

## Return type iterator

## Notes

If the node data is not required, it is simpler and equivalent to use the expression for $n$ in $G$, or list ( $G$ ).

## Examples

There are two simple ways of getting a list of all nodes in the graph:

```
>>> G = nx.path_graph(3)
>>> list(G.nodes())
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```
>>> G.add_node(1, time='5pm')
>>> G.node[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

$\qquad$

MultiGraph.__iter__()
Iterate over the nodes. Use the expression 'for n in G '.
Returns niter - An iterator over all nodes in the graph.
Return type iterator

Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G]
[0, 1, 2, 3]
```


## edges

MultiGraph.edges (nbunch=None, data=False, keys=False, default=None)
Return an iterator over the edges.
Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.

Returns edge - An iterator over (u,v), (u,v,d) or (u,v,key,d) tuples of edges.
Return type iterator

## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add__edge (2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges(keys=True)) # default keys are integers
[(0, 1, 0), (1, 2, 0), (2, 3, 0)]
>>> list(G.edges(data=True,keys=True)) # default keys are integers
[(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})]
>>> list(G.edges(data='weight',default=1,keys=True))
[(0, 1, 0, 1), (1, 2, 0, 1), (2, 3, 0, 5)]
>>> list(G.edges([0,3]))
[(0, 1), (3, 2)]
>>> list(G.edges(0))
[(0, 1)]
```

get_edge_data

MultiGraph.get_edge_data ( $u, v$, key=None, default=None)
Return the attribute dictionary associated with edge ( $u, v$ ).

## Parameters

- u, $\mathbf{v}$ (nodes)
- default (any Python object (default=None)) - Value to return if the edge ( $u, v$ ) is not found.
- key (hashable identifier, optional (default=None)) - Return data only for the edge with specified key.

Returns edge_dict - The edge attribute dictionary.
Return type dictionary

## Notes

It is faster to use $G[u][v][k e y]$.

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> key = G.add_edge(0,1,key='a',weight=7)
>>> G[0][1]['a'] # key='a'
{'weight': 7}
```

Warning: Assigning $G[u][v][k e y]$ corrupts the graph data structure. But it is safe to assign attributes to that dictionary,

```
>>> G[0][1]['a']['weight'] = 10
>>> G[0][1]['a']['weight']
10
```

```
>>> G[1][0]['a']['weight']
10
```


## Examples

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.get_edge_data (0,1)
{0: {}}
>>> e = (0,1)
>>> G.get_edge_data(*e) # tuple form
{0: {}}
>>> G.get_edge_data('a','b',default=0) # edge not in graph, return 0
0
```


## neighbors

MultiGraph.neighbors (n)
Return an iterator over all neighbors of node n .
Parameters $\mathbf{n}$ (node) - A node in the graph
Returns neighbors - An iterator over all neighbors of node n
Return type iterator
Raises NetworkXError - If the node n is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G.neighbors(0)]
[1]
```


## Notes

It is usually more convenient (and faster) to access the adjacency dictionary as $G[n]$ :

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=7)
>>> G['a']
{'b': {'weight': 7} }
>>> G = nx.path_graph(4)
>>> [n for n in G[0]]
[1]
```

__getitem

MultiGraph.__getitem__( $n$ )
Return a dict of neighbors of node $n$. Use the expression ' $\mathrm{G}[\mathrm{n}]$ '.

Parameters $\mathbf{n}$ (node) - A node in the graph.
Returns adj_dict - The adjacency dictionary for nodes connected to n .
Return type dictionary

## Notes

$\mathrm{G}[\mathrm{n}]$ is similar to G.neighbors( n ) but the internal data dictionary is returned instead of an iterator. Assigning $\mathrm{G}[\mathrm{n}]$ will corrupt the internal graph data structure. Use $\mathrm{G}[\mathrm{n}]$ for reading data only.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
{1: {}}
```


## adjacency

## MultiGraph.adjacency()

Return an iterator over (node, adjacency dict) tuples for all nodes.
This is the fastest way to look at every edge. For directed graphs, only outgoing adjacencies are included.
Returns adj_iter - An iterator over (node, adjacency dictionary) for all nodes in the graph.
Return type iterator

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n,nbrdict) for n, nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

```
nbunch_iter
```


## MultiGraph.nbunch_iter(nbunch=None)

Return an iterator over nodes contained in nbunch that are also in the graph.
The nodes in nbunch are checked for membership in the graph and if not are silently ignored.
Parameters nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.

Returns niter - An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.

Return type iterator
Raises NetworkXError - If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

## See also:

Graph._-iter_()

## Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use "if nbunch in self:", even after processing with this routine.
If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.

Information about graph structure

| MultiGraph.has_node(n) | Return True if the graph contains the node n. |
| :--- | :--- |
| MultiGraph.__contains__(n) | Return True if n is a node, False otherwise. |
| MultiGraph.has_edge(u, v[, key]) | Return True if the graph has an edge between nodes u and <br> v. |
| MultiGraph.order() | Return the number of nodes in the graph. |
| MultiGraph.number_of_nodes() | Return the number of nodes in the graph. |
| MultiGraph.__len_() | Return the number of nodes. |
| MultiGraph.degree([nbunch, weight]) | Return an iterator for (node, degree) or degree for single <br> node. |
| MultiGraph.size([weight]) | Return the number of edges or total of all edge weights. |
| MultiGraph.number_of_edges([u, v]) | Return the number of edges between two nodes. |
| MultiGraph.nodes_with_selfloops() | Returns an iterator over nodes with self loops. |
| MultiGraph.selfloop_edges([data, keys, default]) | Return a list of selfloop edges. |
| MultiGraph.number_of_selfloops() | Return the number of selfloop edges. |

## has_node

MultiGraph.has_node ( $n$ )
Return True if the graph contains the node $n$.
Parameters n (node)

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```
>>> 0 in G
True
```

contains

MultiGraph.__contains__(n)
Return True if n is a node, False otherwise. Use the expression ' n in G '.

Examples
>>> G $=$ nx.path_graph(4) \# or DiGraph, MultiGraph, MultiDiGraph, etc
$\ggg 1$ in $G$
True
has_edge

MultiGraph.has_edge $(u, v$, key=None $)$
Return True if the graph has an edge between nodes $u$ and $v$.

## Parameters

- u, v (nodes) - Nodes can be, for example, strings or numbers.
- key (hashable identifier, optional (default=None)) - If specified return True only if the edge with key is found.
Returns edge_ind - True if edge is in the graph, False otherwise.
Return type bool


## Examples

Can be called either using two nodes $u, v$, an edge tuple ( $u, v$ ), or an edge tuple ( $u, v, k e y$ ).

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.has_edge(0,1) # using two nodes
True
>>> e = (0,1)
>>> G.has_edge(*e) # e is a 2-tuple (u,v)
True
>>> G.add_edge(0,1,key='a')
'a'
>>> G.has_edge(0,1,key='a') # specify key
True
>>> e=(0,1,'a')
>>> G.has_edge(*e) # e is a 3-tuple (u,v,'a')
True
```

The following syntax are equivalent:

```
>>> G.has_edge(0,1)
True
>>> 1 in G[0] # though this gives :exc:`KeyError` if 0 not in G
True
```

order

MultiGraph.order()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
number_of_nodes(),__len_()
number_of_nodes

MultiGraph.number_of_nodes()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
order(),__len_()

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
3
```

$\qquad$
$\qquad$

MultiGraph.__len__()
Return the number of nodes. Use the expression 'len(G)'.
Returns nnodes - The number of nodes in the graph.
Return type int

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

degree

MultiGraph.degree (nbunch=None, weight=None)
Return an iterator for (node, degree) or degree for single node.

The node degree is the number of edges adjacent to the node. This function returns the degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - Degree of the node, if a single node is passed as argument.
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, degree).


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.degree(0) # node 0 with degree 1
1
>>> list(G.degree([0,1]))
[(0, 1), (1, 2)]
```

size

MultiGraph.size (weight=None)
Return the number of edges or total of all edge weights.
Parameters weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

## Returns

size - The number of edges or (if weight keyword is provided) the total weight sum.
If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).

Return type numeric

## See also:

```
number_of_edges()
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3
```

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a','b',weight=2)
>>> G.add_edge('b','C',weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```


## number_of_edges

MultiGraph.number_of_edges ( $u=$ None, $v=$ None)
Return the number of edges between two nodes.
Parameters u,v (nodes, optional (default=all edges)) - If $u$ and $v$ are specified, return the number of edges between $u$ and $v$. Otherwise return the total number of all edges.

Returns nedges - The number of edges in the graph. If nodes $u$ and $v$ are specified return the number of edges between those nodes.

Return type int

## See also:

```
size()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.number_of_edges()
3
>>> G.number_of_edges (0,1)
1
>>> e = (0,1)
>>> G.number_of_edges(*e)
1
```


## nodes_with_selfloops

MultiGraph.nodes_with_selfloops()
Returns an iterator over nodes with self loops.
A node with a self loop has an edge with both ends adjacent to that node.
Returns nodelist - A iterator over nodes with self loops.
Return type iterator
See also:

```
selfloop_edges(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1, 1)
>>> G.add_edge(1, 2)
>>> list(G.nodes_with_selfloops())
[1]
```


## selfloop_edges

```
MultiGraph.selfloop_edges(data=False, keys=False,default=None)
```

Return a list of selfloop edges.
A selfloop edge has the same node at both ends.

## Parameters

- data (bool, optional (default=False)) - Return selfloop edges as two tuples (u,v) (data=False) or three-tuples (u,v,datadict) (data=True) or three-tuples (u,v,datavalue) (data='attrname')
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.

Returns edgelist - A list of all selfloop edges.
Return type list of edge tuples

## See also:

```
nodes_with_selfloops(), number_of_selfloops()
```


## Examples

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> G.add_edge (1,1)
0
>>> G.add_edge (1, 2)
0
>>> list(G.selfloop_edges())
[(1, 1)]
>>> list(G.selfloop_edges(data=True))
[(1, 1, {})]
>>> list(G.selfloop_edges(keys=True))
[(1, 1, 0)]
>>> list(G.selfloop_edges(keys=True, data=True))
[(1, 1, 0, {})]
```

number_of_selfloops

MultiGraph.number_of_selfloops()
Return the number of selfloop edges.
A selfloop edge has the same node at both ends.

Returns nloops - The number of selfloops.
Return type int

## See also:

nodes_with_selfloops(), selfloop_edges ()

## Examples

```
>>> G=nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1,1)
>>> G.add_edge (1, 2)
>>> G.number_of_selfloops()
1
```


## Making copies and subgraphs

| MultiGraph.copy([with_data]) | Return a copy of the graph. |
| :--- | :--- |
| MultiGraph.to_undirected() | Return an undirected copy of the graph. |
| MultiGraph.to_directed() | Return a directed representation of the graph. |
| MultiGraph.subgraph(nbunch) | Return the subgraph induced on nodes in nbunch. |
| MultiGraph.edge_subgraph(edges) | Returns the subgraph induced by the specified edges. |

## copy

```
MultiGraph.copy (with_data=True)
```

Return a copy of the graph.
All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy - The default behavior is a "deepcopy" where the graph structure as well as all data attributes and any objects they might contain are copied. The entire graph object is new so that changes in the copy do not affect the original object.

Data Reference (Shallow) - For a shallow copy (with_data=False) the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other.

Independent Shallow - This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This type of copy is not enabled. Instead use:

```
>>> G = nx.path_graph(5)
>>> H = G.__class__(G)
```

Fresh Data- For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges())
```

See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Parameters with_data (bool, optional (default=True)) - If True, the returned graph will have a deep copy of the graph, node, and edge attributes of this object. Otherwise, the returned graph will be a shallow copy.

Returns G-A copy of the graph.
Return type Graph

## See also:

to_directed() return a directed copy of the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```

to_undirected

MultiGraph.to_undirected()
Return an undirected copy of the graph.
Returns G-A deepcopy of the graph.
Return type Graph/MultiGraph

## See also:

```
copy(), add__edge(), add__edges_from()
```


## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{G}=\mathrm{DiGraph}(\mathrm{D})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

## Examples

```
>>> G = nx.path_graph(2) # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges())
[(0, 1)]
```


## to_directed

## MultiGraph.to_directed()

Return a directed representation of the graph.
Returns G - A directed graph with the same name, same nodes, and with each edge (u,v,data) replaced by two directed edges (u,v,data) and (v,u,data).

## Return type MultiDiGraph

## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar $\mathrm{D}=\mathrm{DiGraph}(\mathrm{G})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Warning: If you have subclassed MultiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiDiGraph created by this method.

## Examples

```
>>> G = nx.Graph() # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```
>>> G = nx.DiGraph() # or MultiDiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1)]
```


## subgraph

MultiGraph.subgraph (nbunch)
Return the subgraph induced on nodes in nbunch.
The induced subgraph of the graph contains the nodes in nbunch and the edges between those nodes.
Parameters nbunch (list, iterable) - A container of nodes which will be iterated through once.
Returns G - A subgraph of the graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge or node attributes just point to the original graph. So changes to the node or edge structure will not be reflected in the original graph while changes to the attributes will.

To create a subgraph with its own copy of the edge/node attributes use: nx.Graph(G.subgraph(nbunch))
If edge attributes are containers, a deep copy can be obtained using: G.subgraph(nbunch).copy()
For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from([ $n$ in $G$ if $n$ not in set(nbunch)])

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> H = G.subgraph([0,1,2])
>>> list(H.edges())
[(0, 1), (1, 2)]
```

```
edge_subgraph
```

MultiGraph.edge_subgraph (edges)

Returns the subgraph induced by the specified edges.
The induced subgraph contains each edge in edges and each node incident to any one of those edges.
Parameters edges (iterable) - An iterable of edges in this graph.
Returns $\mathbf{G}$ - An edge-induced subgraph of this graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge, and node attributes in the returned subgraph are references to the corresponding attributes in the original graph. Thus changes to the node or edge structure of the returned graph will not be reflected in the original graph, but changes to the attributes will.

To create a subgraph with its own copy of the edge or node attributes, use:

```
>>> nx.MultiGraph(G.edge_subgraph(edges))
```

If edge attributes are containers, a deep copy of the attributes can be obtained using:

```
>>> G.edge_subgraph(edges).copy()
```


## Examples

Get a subgraph induced by only those edges that have a certain attribute:

```
>>> # Create a graph in which some edges are "good" and some "bad".
>>> G = nx.MultiGraph()
>>> key = G.add_edge(0, 1, key=0, good=True)
>>> key = G.add_edge(0, 1, key=1, good=False)
>>> key = G.add_edge(1, 2, key=0, good=False)
>>> key = G.add_edge(1, 2, key=1, good=True)
>>> # Keep only those edges that are marked as "good".
>>> edges = G.edges(keys=True, data='good')
>>> edges = ((u, v, k) for (u, v, k, good) in edges if good)
>>> H = G.edge_subgraph(edges)
>>> list(H.edges(keys=True, data=True))
[(0, 1, 0, {'good': True}), (1, 2, 1, {'good': True})]
```


### 3.2.7 MultiDiGraph - Directed graphs with self loops and parallel edges

## Overview

MultiDiGraph (data=None, **attr)
A directed graph class that can store multiedges.
Multiedges are multiple edges between two nodes. Each edge can hold optional data or attributes.
A MultiDiGraph holds directed edges. Self loops are allowed.
Nodes can be arbitrary (hashable) Python objects with optional key/value attributes.
Edges are represented as links between nodes with optional key/value attributes.

## Parameters

- data (input graph) - Data to initialize graph. If data=None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.
- $\operatorname{attr}$ (keyword arguments, optional (default= no attributes)) - Attributes to add to graph as key=value pairs.


## See also:

Graph(), DiGraph(), MultiGraph()

## Examples

Create an empty graph structure (a "null graph") with no nodes and no edges.

```
>>> G = nx.MultiDiGraph()
```

G can be grown in several ways.

## Nodes:

Add one node at a time:

```
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```
>>> G.add_nodes_from([2,3])
>>> G.add_nodes_from(range (100,110))
>>> H=nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```
>>> G.add_node(H)
```


## Edges:

G can also be grown by adding edges.
Add one edge,

```
>>> key = G.add_edge(1, 2)
```

a list of edges,

```
>>> keys = G.add_edges_from([(1,2),(1,3)])
```

or a collection of edges,

```
>>> keys = G.add_edges_from(H.edges())
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. If an edge already exists, an additional edge is created and stored using a key to identify the edge. By default the key is the lowest unused integer.

```
>>> keys = G.add_edges_from([(4,5,dict(route=282)), (4,5,dict(route=37))])
>>> G[4]
{5: {0: {}, 1: {'route': 282}, 2: {'route': 37}}}
```


## Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

```
>>> G = nx.MultiDiGraph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Add node attributes using add_node(), add_nodes_from() or G.node

```
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.node[1]
{'time': '5pm'}
>>> G.node[1]['room'] = 714
>>> del G.node[1]['room'] # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Warning: adding a node to G.node does not add it to the graph.
Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edge.

```
>>> key = G.add_edge(1, 2, weight=4.7 )
>>> keys = G.add_edges_from([(3,4), (4,5)], color='red')
>>> keys = G.add_edges_from([(1,2,{'color':'blue'}), (2,3,{'weight':8})])
>>> G[1][2][0]['weight'] = 4.7
>>> G.edge[1][2][0]['weight'] = 4
```


## Shortcuts:

Many common graph features allow python syntax to speed reporting.

```
>>> 1 in G # check if node in graph
True
>>> [n for n in G if n<3] # iterate through nodes
[1, 2]
>>> len(G) # number of nodes in graph
5
>>> G[1] # adjacency dict keyed by neighbor to edge attributes
... # Note: you should not change this dict manually!
{2: {0: {'weight': 4}, 1: {'color': 'blue'}}}
```

The fastest way to traverse all edges of a graph is via adjacency(), but the edges() method is often more convenient.

```
>>> for n, nbrsdict in G.adjacency():
... for nbr,keydict in nbrsdict.items():
... for key,eattr in keydict.items():
... if 'weight' in eattr:
... (n,nbr,eattr['weight'])
(1, 2, 4)
(2, 3, 8)
>>> list(G.edges(data='weight'))
[(1, 2, 4), (1, 2, None), (2, 3, 8), (3, 4, None), (4, 5, None)]
```


## Reporting:

Simple graph information is obtained using methods. Reporting methods usually return iterators instead of containers to reduce memory usage. Methods exist for reporting nodes(), edges(), neighbors() and degree() as well as the number of nodes and edges.

For details on these and other miscellaneous methods, see below.

## Subclasses (Advanced):

The MultiDiGraph class uses a dict-of-dict-of-dict-of-dict structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge_key dicts keyed by neighbor. The edge_key dict holds each edge_attr dict keyed by edge key. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these four dicts in the dict-of-dict-of-dict-of-dict structure can be replaced by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, adjlist_inner_dict_factory, adjlist_outer_dict_factory, and edge_attr_dict_factory.
node_dict_factory [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object
adjlist_outer_dict_factory [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
adjlist_inner_dict_factory [function, (default: dict)] Factory function to be used to create the adjacency list dict which holds multiedge key dicts keyed by neighbor. It should require no arguments and return a dict-like object.
edge_key_dict_factory [function, (default: dict)] Factory function to be used to create the edge key dict which holds edge data keyed by edge key. It should require no arguments and return a dict-like object.
edge_attr_dict_factory [function, (default: dict)] Factory function to be used to create the edge attribute dict which holds attrbute values keyed by attribute name. It should require no arguments and return a dict-like object.

## Examples

Create a multigraph subclass that tracks the order nodes are added.

```
>>> from collections import OrderedDict
>>> class OrderedGraph(nx.MultiDiGraph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> keys = G.add_edges_from( ( (2,2), (2,1), (2,1), (1,1)) )
>>> list(G.edges())
[(2, 1), (2, 1), (2, 2), (1, 1)]
```

Create a multdigraph object that tracks the order nodes are added and for each node track the order that neighbors are added and for each neighbor tracks the order that multiedges are added.

```
>>> class OrderedGraph(nx.MultiDiGraph):
... node_dict_factory = OrderedDict
... adjlist_outer_dict_factory = OrderedDict
... adjlist_inner_dict_factory = OrderedDict
... edge_key_dict_factory = OrderedDict
>>> G = OrderedGraph()
>>> G.add_nodes_from( (2,1) )
>>> list(G.nodes())
[2, 1]
>>> elist = ( (2, 2), (2,1,2,{'weight':0.1}), (2,1,1,{'weight':0.2}), (1,1))
>>> keys = G.add_edges_from(elist)
>>> list(G.edges(keys=True))
[(2, 2, 0), (2, 1, 2), (2, 1, 1), (1, 1, 0)]
```


### 3.2.8 Methods

## Adding and Removing Nodes and Edges

| MultiDiGraph.__init__([data]) |  |
| :---: | :---: |
| MultiDiGraph.add_node(n, \*\*attr) | Add a single node n and update node attributes. |
| MultiDiGraph.add_nodes_from(nodes, \***attr) | Add multiple nodes. |
| MultiDiGraph.remove_node(n) | Remove node n . |
| MultiDiGraph.remove_nodes_from(nbunch) | Remove multiple nodes. |
|  | Continued on next page |

Table 3.13 - continued from previous page

| MultiDiGraph.add_edge(u, v[, key]) | Add an edge between u and v. |
| :--- | :--- |
| MultiDiGraph.add_edges_from(ebunch, $l^{*} \mid *$ attr) | Add all the edges in ebunch. |
| MultiDiGraph.add_weighted_edges_from(ebunchAdd all the edges in ebunch as weighted edges with speci- |  |
|  | fied weights. |
| MultiDiGraph.new_edge_key(u, v) | Return an unused key for edges between nodes u and v. |
| MultiDiGraph.remove_edge(u, v[, key]) | Remove an edge between u and v. |
| MultiDiGraph.remove_edges_from(ebunch) | Remove all edges specified in ebunch. |
| MultiDiGraph.clear() | Remove all nodes and edges from the graph. |

$\qquad$
init
MultiDiGraph.__init__(data=None, **attr)

## add_node

MultiDiGraph.add_node ( $n$, **attr)
Add a single node n and update node attributes.

## Parameters

- n (node) - A node can be any hashable Python object except None.
- attr (keyword arguments, optional) - Set or change node attributes using key=value.


## See also:

```
    add_nodes_from()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0,1),(1,2),(2,0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:

```
>>> G.add_node(1,size=10)
>>> G.add_node(3,weight=0.4,UTM=('13S',382871,3972649))
```


## Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn't change on mutables.

```
add_nodes_from
```

MultiDiGraph.add_nodes_from (nodes, **attr)
Add multiple nodes.

## Parameters

- nodes (iterable container) - A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- attr (keyword arguments, optional (default= no attributes)) - Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.


## See also:

add_node()

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0,1), (1,2), (2,0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(),key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```
>>> G.add_nodes_from([1,2], size=10)
>>> G.add_nodes_from([3,4], weight=0.4)
```

Use (node, attrdict) tuples to update attributes for specific nodes.

```
>>> G.add_nodes_from([(1,dict(size=11)), (2,{'color':'blue'})])
>>> G.node[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.node[1]['size']
11
```

remove_node

MultiDiGraph. remove_node ( $n$ )
Remove node n .
Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.
Parameters $\mathbf{n}$ (node) - A node in the graph
Raises NetworkXerror - If n is not in the graph.

## See also:

```
remove_nodes_from()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges())
[(0, 1), (1, 2)]
>>> G.remove_node (1)
>>> list(G.edges())
[]
```

remove_nodes_from

MultiDiGraph.remove_nodes_from (nbunch)
Remove multiple nodes.
Parameters nodes (iterable container) - A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

## See also:

```
remove_node()
```


## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes())
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes())
[ ]
```

add_edge

MultiDiGraph.add_edge ( $u, v$, key=None, **attr)
Add an edge between $u$ and $v$.
The nodes $u$ and $v$ will be automatically added if they are not already in the graph.
Edge attributes can be specified with keywords or by directly accessing the edge's attribute dictionary. See examples below.

## Parameters

- u, v (nodes) - Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
- key (hashable identifier, optional (default=lowest unused integer)) - Used to distinguish multiedges between a pair of nodes.
- attr_dict (dictionary, optional (default= no attributes)) - Dictionary of edge attributes. Key/value pairs will update existing data associated with the edge.
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## Returns

Return type The edge key assigned to the edge.

## See also:

add_edges_from () add a collection of edges

## Notes

To replace/update edge data, use the optional key argument to identify a unique edge. Otherwise a new edge will be created.

NetworkX algorithms designed for weighted graphs cannot use multigraphs directly because it is not clear how to handle multiedge weights. Convert to Graph using edge attribute 'weight' to enable weighted graph algorithms.
Default keys are generated using the method new_edge_key (). This method can be overridden by subclassing the base class and providing a custom new_edge_key () method.

## Examples

The following all add the edge $\mathrm{e}=(1,2)$ to graph G :

```
>>> G = nx.MultiDiGraph()
>>> e = (1,2)
>>> key = G.add_edge(1, 2) # explicit two-node form
>>> G.add_edge(*e) # single edge as tuple of two nodes
1
>>> G.add_edges_from( [(1,2)] ) # add edges from iterable container
[2]
```

Associate data to edges using keywords:

```
>>> key = G.add_edge(1, 2, weight=3)
>>> key = G.add_edge(1, 2, key=0, weight=4) # update data for key=0
>>> key = G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string associations, directly access the edge's attribute dictionary.
add_edges_from

MultiDiGraph.add_edges_from (ebunch, **attr)
Add all the edges in ebunch.

## Parameters

- ebunch (container of edges) - Each edge given in the container will be added to the graph. The edges can be:
- 2-tuples (u,v) or
- 3-tuples (u,v,d) for an edge attribute dict d, or
- 4-tuples ( $u, v, k, d$ ) for an edge identified by key $k$
- attr (keyword arguments, optional) - Edge data (or labels or objects) can be assigned using keyword arguments.


## Returns

Return type A list of edge keys assigned to the edges in ebunch.

## See also:

add_edge () add a single edge
add_weighted_edges_from() convenient way to add weighted edges

## Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added.
Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments.
Default keys are generated using the method new_edge_key (). This method can be overridden by subclassing the base class and providing a custom new_edge_key () method.

## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0,1),(1,2)]) # using a list of edge tuples
>>> e = zip(range(0,3),range(1,4))
>>> G.add_edges_from(e) # Add the path graph 0-1-2-3
```

Associate data to edges

```
>>> G.add_edges_from([(1,2), (2,3)], weight=3)
>>> G.add_edges_from([(3,4), (1,4)], label='WN2898')
```

add_weighted_edges_from

MultiDiGraph.add_weighted_edges_from (ebunch, weight='weight', **attr)
Add all the edges in ebunch as weighted edges with specified weights.

## Parameters

- ebunch (container of edges) - Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples ( $u, v, w$ ) where $w$ is a number.
- weight (string, optional (default= 'weight')) - The attribute name for the edge weights to be added.
- attr (keyword arguments, optional (default= no attributes)) - Edge attributes to add/update for all edges.


## See also:

add_edge () add a single edge
add_edges_from() add multiple edges

## Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

Examples
>>> G $=$ nx.Graph() \# or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from ([(0,1,3.0), (1, 2, 7.5)])
new_edge_key

MultiDiGraph.new_edge_key $(u, v)$
Return an unused key for edges between nodes $u$ and $v$.
The nodes $u$ and $v$ do not need to be already in the graph.

## Notes

In the standard MultiGraph class the new key is the number of existing edges between $u$ and $v$ (increased if necessary to ensure unused). The first edge will have key 0 , then 1 , etc. If an edge is removed further new_edge_keys may not be in this order.

Parameters u, v (nodes)
Returns key
Return type int
remove_edge

MultiDiGraph. remove_edge ( $u, v$, key=None)
Remove an edge between $u$ and $v$.

## Parameters

- $\mathbf{u}, \mathbf{v}$ (nodes) - Remove an edge between nodes $u$ and $v$.
- key (hashable identifier, optional (default=None)) - Used to distinguish multiple edges between a pair of nodes. If None remove a single (arbitrary) edge between $u$ and $v$.

Raises NetworkXError - If there is not an edge between $u$ and $v$, or if there is no edge with the specified key.
See also:
remove_edges_from () remove a collection of edges

Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge (0,1)
>>> e = (1,2)
>>> G.remove_edge(*e) # unpacks e from an edge tuple
```

For multiple edges

```
>>> G = nx.MultiDiGraph()
>>> G.add_edges_from([(1,2),(1,2),(1,2)]) # key_list returned
[0, 1, 2]
>>> G.remove_edge(1,2) # remove a single (arbitrary) edge
```

For edges with keys

```
>>> G = nx.MultiDiGraph()
>>> G.add_edge(1,2,key='first')
'first'
>>> G.add_edge(1,2,key='second')
'second'
>>> G.remove_edge(1,2,key='second')
```


## remove_edges_from

MultiDiGraph.remove_edges_from (ebunch)
Remove all edges specified in ebunch.
Parameters ebunch (list or container of edge tuples) - Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples ( $u, v$ ) All edges between $u$ and $v$ are removed.
- 3-tuples (u,v,key) The edge identified by key is removed.
- 4-tuples (u,v,key,data) where data is ignored.


## See also:

remove_edge () remove a single edge

## Notes

Will fail silently if an edge in ebunch is not in the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1,2), (2,3)]
>>> G.remove_edges_from(ebunch)
```

Removing multiple copies of edges

```
>>> G = nx.MultiGraph()
>>> keys = G.add_edges_from([(1,2),(1,2),(1, 2)])
>>> G.remove_edges_from([(1,2), (1,2)])
>>> list(G.edges())
[(1, 2)]
>>> G.remove_edges_from([(1,2), (1,2)]) # silently ignore extra copy
>>> list(G.edges()) # now empty graph
[ ]
```

clear

MultiDiGraph.clear()
Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes())
[]
>>> list(G.edges())
[]
```


## Iterating over nodes and edges

| MultiDiGraph._nodes([data, default]) | Returns an iterator over the nodes. |
| :--- | :--- |
| MultiDiGraph._iter_() | Iterate over the nodes. |
| MultiDiGraph.edges([nbunch, data, keys, default]) | Return an iterator over the edges. |
| MultiDiGraph.out_edges([nbunch, data, keys, ..]) | Return an iterator over the edges. |
| MultiDiGraph.in_edges([nbunch, data, keys, ..]) | Return an iterator over the incoming edges. |
| MultiDiGraph.get_edge_data(u, v[, key, default]) | Return the attribute dictionary associated with edge (u,v). |
| MultiDiGraph.neighbors(n) | Return an iterator over successor nodes of n. |
| MultiDiGraph.__getitem_(n) | Return a dict of neighbors of node n. |
| MultiDiGraph.successors(n) | Return an iterator over successor nodes of n. |
| MultiDiGraph.predecessors(n) | Return an iterator over predecessor nodes of n. |
| MultiDiGraph.adjacency() | Return an iterator over (node, adjacency dict) tuples for all <br> nodes. |
| MultiDiGraph.nbunch_iter([nbunch]) | Return an iterator over nodes contained in nbunch that are <br> also in the graph. |

## nodes

MultiDiGraph.nodes (data=False, default=None)
Returns an iterator over the nodes.

## Parameters

- data (string or bool, optional (default=False)) - The node attribute returned in 2-tuple
( n , ddict[data]). If True, return entire node attribute dict as ( n , ddict). If False, return just the nodes $n$.
- default (value, optional (default=None)) - Value used for nodes that dont have the requested attribute. Only relevant if data is not True or False.

Returns An iterator over nodes, or ( $\mathrm{n}, \mathrm{d}$ ) tuples of node with data. If data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

## Return type iterator

## Notes

If the node data is not required, it is simpler and equivalent to use the expression for $n$ in $G$, or list ( $G$ ).

## Examples

There are two simple ways of getting a list of all nodes in the graph:

```
>>> G = nx.path_graph(3)
>>> list (G.nodes())
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```
>>> G.add_node(1, time='5pm')
>>> G.node[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

$\qquad$
iter

MultiDiGraph.__iter__()
Iterate over the nodes. Use the expression 'for n in G '.

Returns niter - An iterator over all nodes in the graph.
Return type iterator

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>> [n for }n\mathrm{ in G]
[0, 1, 2, 3]
```

edges

MultiDiGraph.edges (nbunch=None, data=False, keys=False, default=None)
Return an iterator over the edges.
Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns edge - An iterator over (u,v), (u,v,d) or (u,v,key,d) tuples of edges.
Return type iterator

## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add_edge(2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges(keys=True)) # default keys are integers
[(0, 1, 0), (1, 2, 0), (2, 3, 0)]
>>> list(G.edges(data=True,keys=True)) # default keys are integers
[(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})]
>>> list(G.edges(data='weight',default=1,keys=True))
```

```
[(0, 1, 0, 1), (1, 2, 0, 1), (2, 3, 0, 5)]
>>> list(G.edges([0,2]))
[(0, 1), (2, 3)]
>>> list(G.edges(0))
[(0, 1)]
```


## See also:

```
in_edges(),out_edges()
```


## out_edges

MultiDiGraph.out_edges (nbunch=None, data=False, keys=False, default=None)
Return an iterator over the edges.
Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.
Returns edge - An iterator over (u,v), (u,v,d) or (u,v,key,d) tuples of edges.
Return type iterator


## Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

## Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add_edge(2,3,weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True)) # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges(keys=True)) # default keys are integers
[(0, 1, 0), (1, 2, 0), (2, 3, 0)]
>>> list(G.edges(data=True,keys=True)) # default keys are integers
[(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})]
>>> list(G.edges(data='weight',default=1,keys=True))
[(0, 1, 0, 1), (1, 2, 0, 1), (2, 3, 0, 5)]
```

```
>>> list(G.edges([0,2]))
[(0, 1), (2, 3)]
>>> list(G.edges(0))
[(0, 1)]
```

See also:

```
in_edges(),out_edges()
```

in_edges

MultiDiGraph.in_edges $($ nbunch=None, data=False, keys=False, default=None)
Return an iterator over the incoming edges.

## Parameters

- nbunch (iterable container, optional (default= all nodes)) - A container of nodes. The container will be iterated through once.
- data (string or bool, optional (default=False)) - The edge attribute returned in 3-tuple (u,v,ddict[data]). If True, return edge attribute dict in 3-tuple (u,v,ddict). If False, return 2-tuple (u,v).
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.

Returns in_edge - An iterator over (u,v), (u,v,d) or (u,v,key,d) tuples of edges.
Return type iterator
See also:
edges () return an iterator over edges
get_edge_data

MultiDiGraph.get_edge_data ( $u, v, k e y=N o n e$, default=None)
Return the attribute dictionary associated with edge ( $u, v$ ).

## Parameters

- u, $\mathbf{v}$ (nodes)
- default (any Python object (default=None)) - Value to return if the edge (u,v) is not found.
- key (hashable identifier, optional (default=None)) - Return data only for the edge with specified key.

Returns edge_dict - The edge attribute dictionary.
Return type dictionary

## Notes

It is faster to use $\mathrm{G}[\mathrm{u}][\mathrm{v}][\mathrm{key}]$.

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> key = G.add_edge(0,1,key='a',weight=7)
>>> G[0][1]['a'] # key='a'
{'weight': 7}
```

Warning: Assigning $G[u][v][k e y]$ corrupts the graph data structure. But it is safe to assign attributes to that dictionary,

```
>>> G[0][1]['a']['weight'] = 10
>>> G[0][1]['a']['weight']
10
>>> G[1][0]['a']['weight']
10
```


## Examples

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.get_edge_data (0,1)
{0: {}}
>>> e = (0,1)
>>> G.get_edge_data(*e) # tuple form
{0: {}}
>>> G.get_edge_data('a','b',default=0) # edge not in graph, return 0
0
```


## neighbors

MultiDiGraph.neighbors (n)
Return an iterator over successor nodes of $n$.
neighbors() and successors() are the same.

## getitem

MultiDiGraph.__getitem__( $n$ )
Return a dict of neighbors of node $n$. Use the expression ' $\mathrm{G}[\mathrm{n}]$ '.
Parameters $\mathbf{n}$ (node) - A node in the graph.
Returns adj_dict - The adjacency dictionary for nodes connected to n .
Return type dictionary

## Notes

$\mathrm{G}[\mathrm{n}]$ is similar to G.neighbors(n) but the internal data dictionary is returned instead of an iterator.
Assigning $\mathrm{G}[\mathrm{n}]$ will corrupt the internal graph data structure. Use $\mathrm{G}[\mathrm{n}]$ for reading data only.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
{1: {}}
```


## successors

MultiDiGraph.successors (n)
Return an iterator over successor nodes of $n$.
neighbors() and successors() are the same.

```
predecessors
```

```
MultiDiGraph.predecessors(n)
```

Return an iterator over predecessor nodes of n .

## adjacency

MultiDiGraph.adjacency()
Return an iterator over (node, adjacency dict) tuples for all nodes.
This is the fastest way to look at every edge. For directed graphs, only outgoing adjacencies are included.
Returns adj_iter - An iterator over (node, adjacency dictionary) for all nodes in the graph.
Return type iterator

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
```

$\ggg$ [(n, nbrdict) for $n, n b r d i c t$ in G.adjacency()]
$[(0,\{1:\{ \}\}),(1,\{0:\{ \}, 2:\{ \}\}),(2,\{1:\{ \}, 3:\{ \}\}),(3,\{2:\{ \}\})]$
nbunch_iter

MultiDiGraph.nbunch_iter (nbunch=None)
Return an iterator over nodes contained in nbunch that are also in the graph.
The nodes in nbunch are checked for membership in the graph and if not are silently ignored.
Parameters nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.

Returns niter - An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.

Return type iterator
Raises NetworkXError - If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

## See also:

Graph.__iter__()

## Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use "if nbunch in self:", even after processing with this routine.
If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.

Information about graph structure

| MultiDiGraph.has_node(n) | Return True if the graph contains the node n. |
| :--- | :--- |
| MultiDiGraph.__contains__(n) | Return True if n is a node, False otherwise. |
| MultiDiGraph.has_edge(u, v[, key]) | Return True if the graph has an edge between nodes u and <br> v. |
| MultiDiGraph.order() | Return the number of nodes in the graph. |
| MultiDiGraph.number_of_nodes() | Return the number of nodes in the graph. |
| MultiDiGraph.__len__() | Return the number of nodes. |
| MultiDiGraph.degree([nbunch, weight]) | Return an iterator for (node, degree) or degree for single <br> node. |
| MultiDiGraph.in_degree([nbunch, weight]) | Return an iterator for (node, in-degree) or in-degree for sin- <br> gle node. |
| MultiDiGraph.out_degree([nbunch, weight]) | Return an iterator for (node, out-degree) or out-degree for <br> single node. |
| MultiDiGraph.size([weight]) | Return the number of edges or total of all edge weights. |
| MultiDiGraph.number_of_edges([u, v]) | Return the number of edges between two nodes. |
| MultiDiGraph.nodes_with_selfloops() | Returns an iterator over nodes with self loops. |
| MultiDiGraph.selfloop_edges([data, keys, ...]) | Return a list of selfloop edges. |
| MultiDiGraph.number_of_selfloops() | Return the number of selfloop edges. |

has_node

MultiDiGraph.has_node ( $n$ )
Return True if the graph contains the node $n$.
Parameters n(node)

## Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```
>>> 0 in G
True
```


## __contains

MultiDiGraph.__contains__( $n$ )
Return True if n is a node, False otherwise. Use the expression ' n in G '.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

has_edge
MultiDiGraph.has_edge ( $u, v$, key=None)

Return True if the graph has an edge between nodes $u$ and $v$.

## Parameters

- u, v (nodes) - Nodes can be, for example, strings or numbers.
- key (hashable identifier, optional (default=None)) - If specified return True only if the edge with key is found.

Returns edge_ind - True if edge is in the graph, False otherwise.
Return type bool

## Examples

Can be called either using two nodes $u, v$, an edge tuple ( $u, v$ ), or an edge tuple ( $u, v, k e y$ ).

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.has_edge(0,1) # using two nodes
True
>>> e = (0,1)
>>> G.has_edge(*e) # e is a 2-tuple (u,v)
True
>>> G.add_edge(0,1,key='a')
'a'
>>> G.has_edge(0,1,key='a') # specify key
True
>>> e=(0,1,'a')
>>> G.has_edge(*e) # e is a 3-tuple (u,v,'a')
True
```

The following syntax are equivalent:

```
>>> G.has_edge (0,1)
True
>>> 1 in G[0] # though this gives :exc:`KeyError` if 0 not in G
True
```

order

MultiDiGraph.order()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
number_of_nodes(),__len__()
number_of_nodes

MultiDiGraph.number_of_nodes()
Return the number of nodes in the graph.
Returns nnodes - The number of nodes in the graph.
Return type int
See also:
order(),__len__()

Examples

```
>>> G = nx.path_graph(3) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
3
```

_len $\qquad$

MultiDiGraph.__len__()
Return the number of nodes. Use the expression 'len(G)'.
Returns nnodes - The number of nodes in the graph.
Return type int

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

degree

MultiDiGraph. degree (nbunch=None, weight=None)
Return an iterator for (node, degree) or degree for single node.
The node degree is the number of edges adjacent to the node. This function returns the degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights.


## Returns

- If a single nodes is requested
- deg (int) - Degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, degree).


## See also:

out_degree(), in_degree()

## Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.degree(0) # node 0 with degree 1
1
>>> list(G.degree([0,1]))
[(0, 1), (1, 2)]
```


## in_degree

MultiDiGraph.in_degree (nbunch=None, weight=None)
Return an iterator for (node, in-degree) or in-degree for single node.
The node in-degree is the number of edges pointing in to the node. This function returns the in-degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.


## Returns

- If a single node is requested
- deg (int) - Degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, in-degree).


## See also:

degree(), out_degree()

Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.in_degree(0) # node 0 with degree 0
0
>>> list(G.in_degree([0,1]))
[(0, 0), (1, 1)]
```


## out_degree

MultiDiGraph.out_degree (nbunch=None, weight=None)
Return an iterator for (node, out-degree) or out-degree for single node.
The node out-degree is the number of edges pointing out of the node. This function returns the out-degree for a single node or an iterator for a bunch of nodes or if nothing is passed as argument.

## Parameters

- nbunch (iterable container, optional (default=all nodes)) - A container of nodes. The container will be iterated through once.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights.


## Returns

- If a single node is requested
- deg (int) - Degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) - The iterator returns two-tuples of (node, out-degree).


## See also:

```
degree(),in_degree()
```


## Examples

```
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.out_degree(0) # node 0 with degree 1
1
>>> list(G.out_degree([0,1]))
[(0, 1), (1, 1)]
```

size

MultiDiGraph.size (weight=None)
Return the number of edges or total of all edge weights.
Parameters weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

## Returns

size - The number of edges or (if weight keyword is provided) the total weight sum.
If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).
Return type numeric

## See also:

```
number_of_edges()
```


## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3
```

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a','b',weight=2)
>>> G.add_edge('b','c',weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```

number_of_edges

MultiDiGraph.number_of_edges ( $u=$ None, $v=$ None)
Return the number of edges between two nodes.
Parameters $\mathbf{u}, \mathbf{v}$ (nodes, optional (default=all edges)) - If $\mathbf{u}$ and v are specified, return the number of edges between $u$ and $v$. Otherwise return the total number of all edges.
Returns nedges - The number of edges in the graph. If nodes $u$ and $v$ are specified return the number of edges between those nodes.
Return type int

## See also:

size()

Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.number_of_edges()
3
>>> G.number_of_edges (0,1)
1
>>> e = (0,1)
>>> G.number_of_edges(*e)
1
```


## nodes_with_selfloops

MultiDiGraph.nodes_with_selfloops()
Returns an iterator over nodes with self loops.
A node with a self loop has an edge with both ends adjacent to that node.
Returns nodelist - A iterator over nodes with self loops.
Return type iterator

## See also:

```
selfloop_edges(), number_of_selfloops()
```


## Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge(1, 1)
>>> G.add_edge(1, 2)
>>> list(G.nodes_with_selfloops())
[1]
```

selfloop_edges

MultiDiGraph.selfloop_edges (data=False, keys=False, default=None)
Return a list of selfloop edges.
A selfloop edge has the same node at both ends.

## Parameters

- data (bool, optional (default=False)) - Return selfloop edges as two tuples (u,v) (data=False) or three-tuples (u,v,datadict) (data=True) or three-tuples (u,v,datavalue) (data='attrname')
- default (value, optional (default=None)) - Value used for edges that dont have the requested attribute. Only relevant if data is not True or False.
- keys (bool, optional (default=False)) - If True, return edge keys with each edge.

Returns edgelist - A list of all selfloop edges.
Return type list of edge tuples

## See also:

```
nodes_with_selfloops(), number_of_selfloops()
```


## Examples

```
>>> G = nx.MultiGraph() # or MultiDiGraph
>>> G.add_edge (1,1)
0
>>> G.add_edge (1, 2)
0
>>> list(G.selfloop_edges())
[(1, 1)]
>>> list(G.selfloop_edges(data=True))
[(1, 1, {})]
>>> list(G.selfloop_edges(keys=True))
[(1, 1, 0)]
>>> list(G.selfloop_edges(keys=True, data=True))
[(1, 1, 0, {})]
```


## number_of_selfloops

MultiDiGraph.number_of_selfloops()
Return the number of selfloop edges.
A selfloop edge has the same node at both ends.
Returns nloops - The number of selfloops.
Return type int

## See also:

```
nodes_with_selfloops(),selfloop_edges()
```


## Examples

```
>>> G=nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge (1,1)
>>> G.add__edge (1, 2)
>>> G.number_of_selfloops()
1
```


## Making copies and subgraphs

| MultiDiGraph.copy([with_data]) | Return a copy of the graph. |
| :--- | :--- |
| MultiDiGraph.to_undirected([reciprocal]) | Return an undirected representation of the digraph. |
| MultiDiGraph.to_directed() | Return a directed copy of the graph. |
| MultiDiGraph.edge_subgraph(edges) | Returns the subgraph induced by the specified edges. |
| MultiDiGraph.subgraph(nbunch) | Return the subgraph induced on nodes in nbunch. |
| MultiDiGraph.reverse([copy]) | Return the reverse of the graph. |

## copy

MultiDiGraph.copy (with_data=True)
Return a copy of the graph.
All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy - The default behavior is a "deepcopy" where the graph structure as well as all data attributes and any objects they might contain are copied. The entire graph object is new so that changes in the copy do not affect the original object.

Data Reference (Shallow) - For a shallow copy (with_data=False) the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other.

Independent Shallow - This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This type of copy is not enabled. Instead use:

```
>>> G = nx.path_graph(5)
>>> H = G.__class__(G)
```

Fresh Data- For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges())
```

See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

Parameters with_data (bool, optional (default=True)) - If True, the returned graph will have a deep copy of the graph, node, and edge attributes of this object. Otherwise, the returned graph will be a shallow copy.

Returns G-A copy of the graph.
Return type Graph

## See also:

to_directed() return a directed copy of the graph.

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```


## to_undirected

MultiDiGraph.to_undirected (reciprocal=False)
Return an undirected representation of the digraph.

Parameters reciprocal (bool (optional)) - If True only keep edges that appear in both directions in the original digraph.
Returns G - An undirected graph with the same name and nodes and with edge (u,v,data) if either ( $u, v$, data) or ( $v, u, d a t a)$ is in the digraph. If both edges exist in digraph and their edge data is different, only one edge is created with an arbitrary choice of which edge data to use. You must check and correct for this manually if desired.

## Return type MultiGraph

## Notes

This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.
This is in contrast to the similar $\mathrm{D}=\mathrm{DiGraph}(\mathrm{G})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.
Warning: If you have subclassed MultiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiDiGraph created by this method.

## to_directed

MultiDiGraph.to_directed()
Return a directed copy of the graph.
Returns G-A deepcopy of the graph.
Return type MultiDiGraph

## Notes

If edges in both directions ( $u, v$ ) and ( $v, u$ ) exist in the graph, attributes for the new undirected edge will be a combination of the attributes of the directed edges. The edge data is updated in the (arbitrary) order that the edges are encountered. For more customized control of the edge attributes use add_edge().
This returns a "deepcopy" of the edge, node, and graph attributes which attempts to completely copy all of the data and references.
This is in contrast to the similar $\mathrm{G}=\mathrm{DiGraph}(\mathrm{D})$ which returns a shallow copy of the data.
See the Python copy module for more information on shallow and deep copies, http://docs.python.org/library/ copy.html.

## Examples

```
>>> G = nx.Graph() # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```
>>> G = nx.MultiDiGraph()
>>> key = G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges())
[(0, 1)]
```


## edge_subgraph

MultiDiGraph.edge_subgraph (edges)
Returns the subgraph induced by the specified edges.
The induced subgraph contains each edge in edges and each node incident to any one of those edges.
Parameters edges (iterable) - An iterable of edges in this graph.
Returns $\mathbf{G}$ - An edge-induced subgraph of this graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge, and node attributes in the returned subgraph are references to the corresponding attributes in the original graph. Thus changes to the node or edge structure of the returned graph will not be reflected in the original graph, but changes to the attributes will.

To create a subgraph with its own copy of the edge or node attributes, use:

```
>>> nx.MultiDiGraph(G.edge_subgraph(edges))
```

If edge attributes are containers, a deep copy of the attributes can be obtained using:

```
>>> G.edge_subgraph(edges).copy()
```


## Examples

Get a subgraph induced by only those edges that have a certain attribute:

```
>>> # Create a graph in which some edges are "good" and some "bad".
>>> G = nx.MultiDiGraph()
>>> key = G.add_edge(0, 1, key=0, good=True)
>>> key = G.add_edge(0, 1, key=1, good=False)
>>> key = G.add_edge(1, 2, key=0, good=False)
>>> key = G.add_edge(1, 2, key=1, good=True)
>>> # Keep only those edges that are marked as "good".
>>> edges = G.edges(keys=True, data='good')
>>> edges = ((u, v, k) for (u, v, k, good) in edges if good)
>>> H = G.edge_subgraph(edges)
>>> list(H.edges(keys=True, data=True))
[(0, 1, 0, {'good': True}), (1, 2, 1, {'good': True})]
```


## subgraph

MultiDiGraph.subgraph (nbunch)
Return the subgraph induced on nodes in nbunch.
The induced subgraph of the graph contains the nodes in nbunch and the edges between those nodes.
Parameters nbunch (list, iterable) - A container of nodes which will be iterated through once.
Returns $\mathbf{G}$ - A subgraph of the graph with the same edge attributes.
Return type Graph

## Notes

The graph, edge or node attributes just point to the original graph. So changes to the node or edge structure will not be reflected in the original graph while changes to the attributes will.

To create a subgraph with its own copy of the edge/node attributes use: nx.Graph(G.subgraph(nbunch))
If edge attributes are containers, a deep copy can be obtained using: G.subgraph(nbunch).copy()
For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from([ $n$ in $G$ if $n$ not in set(nbunch)])

## Examples

```
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0,1,2])
>>> list(H.edges())
[(0, 1), (1, 2)]
```


## reverse

## MultiDiGraph.reverse (copy=True)

Return the reverse of the graph.
The reverse is a graph with the same nodes and edges but with the directions of the edges reversed.
Parameters copy (bool optional (default=True)) - If True, return a new DiGraph holding the reversed edges. If False, reverse the reverse graph is created using the original graph (this changes the original graph).

## Algorithms

### 4.1 Approximation

Warning: The approximation submodule is not imported automatically with networkx.

Approximate algorithms can be imported with from networkx.algorithms import approximation.

### 4.1.1 Connectivity

Fast approximation for node connectivity
all_pairs_node_connectivity(G[, nbunch, cut- Compute node connectivity between all pairs of nodes. off])
local_node_connectivity(G, source, target[, ...]) Compute node connectivity between source and target. node_connectivity $(\mathrm{G}[, \mathrm{s}, \mathrm{t}]) \quad$ Returns an approximation for node connectivity for a graph or digraph G.

```
all_pairs_node_connectivity
```

all_pairs_node_connectivity (G, nbunch=None, cutoff=None)
Compute node connectivity between all pairs of nodes.
Pairwise or local node connectivity between two distinct and nonadjacent nodes is the minimum number of nodes that must be removed (minimum separating cutset) to disconnect them. By Menger's theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target). Which is what we compute in this function.

This algorithm is a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes ${ }^{1}$. It works for both directed and undirected graphs.

## Parameters

- G (NetworkX graph)
- nbunch (container) - Container of nodes. If provided node connectivity will be computed only over pairs of nodes in nbunch.

[^1]- cutoff (integer) - Maximum node connectivity to consider. If None, the minimum degree of source or target is used as a cutoff in each pair of nodes. Default value None.
Returns K - Dictionary, keyed by source and target, of pairwise node connectivity
Return type dictionary


## See also:

local_node_connectivity(), all_pairs_node_connectivity()

## References

## local_node_connectivity

local_node_connectivity ( $G$, source, target, cutoff=None)
Compute node connectivity between source and target.
Pairwise or local node connectivity between two distinct and nonadjacent nodes is the minimum number of nodes that must be removed (minimum separating cutset) to disconnect them. By Menger's theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target). Which is what we compute in this function.

This algorithm is a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes ${ }^{1}$. It works for both directed and undirected graphs.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for node connectivity
- target (node) - Ending node for node connectivity
- cutoff (integer) - Maximum node connectivity to consider. If None, the minimum degree of source or target is used as a cutoff. Default value None.

Returns $\mathbf{k}$ - pairwise node connectivity
Return type integer

## Examples

```
>>> # Platonic icosahedral graph has node connectivity 5
>>> # for each non adjacent node pair
>>> from networkx.algorithms import approximation as approx
>>> G = nx.icosahedral_graph()
>>> approx.local_node_connectivity(G, 0, 6)
5
```


## Notes

This algorithm ${ }^{1}$ finds node independents paths between two nodes by computing their shortest path using BFS, marking the nodes of the path found as 'used' and then searching other shortest paths excluding the nodes marked as used until no more paths exist. It is not exact because a shortest path could use nodes that, if the path

[^2]were longer, may belong to two different node independent paths. Thus it only guarantees an strict lower bound on node connectivity.
Note that the authors propose a further refinement, losing accuracy and gaining speed, which is not implemented yet.

## See also:

all_pairs_node_connectivity(), node_connectivity()

## References

## node_connectivity

node_connectivity ( $G, s=$ None, $t=$ None)
Returns an approximation for node connectivity for a graph or digraph G.
Node connectivity is equal to the minimum number of nodes that must be removed to disconnect G or render it trivial. By Menger's theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target).

If source and target nodes are provided, this function returns the local node connectivity: the minimum number of nodes that must be removed to break all paths from source to target in G.

This algorithm is based on a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes ${ }^{1}$. It works for both directed and undirected graphs.

## Parameters

- G (NetworkX graph) - Undirected graph
- s (node) - Source node. Optional. Default value: None.
- t (node) - Target node. Optional. Default value: None.

Returns K - Node connectivity of G, or local node connectivity if source and target are provided.
Return type integer

## Examples

```
>>> # Platonic icosahedral graph is 5-node-connected
>>> from networkx.algorithms import approximation as approx
>>> G = nx.icosahedral_graph()
>>> approx.node_connectivity(G)
5
```


## Notes

This algorithm ${ }^{1}$ finds node independents paths between two nodes by computing their shortest path using BFS, marking the nodes of the path found as 'used' and then searching other shortest paths excluding the nodes marked as used until no more paths exist. It is not exact because a shortest path could use nodes that, if the path were longer, may belong to two different node independent paths. Thus it only guarantees an strict lower bound on node connectivity.

[^3]
## See also:

```
all_pairs_node_connectivity(),local_node_connectivity()
```


## References

### 4.1.2 K-components

Fast approximation for k-component structure
$\qquad$
k_components(G[, min_density]) Returns the approximate k-component structure of a graph G.

## k_components

$\mathbf{k}$ _components ( $G$, min_density=0.95)
Returns the approximate k-component structure of a graph G.
A k -component is a maximal subgraph of a graph G that has, at least, node connectivity k : we need to remove at least $k$ nodes to break it into more components. $k$-components have an inherent hierarchical structure because they are nested in terms of connectivity: a connected graph can contain several 2-components, each of which can contain one or more 3-components, and so forth.

This implementation is based on the fast heuristics to approximate the k-component sturcture of a graph ${ }^{1}$. Which, in turn, it is based on a fast approximation algorithm for finding good lower bounds of the number of node independent paths between two nodes ${ }^{2}$.

## Parameters

- G (NetworkX graph) - Undirected graph
- min_density (Float) - Density relaxation treshold. Default value 0.95

Returns k_components - Dictionary with connectivity level k as key and a list of sets of nodes that form a k -component of level k as values.

Return type dict

## Examples

```
>>> # Petersen graph has 10 nodes and it is triconnected, thus all
>>> # nodes are in a single component on all three connectivity levels
>>> from networkx.algorithms import approximation as apxa
>>> G = nx.petersen_graph()
>>> k_components = apxa.k_components(G)
```


## Notes

The logic of the approximation algorithm for computing the k -component structure ${ }^{1}$ is based on repeatedly applying simple and fast algorithms for k -cores and biconnected components in order to narrow down the

[^4]number of pairs of nodes over which we have to compute White and Newman's approximation algorithm for finding node independent paths ${ }^{2}$. More formally, this algorithm is based on Whitney's theorem, which states an inclusion relation among node connectivity, edge connectivity, and minimum degree for any graph G. This theorem implies that every k -component is nested inside a k -edge-component, which in turn, is contained in a k -core. Thus, this algorithm computes node independent paths among pairs of nodes in each biconnected part of each $k$-core, and repeats this procedure for each $k$ from 3 to the maximal core number of a node in the input graph.

Because, in practice, many nodes of the core of level $k$ inside a bicomponent actually are part of a component of level k , the auxiliary graph needed for the algorithm is likely to be very dense. Thus, we use a complement graph data structure (see AntiGraph) to save memory. AntiGraph only stores information of the edges that are not present in the actual auxiliary graph. When applying algorithms to this complement graph data structure, it behaves as if it were the dense version.

## See also:

```
k_components()
```


## References

### 4.1.3 Clique

Cliques.

| max_clique $(\mathrm{G})$ | Find the Maximum Clique |
| :--- | :--- |
| clique_removal $(\mathrm{G})$ | Repeatedly remove cliques from the graph. |

## max_clique

max_clique ( $G$ )
Find the Maximum Clique
Finds the $O\left(|V| /(\log |V|)^{\wedge} 2\right)$ apx of maximum clique/independent set in the worst case.
Parameters G (NetworkX graph) - Undirected graph
Returns clique - The apx-maximum clique of the graph
Return type set

## Notes

A clique in an undirected graph $G=(V, E)$ is a subset of the vertex set $C$ subseteq $V$, such that for every two vertices in C , there exists an edge connecting the two. This is equivalent to saying that the subgraph induced by C is complete (in some cases, the term clique may also refer to the subgraph).
A maximum clique is a clique of the largest possible size in a given graph. The clique number omega ( G ) of a graph $G$ is the number of vertices in a maximum clique in $G$. The intersection number of $G$ is the smallest number of cliques that together cover all edges of $G$.
http://en.wikipedia.org/wiki/Maximum_clique

## References

```
clique_removal
```

```
clique_removal(G)
```

Repeatedly remove cliques from the graph.
Results in a $O\left(|\mathrm{~V}| /(\log |\mathrm{V}|)^{\wedge} 2\right)$ approximation of maximum clique \& independent set. Returns the largest independent set found, along with found maximal cliques.

Parameters G (NetworkX graph) - Undirected graph
Returns max_ind_cliques - Maximal independent set and list of maximal cliques (sets) in the graph.
Return type (set, list) tuple

## References

### 4.1.4 Clustering

average_clustering(G[, trials]) Estimates the average clustering coefficient of G.

## average_clustering

average_clustering ( $G$, trials=1000)
Estimates the average clustering coefficient of G.
The local clustering of each node in G is the fraction of triangles that actually exist over all possible triangles in its neighborhood. The average clustering coefficient of a graph G is the mean of local clusterings.

This function finds an approximate average clustering coefficient for $G$ by repeating $n$ times (defined in trials) the following experiment: choose a node at random, choose two of its neighbors at random, and check if they are connected. The approximate coefficient is the fraction of triangles found over the number of trials ${ }^{1}$.

## Parameters

- G (NetworkX graph)
- trials (integer) - Number of trials to perform (default 1000).

Returns c-Approximated average clustering coefficient.
Return type float

## References

### 4.1.5 Dominating Set

Functions for finding node and edge dominating sets.

[^5]A 'dominating set'_[1] for an undirected graph ${ }^{*} G$ with vertex set $V$ and edge set $E$ is a subset $D$ of $V$ such that every vertex not in $D$ is adjacent to at least one member of $D$. An 'edge dominating set'_[2] is a subset $* F$ of $E$ such that every edge not in $F$ is incident to an endpoint of at least one edge in $F$.

| min_weighted_dominating_set(G[, weight]) | Returns a dominating set that approximates the minimum <br> weight node dominating set. |
| :--- | :--- |
| min_edge_dominating_set $(\mathbf{G})$ | Return minimum cardinality edge dominating set. |

## min_weighted_dominating_set

min_weighted_dominating_set ( $G$, weight=None)
Returns a dominating set that approximates the minimum weight node dominating set.

## Parameters

- G (NetworkX graph) - Undirected graph.
- weight (string) - The node attribute storing the weight of an edge. If provided, the node attribute with this key must be a number for each node. If not provided, each node is assumed to have weight one.

Returns min_weight_dominating_set - A set of nodes, the sum of whose weights is no more than $(\log \mathrm{w}(\mathrm{V})) \mathrm{w}\left(\mathrm{V}^{\wedge} \star\right)$, where $\mathrm{w}(\mathrm{V})$ denotes the sum of the weights of each node in the graph and $w\left(V^{\wedge} \star\right)$ denotes the sum of the weights of each node in the minimum weight dominating set.

## Return type set

## Notes

This algorithm computes an approximate minimum weighted dominating set for the graph G. The returned solution has weight $(\log w(V)) \quad w\left(V^{\wedge} \star\right)$, where $w(V)$ denotes the sum of the weights of each node in the graph and $w\left(V^{\wedge} \star\right)$ denotes the sum of the weights of each node in the minimum weight dominating set for the graph.

This implementation of the algorithm runs in $O(m)$ time, where $m$ is the number of edges in the graph.

## References

```
min_edge_dominating_set
```

min_edge_dominating_set $(G)$
Return minimum cardinality edge dominating set.
Parameters G (NetworkX graph) - Undirected graph
Returns min_edge_dominating_set - Returns a set of dominating edges whose size is no more than $2 *$ OPT.

## Return type set

## Notes

The algorithm computes an approximate solution to the edge dominating set problem. The result is no more than 2 * OPT in terms of size of the set. Runtime of the algorithm is $\mathrm{O}(|\mathrm{E}|)$.

### 4.1.6 Independent Set

## Independent Set

Independent set or stable set is a set of vertices in a graph, no two of which are adjacent. That is, it is a set I of vertices such that for every two vertices in I, there is no edge connecting the two. Equivalently, each edge in the graph has at most one endpoint in I. The size of an independent set is the number of vertices it contains.

A maximum independent set is a largest independent set for a given graph G and its size is denoted $\alpha(\mathrm{G})$. The problem of finding such a set is called the maximum independent set problem and is an NP-hard optimization problem. As such, it is unlikely that there exists an efficient algorithm for finding a maximum independent set of a graph.
http://en.wikipedia.org/wiki/Independent_set_(graph_theory)
Independent set algorithm is based on the following paper:
$0\left(|\mathrm{~V}| /(\log |\mathrm{V}|)^{\wedge} 2\right)$ apx of maximum clique/independent set.
Boppana, R., \& Halldórsson, M. M. (1992). Approximating maximum independent sets by excluding subgraphs. BIT Numerical Mathematics, 32(2), 180-196. Springer. doi:10.1007/BF01994876

```
maximum_independent_set
```

maximum_independent_set $(G)$

Return an approximate maximum independent set.
Parameters G (NetworkX graph) - Undirected graph
Returns iset - The apx-maximum independent set
Return type Set

## Notes

Finds the $\mathrm{O}\left(|\mathrm{V}| /(\log |\mathrm{V}|)^{\wedge} 2\right)$ apx of independent set in the worst case.

## References

### 4.1.7 Matching

## Graph Matching

Given a graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$, a matching M in G is a set of pairwise non-adjacent edges; that is, no two edges share a common vertex.
http://en.wikipedia.org/wiki/Matching_(graph_theory)
min_maximal_matching(G) $\quad$ Returns the minimum maximal matching of G.

## min_maximal_matching

```
min_maximal_matching (G)
```

Returns the minimum maximal matching of G. That is, out of all maximal matchings of the graph G, the smallest is returned.

Parameters G (NetworkX graph) - Undirected graph
Returns min_maximal_matching - Returns a set of edges such that no two edges share a common endpoint and every edge not in the set shares some common endpoint in the set. Cardinality will be $2 *$ OPT in the worst case.

## Return type set

## Notes

The algorithm computes an approximate solution fo the minimum maximal cardinality matching problem. The solution is no more than $2 *$ OPT in size. Runtime is $\mathrm{O}(|\mathrm{E}|)$.

## References

### 4.1.8 Ramsey

Ramsey numbers.

| ramsey_R2(G) | Approximately computes the Ramsey number $R(2 ; s, t)$ <br> for graph. |
| :--- | :--- |

## ramsey_R2

ramsey_R2 (G)
Approximately computes the Ramsey number $R(2 ; s, t)$ for graph.
Parameters G (NetworkX graph) - Undirected graph
Returns max_pair - Maximum clique, Maximum independent set.
Return type (set, set) tuple

### 4.1.9 Vertex Cover

Functions for computing an approximate minimum weight vertex cover.
A vertex cover is a subset of nodes such that each edge in the graph is incident to at least one node in the subset.
min_weighted_vertex_cover(G[, weight]) $\quad$ Returns an approximate minimum weighted vertex cover.

## min_weighted_vertex_cover

## min_weighted_vertex_cover ( $G$, weight=None)

Returns an approximate minimum weighted vertex cover.
The set of nodes returned by this function is guaranteed to be a vertex cover, and the total weight of the set is guaranteed to be at most twice the total weight of the minimum weight vertex cover. In other words,

$$
w(S) \leq 2 * w\left(S^{*}\right),
$$

where $S$ is the vertex cover returned by this function, $S^{*}$ is the vertex cover of minimum weight out of all vertex covers of the graph, and $w$ is the function that computes the sum of the weights of each node in that given set.

## Parameters

- G (NetworkX graph)
- weight (string, optional (default $=$ None $)$ - If None, every edge has weight 1 . If a string, use this node attribute as the node weight. A node without this attribute is assumed to have weight 1.

Returns min_weighted_cover - Returns a set of nodes whose weight sum is no more than twice the weight sum of the minimum weight vertex cover.

## Return type set

## Notes

For a directed graph, a vertex cover has the same definition: a set of nodes such that each edge in the graph is incident to at least one node in the set. Whether the node is the head or tail of the directed edge is ignored.
This is the local-ratio algorithm for computing an approximate vertex cover. The algorithm greedily reduces the costs over edges, iteratively building a cover. The worst-case runtime of this implementation is $O(m \log n)$, where $n$ is the number of nodes and $m$ the number of edges in the graph.

## References

### 4.2 Assortativity

### 4.2.1 Assortativity

| degree_assortativity_coefficient(G[, <br> ...]) | y, |
| :--- | :--- | Compute degree assortativity of graph..

```
degree_assortativity_coefficient
```

degree_assortativity_coefficient ( $G, x=$ 'out', $y=$ 'in', weight $=$ None, nodes $=$ None)
Compute degree assortativity of graph.
Assortativity measures the similarity of connections in the graph with respect to the node degree.

## Parameters

- G (NetworkX graph)
- x (string ('in','out')) - The degree type for source node (directed graphs only).
- $\mathbf{y}$ (string ('in','out')) - The degree type for target node (directed graphs only).
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.
- nodes (list or iterable (optional)) - Compute degree assortativity only for nodes in container. The default is all nodes.

Returns $\mathbf{r}$ - Assortativity of graph by degree.
Return type float

Examples

```
>>> G=nx.path_graph(4)
>>> r=nx.degree_assortativity_coefficient(G)
>>> print("%3.1f"%r)
-0.5
```


## See also:

```
attribute_assortativity_coefficient(), numeric_assortativity_coefficient(),
neighbor_connectivity(), degree_mixing_dict(), degree_mixing_matrix()
```


## Notes

This computes Eq. (21) in Ref. ${ }^{1}$, where e is the joint probability distribution (mixing matrix) of the degrees. If $G$ is directed than the matrix $e$ is the joint probability of the user-specified degree type for the source and target.

## References

## attribute_assortativity_coefficient

```
attribute_assortativity_coefficient (G,attribute, nodes=None)
```

Compute assortativity for node attributes.
Assortativity measures the similarity of connections in the graph with respect to the given attribute.

## Parameters

- G (NetworkX graph)

[^6]- attribute (string) - Node attribute key
- nodes (list or iterable (optional)) - Compute attribute assortativity for nodes in container. The default is all nodes.

Returns $\mathbf{r}$ - Assortativity of graph for given attribute
Return type float

## Examples

```
>>> G=nx.Graph()
>>> G.add_nodes_from([0,1],color='red')
>>> G.add_nodes_from([2,3],color='blue')
>>> G.add_edges_from([(0,1), (2, 3)])
>>> print(nx.attribute_assortativity_coefficient(G,'color'))
1.0
```


## Notes

This computes Eq. (2) in Ref. ${ }^{1}$, trace(M)-sum(M))/(1-sum(M), where $M$ is the joint probability distribution (mixing matrix) of the specified attribute.

## References

numeric_assortativity_coefficient
numeric_assortativity_coefficient (G, attribute, nodes=None)
Compute assortativity for numerical node attributes.
Assortativity measures the similarity of connections in the graph with respect to the given numeric attribute. The numeric attribute must be an integer.

## Parameters

- G (NetworkX graph)
- attribute (string) - Node attribute key. The corresponding attribute value must be an integer.
- nodes (list or iterable (optional)) - Compute numeric assortativity only for attributes of nodes in container. The default is all nodes.

Returns $\mathbf{r}$ - Assortativity of graph for given attribute
Return type float

Examples

```
>>> G=nx.Graph()
>>> G.add_nodes_from([0,1],size=2)
>>> G.add_nodes_from([2,3],size=3)
>>> G.add_edges_from([(0,1),(2,3)])
```

[^7]```
>>> print(nx.numeric_assortativity_coefficient(G,'size'))
1.0
```


## Notes

This computes Eq. (21) in Ref. ${ }^{1}$, for the mixing matrix of of the specified attribute.

## References

degree_pearson_correlation_coefficient
degree_pearson_correlation_coefficient ( $G, x=$ 'out', $y=$ 'in', weight=None, nodes=None)
Compute degree assortativity of graph.
Assortativity measures the similarity of connections in the graph with respect to the node degree.
This is the same as degree_assortativity_coefficient but uses the potentially faster scipy.stats.pearsonr function.

## Parameters

- G (NetworkX graph)
- $\mathbf{x}$ (string ('in','out')) - The degree type for source node (directed graphs only).
- $\mathbf{y}$ (string ( 'in','out')) - The degree type for target node (directed graphs only).
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.
- nodes (list or iterable (optional)) - Compute pearson correlation of degrees only for specified nodes. The default is all nodes.

Returns $\mathbf{r}$ - Assortativity of graph by degree.
Return type float

## Examples

```
>>> G=nx.path_graph(4)
>>> r=nx.degree_pearson_correlation_coefficient(G)
>>> print("%3.1f"%r)
-0.5
```


## Notes

This calls scipy.stats.pearsonr.

[^8]
## References

### 4.2.2 Average neighbor degree

average_neighbor_degree(G[, source, target, ...]) $\quad$| Returns the average degree of the neighborhood of each |
| :--- |
| node | node.

## average_neighbor_degree

average_neighbor_degree ( $G$, source='out', target='out', nodes=None, weight=None)
Returns the average degree of the neighborhood of each node.
The average degree of a node $i$ is

$$
k_{n n, i}=\frac{1}{|N(i)|} \sum_{j \in N(i)} k_{j}
$$

where $N$ (i) are the neighbors of node $i$ and $k \_j$ is the degree of node $j$ which belongs to $N(i)$. For weighted graphs, an analogous measure can be defined ${ }^{1}$,

$$
k_{n n, i}^{w}=\frac{1}{s_{i}} \sum_{j \in N(i)} w_{i j} k_{j}
$$

where s_i is the weighted degree of node $i$, $w_{\text {_ }}\{\mathrm{ij}\}$ is the weight of the edge that links $i$ and $j$ and $N$ (i) are the neighbors of node $i$.

## Parameters

- G (NetworkX graph)
- source (string ("in"|"out")) - Directed graphs only. Use "in"- or "out"-degree for source node.
- target (string ("in"|"out")) - Directed graphs only. Use "in"- or "out"-degree for target node.
- nodes (list or iterable, optional) - Compute neighbor degree for specified nodes. The default is all nodes in the graph.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns d-A dictionary keyed by node with average neighbors degree value.
Return type dict

## Examples

```
>>> G=nx.path_graph(4)
>>> G.edge[0][1]['weight'] = 5
>>> G.edge[2][3]['weight'] = 3
```

[^9]```
>>> nx.average_neighbor_degree(G)
{0:2.0, 1: 1.5, 2: 1.5, 3: 2.0}
>>> nx.average_neighbor_degree(G, weight='weight')
{0:2.0, 1: 1.1666666666666667, 2: 1.25, 3: 2.0}
```

```
>>> G=nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> nx.average_neighbor_degree(G, source='in', target='in')
{0: 1.0, 1: 1.0, 2: 1.0, 3: 0.0}
```

>>> nx.average_neighbor_degree(G, source='out', target='out')
$\{0: 1.0,1: 1.0,2: 0.0,3: 0.0\}$

## Notes

For directed graphs you can also specify in-degree or out-degree by passing keyword arguments.
See also:

```
average_degree_connectivity()
```


## References

### 4.2.3 Average degree connectivity

| average_degree_connectivity(G[, source, ...]) | Compute the average degree connectivity of graph. |
| :--- | :--- |
| k_nearest_neighbors(G[, source, target, ...]) | Compute the average degree connectivity of graph. |

## average_degree_connectivity

average_degree_connectivity ( $G$, source='in+out', target='in+out', nodes=None, weight $=$ None)
Compute the average degree connectivity of graph.
The average degree connectivity is the average nearest neighbor degree of nodes with degree k. For weighted graphs, an analogous measure can be computed using the weighted average neighbors degree defined in ${ }^{1}$, for a node $i$, as

$$
k_{n n, i}^{w}=\frac{1}{s_{i}} \sum_{j \in N(i)} w_{i j} k_{j}
$$

where $s_{-} i$ is the weighted degree of node $i, w_{-}\{j\}$ is the weight of the edge that links $i$ and $j$, and $N$ (i) are the neighbors of node $i$.

## Parameters

- G (NetworkX graph)
- source ("in"|"out"|"in+out" (default:"in+out")) - Directed graphs only. Use "in"- or "out"-degree for source node.

[^10]- target ("in"|"out"|"in+out" (default:"in+out") - Directed graphs only. Use "in"- or "out"-degree for target node.
- nodes (list or iterable (optional)) - Compute neighbor connectivity for these nodes. The default is all nodes.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns d - A dictionary keyed by degree k with the value of average connectivity.
Return type dict
Raises ValueError - If either source or target are not one of 'in', 'out', or 'in+out'.

## Examples

```
>>> G=nx.path_graph(4)
>>> G.edge[1][2]['weight'] = 3
>>> nx.k_nearest_neighbors(G)
{1:2.0, 2: 1.5}
>>> nx.k_nearest_neighbors(G, weight='weight')
{1:2.0, 2: 1.75}
```


## See also:

neighbors_average_degree()

## Notes

This algorithm is sometimes called "k nearest neighbors" and is also available as k_nearest_neighbors.

## References

k_nearest_neighbors
$\mathbf{k}$ _nearest_neighbors ( $G$, source ='in+out', target='in+out', nodes $=$ None, weight $=$ None )
Compute the average degree connectivity of graph.
The average degree connectivity is the average nearest neighbor degree of nodes with degree k. For weighted graphs, an analogous measure can be computed using the weighted average neighbors degree defined in ${ }^{1}$, for a node $i$, as

$$
k_{n n, i}^{w}=\frac{1}{s_{i}} \sum_{j \in N(i)} w_{i j} k_{j}
$$

where $s \_i$ is the weighted degree of node $i, w_{-}\{i j\}$ is the weight of the edge that links $i$ and $j$, and $N$ (i) are the neighbors of node $i$.

## Parameters

- G (NetworkX graph)

[^11]- source ("in"|"out"|"in+out" (default:"in+out")) - Directed graphs only. Use "in"- or "out"-degree for source node.
- target ("in"|"out"|"in+out" (default:"in+out") - Directed graphs only. Use "in"- or "out"-degree for target node.
- nodes (list or iterable (optional)) - Compute neighbor connectivity for these nodes. The default is all nodes.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns d - A dictionary keyed by degree k with the value of average connectivity.
Return type dict
Raises ValueError - If either source or target are not one of 'in', 'out', or 'in+out'.

## Examples

```
>>> G=nx.path_graph(4)
>>> G.edge[1][2]['weight'] = 3
>>> nx.k_nearest_neighbors(G)
{1: 2.0, 2: 1.5}
>>> nx.k_nearest_neighbors(G, weight='weight')
{1: 2.0, 2: 1.75}
```


## See also:

```
neighbors_average_degree()
```


## Notes

This algorithm is sometimes called "k nearest neighbors" and is also available as k_nearest_neighbors.

## References

### 4.2.4 Mixing

| attribute_mixing_matrix(G, attribute[, ...]) | Return mixing matrix for attribute. |
| :--- | :--- |
| degree_mixing_matrix $(\mathrm{G}[, \mathrm{x}, \mathrm{y}$, weight, ..]) | Return mixing matrix for attribute. |
| degree_mixing_dict $(\mathrm{G}[, \mathrm{x}, \mathrm{y}$, weight, nodes, ...]) | Return dictionary representation of mixing matrix for de- <br> gree. |
| attribute_mixing_dict(G, attribute[, nodes, ...]) | Return dictionary representation of mixing matrix for at- <br> tribute. |

## attribute_mixing_matrix

attribute_mixing_matrix (G, attribute, nodes=None, mapping $=$ None, normalized $=$ True )
Return mixing matrix for attribute.

## Parameters

- G (graph ) - NetworkX graph object.
- attribute (string) - Node attribute key.
- nodes (list or iterable (optional)) - Use only nodes in container to build the matrix. The default is all nodes.
- mapping (dictionary, optional) - Mapping from node attribute to integer index in matrix. If not specified, an arbitrary ordering will be used.
- normalized (bool (default=False)) - Return counts if False or probabilities if True.

Returns $\mathbf{m}$ - Counts or joint probability of occurrence of attribute pairs.
Return type numpy array
degree_mixing_matrix
degree_mixing_matrix ( $G, x=$ 'out', $y=$ 'in', weight $=$ None, nodes $=$ None, normalized $=$ True )
Return mixing matrix for attribute.

## Parameters

- G (graph) - NetworkX graph object.
- $\mathbf{x}$ (string ( 'in','out')) - The degree type for source node (directed graphs only).
- $\mathbf{y}$ (string ('in','out')) - The degree type for target node (directed graphs only).
- nodes (list or iterable (optional)) - Build the matrix using only nodes in container. The default is all nodes.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.
- normalized (bool (default=False)) - Return counts if False or probabilities if True.

Returns $\mathbf{m}$ - Counts, or joint probability, of occurrence of node degree.
Return type numpy array
degree_mixing_dict
degree_mixing_dict ( $G, x=$ 'out', $y=$ 'in', weight $=$ None, nodes $=$ None, normalized $=$ False )
Return dictionary representation of mixing matrix for degree.

## Parameters

- G (graph) - NetworkX graph object.
- $\mathbf{x}$ (string ( 'in','out')) - The degree type for source node (directed graphs only).
- $\mathbf{y}$ (string ('in','out')) - The degree type for target node (directed graphs only).
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.
- normalized (bool (default=False)) - Return counts if False or probabilities if True.

Returns d-Counts or joint probability of occurrence of degree pairs.
Return type dictionary

## attribute_mixing_dict

## attribute_mixing_dict (G, attribute, nodes=None, normalized=False)

Return dictionary representation of mixing matrix for attribute.

## Parameters

- G (graph) - NetworkX graph object.
- attribute (string) - Node attribute key.
- nodes (list or iterable (optional)) - Unse nodes in container to build the dict. The default is all nodes.
- normalized (bool (default=False)) - Return counts if False or probabilities if True.


## Examples

```
>>> G=nx.Graph()
>>> G.add_nodes_from([0,1],color='red')
>>> G.add_nodes_from([2,3],color='blue')
>>> G.add_edge (1,3)
>>> d=nx.attribute_mixing_dict(G,'color')
>>> print(d['red']['blue'])
1
>>> print(d['blue']['red']) # d symmetric for undirected graphs
1
```

Returns d - Counts or joint probability of occurrence of attribute pairs.
Return type dictionary

### 4.3 Bipartite

This module provides functions and operations for bipartite graphs. Bipartite graphs $B=(U, V, E)$ have two node sets $U, V$ and edges in $E$ that only connect nodes from opposite sets. It is common in the literature to use an spatial analogy referring to the two node sets as top and bottom nodes.

The bipartite algorithms are not imported into the networkx namespace at the top level so the easiest way to use them is with:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
```

NetworkX does not have a custom bipartite graph class but the Graph() or DiGraph() classes can be used to represent bipartite graphs. However, you have to keep track of which set each node belongs to, and make sure that there is no edge between nodes of the same set. The convention used in NetworkX is to use a node attribute named "bipartite" with values 0 or 1 to identify the sets each node belongs to.

For example:

```
>>> B = nx.Graph()
>>> B.add_nodes_from([1,2,3,4], bipartite=0) # Add the node attribute "bipartite"
>>> B.add_nodes_from(['a','b','c'], bipartite=1)
>>> B.add_edges_from([(1,'a'), (1,'b'), (2,'b'), (2,'c'), (3,'c'), (4,'a')])
```

Many algorithms of the bipartite module of NetworkX require, as an argument, a container with all the nodes that belong to one set, in addition to the bipartite graph B. If B is connected, you can find the node sets using a twocoloring algorithm:

```
>>> nx.is_connected(B)
True
>>> bottom_nodes, top_nodes = bipartite.sets(B)
```

list(top_nodes) [1, 2, 3, 4] list(bottom_nodes) ['a', 'c', 'b']
However, if the input graph is not connected, there are more than one possible colorations. Thus, the following result is correct:

```
>>> B.remove_edge (2,'C')
>>> nx.is_connected(B)
False
>>> bottom_nodes, top_nodes = bipartite.sets(B)
```

list(top_nodes) [1, 2, 4, 'c'] list(bottom_nodes) ['a', 3, 'b']
Using the "bipartite" node attribute, you can easily get the two node sets:

```
>>> top_nodes = set(n for n,d in B.nodes(data=True) if d['bipartite']==0)
>>> bottom_nodes = set(B) - top_nodes
```

list(top_nodes) [1, 2, 3, 4] list(bottom_nodes) ['a', 'c', 'b']
So you can easily use the bipartite algorithms that require, as an argument, a container with all nodes that belong to one node set:

```
>>> print(round(bipartite.density(B, bottom_nodes), 2))
0.42
>>> G = bipartite.projected_graph(B, top_nodes)
>>> list(G.edges())
[(1, 2), (1, 4)]
```

All bipartite graph generators in NetworkX build bipartite graphs with the "bipartite" node attribute. Thus, you can use the same approach:

```
>>> RB = bipartite.random_graph(5, 7, 0.2)
>>> RB_top = set(n for n,d in RB.nodes(data=True) if d['bipartite']==0)
>>> RB_bottom = set(RB) - RB_top
>>> list(RB_top)
[0, 1, 2, 3, 4]
>>> list(RB_bottom)
[5, 6, 7, 8, 9, 10, 11]
```

For other bipartite graph generators see the bipartite section of Graph generators.

### 4.3.1 Basic functions

## Bipartite Graph Algorithms

| is_bipartite $(\mathrm{G})$ | Returns True if graph G is bipartite, False if not. |
| :--- | :--- |
| is_bipartite_node_set(G, nodes) | Returns True if nodes and G/nodes are a bipartition of G. |
|  | Continued on next page |

Table 4.15 - continued from previous page

| $\operatorname{sets}(\mathbf{G})$ | Returns bipartite node sets of graph $\mathbf{G .}$ |
| :--- | :--- |
| $\operatorname{color}(\mathbf{G})$ | Returns a two-coloring of the graph. |
| density $(\mathbf{B}$, nodes $)$ | Return density of bipartite graph $\mathbf{B}$. |
| degrees(B, nodes[, weight]) | Return the degrees of the two node sets in the bipartite <br> graph B. |

## is_bipartite

## is_bipartite (G)

Returns True if graph G is bipartite, False if not.
Parameters G (NetworkX graph)

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> print(bipartite.is_bipartite(G))
True
```


## See also:

```
color(), is_bipartite_node_set()
```

```
is_bipartite_node_set
```

is_bipartite_node_set ( $G$, nodes)
Returns True if nodes and $\mathrm{G} /$ nodes are a bipartition of G .

## Parameters

- G (NetworkX graph)
- nodes (list or container) - Check if nodes are a one of a bipartite set.


## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> X = set([1,3])
>>> bipartite.is_bipartite_node_set (G,X)
True
```


## Notes

For connected graphs the bipartite sets are unique. This function handles disconnected graphs.

## sets

```
sets(G)
```

Returns bipartite node sets of graph G.
Raises an exception if the graph is not bipartite.
Parameters G (NetworkX graph)
Returns (X,Y) - One set of nodes for each part of the bipartite graph.
Return type two-tuple of sets

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> X, Y = bipartite.sets(G)
>>> list(X)
[0, 2]
>>> list(Y)
[1, 3]
```


## See also:

color()
color
color ( $G$ )
Returns a two-coloring of the graph.
Raises an exception if the graph is not bipartite.
Parameters G (NetworkX graph)
Returns color - A dictionary keyed by node with a 1 or 0 as data for each node color.
Return type dictionary
Raises exc:NetworkXError if the graph is not two-colorable.

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> c = bipartite.color(G)
>>> print(c)
{0: 1, 1: 0, 2: 1, 3: 0}
```

You can use this to set a node attribute indicating the biparite set:

```
>>> nx.set_node_attributes(G, 'bipartite', c)
>>> print(G.node[0]['bipartite'])
1
>>> print(G.node[1]['bipartite'])
0
```


## density

density ( $B$, nodes)
Return density of bipartite graph B.

## Parameters

- G (NetworkX graph)
- nodes (list or container) - Nodes in one set of the bipartite graph.

Returns d-The bipartite density
Return type float

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.complete_bipartite_graph (3,2)
>>> X=\operatorname{set ([0,1,2])}
>>> bipartite.density(G,X)
1.0
>>> Y=set([3,4])
>>> bipartite.density(G,Y)
1.0
```


## See also:

```
color()
```


## degrees

## degrees ( $B$, nodes, weight=None)

Return the degrees of the two node sets in the bipartite graph B.

## Parameters

- G (NetworkX graph)
- nodes (list or container) - Nodes in one set of the bipartite graph.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1 . The degree is the sum of the edge weights adjacent to the node.

Returns (degX,degY) - The degrees of the two bipartite sets as dictionaries keyed by node.
Return type tuple of dictionaries

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.complete_bipartite_graph (3,2)
>>> Y=set ([3,4])
>>> degX, degY=bipartite.degrees(G,Y)
>>> dict(degX)
{0: 2, 1: 2, 2: 2}
```


## See also:

```
color(),density()
```


### 4.3.2 Matching

Provides functions for computing a maximum cardinality matching in a bipartite graph.
If you don't care about the particular implementation of the maximum matching algorithm, simply use the maximum_matching (). If you do care, you can import one of the named maximum matching algorithms directly.

For example, to find a maximum matching in the complete bipartite graph with two vertices on the left and three vertices on the right:

```
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph (2, 3)
>>> left, right = nx.bipartite.sets(G)
>>> list(left)
[0, 1]
>>> list(right)
[2, 3, 4]
>>> nx.bipartite.maximum_matching(G)
{0: 2, 1: 3, 2: 0, 3: 1}
```

The dictionary returned by maximum_matching () includes a mapping for vertices in both the left and right vertex sets.

| eppstein_matching(G) | Returns the maximum cardinality matching of the bipartite <br> graph $G$. |
| :--- | :--- |
| hopcroft_karp_matching(G) | Returns the maximum cardinality matching of the bipartite <br> graph $G$. |
| to_vertex_cover(G, matching) | Returns the minimum vertex cover corresponding to the <br> given maximum matching of the bipartite graph G. |

## eppstein_matching

## eppstein_matching ( $G$ )

Returns the maximum cardinality matching of the bipartite graph G .
Parameters G (NetworkX graph) - Undirected bipartite graph
Returns matches - The matching is returned as a dictionary, matching, such that matching [v] $==\mathrm{w}$ if node v is matched to node w . Unmatched nodes do not occur as a key in mate.

Return type dictionary

## Notes

This function is implemented with David Eppstein's version of the algorithm Hopcroft-Karp algorithm (see hopcroft_karp_matching()), which originally appeared in the Python Algorithms and Data Structures library (PADS).

## See also:

hopcroft_karp_matching()
hopcroft_karp_matching
hopcroft_karp_matching ( $G$ )
Returns the maximum cardinality matching of the bipartite graph G .
Parameters G (NetworkX graph) - Undirected bipartite graph
Returns matches - The matching is returned as a dictionary, matches, such that matches [v] $==\mathrm{w}$ if node v is matched to node w . Unmatched nodes do not occur as a key in mate.
Return type dictionary

## Notes

This function is implemented with the Hopcroft-Karp matching algorithm for bipartite graphs.
See also:
eppstein_matching()

## References

to_vertex_cover
to_vertex_cover ( $G$, matching)
Returns the minimum vertex cover corresponding to the given maximum matching of the bipartite graph G .

## Parameters

- G (NetworkX graph) - Undirected bipartite graph
- matching (dictionary) - A dictionary whose keys are vertices in $G$ and whose values are the distinct neighbors comprising the maximum matching for $G$, as returned by, for example, maximum_matching(). The dictionary must represent the maximum matching.

Returns vertex_cover - The minimum vertex cover in G .
Return type set

## Notes

This function is implemented using the procedure guaranteed by Konig's theorem, which proves an equivalence between a maximum matching and a minimum vertex cover in bipartite graphs.
Since a minimum vertex cover is the complement of a maximum independent set for any graph, one can compute the maximum independent set of a bipartite graph this way:

```
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph (2, 3)
>>> matching = nx.bipartite.maximum_matching(G)
>>> vertex_cover = nx.bipartite.to_vertex_cover(G, matching)
>>> independent_set = set(G) - vertex_cover
>>> print(list(independent_set))
[2, 3, 4]
```


### 4.3.3 Matrix

## Biadjacency matrices

| biadjacency_matrix $(\mathbf{G}$, row_order[, ...]) | Return the biadjacency matrix of the bipartite graph G. |
| :--- | :--- |
| from_biadjacency_matrix(A[, create_using, ...]) | Creates a new bipartite graph from a biadjacency matrix <br> given as a SciPy sparse matrix. |

## biadjacency_matrix

biadjacency_matrix (G, row_order, column_order=None, dtype=None, weight='weight',format='csr') Return the biadjacency matrix of the bipartite graph $G$.

Let $G=(U, V, E)$ be a bipartite graph with node sets $U=u_{\_}\{1\}, \ldots, u_{\_}\{r\}$ and $V=$ $v_{-}\{1\}, \ldots, v_{-}\{s\}$. The biadjacency matrix ${ }^{1}$ is the $r x s$ matrix $B$ in which $b_{-}\{i, j\}=1$ if, and only if, (u_i,v_j) in E. If the parameter weight is not None and matches the name of an edge attribute, its value is used instead of 1 .

## Parameters

- G (graph) - A NetworkX graph
- row_order (list of nodes) - The rows of the matrix are ordered according to the list of nodes.
- column_order (list, optional) - The columns of the matrix are ordered according to the list of nodes. If column_order is None, then the ordering of columns is arbitrary.
- dtype (NumPy data-type, optional) - A valid NumPy dtype used to initialize the array. If None, then the NumPy default is used.
- weight (string or None, optional (default='weight')) - The edge data key used to provide each value in the matrix. If None, then each edge has weight 1.
- format (str in \{'bsr', 'csr', 'csc', 'coo', 'lil', 'dia', 'dok'\}) - The type of the matrix to be returned (default 'csr'). For some algorithms different implementations of sparse matrices can perform better. See ${ }^{2}$ for details.

Returns M - Biadjacency matrix representation of the bipartite graph G.
Return type SciPy sparse matrix

## Notes

No attempt is made to check that the input graph is bipartite.
For directed bipartite graphs only successors are considered as neighbors. To obtain an adjacency matrix with ones (or weight values) for both predecessors and successors you have to generate two biadjacency matrices where the rows of one of them are the columns of the other, and then add one to the transpose of the other.

## See also:

adjacency_matrix(), from_biadjacency_matrix()

[^12]
## References

from_biadjacency_matrix

from_biadjacency_matrix (A, create_using=None, edge_attribute='weight')
Creates a new bipartite graph from a biadjacency matrix given as a SciPy sparse matrix.

## Parameters

- A (scipy sparse matrix) - A biadjacency matrix representation of a graph
- create_using (NetworkX graph) - Use specified graph for result. The default is Graph()
- edge_attribute (string) - Name of edge attribute to store matrix numeric value. The data will have the same type as the matrix entry (int, float, (real,imag)).


## Notes

The nodes are labeled with the attribute bipartite set to an integer 0 or 1 representing membership in part 0 or part 1 of the bipartite graph.

If create_using is an instance of networkx.MultiGraph or networkx.MultiDiGraph and the entries of A are of type int, then this function returns a multigraph (of the same type as create_using) with parallel edges. In this case, edge_attribute will be ignored.

## See also:

```
biadjacency_matrix(), from_numpy_matrix()
```


## References

[1] http://en.wikipedia.org/wiki/Adjacency_matrix\#Adjacency_matrix_of_a_bipartite_graph

### 4.3.4 Projections

One-mode (unipartite) projections of bipartite graphs.

| projected_graph(B, nodes[, multigraph] $)$ | Returns the projection of B onto one of its node sets. |
| :--- | :--- |
| weighted_projected_graph(B, nodes[, ratio]) | Returns a weighted projection of B onto one of its node <br> sets. |
| collaboration_weighted_projected_graph(B, <br> nodes $)$ | Newman's weighted projection of B onto one of its node <br> sets. |
| overlap_weighted_projected_graph(B,, <br> nodes[, ..]) | Overlap weighted projection of B onto one of its node sets. |
| generic_weighted_projected_graph(B, <br> nodes[, ...]) | Weighted projection of B with a user-specified weight func- <br> tion. |

## projected_graph

```
projected_graph( }B\mathrm{ , nodes, multigraph=False)
```

Returns the projection of $B$ onto one of its node sets.
Returns the graph G that is the projection of the bipartite graph B onto the specified nodes. They retain their
attributes and are connected in G if they have a common neighbor in B.

## Parameters

- B (NetworkX graph) - The input graph should be bipartite.
- nodes (list or iterable) - Nodes to project onto (the "bottom" nodes).
- multigraph (bool (default=False)) - If True return a multigraph where the multiple edges represent multiple shared neighbors. They edge key in the multigraph is assigned to the label of the neighbor.

Returns Graph - A graph that is the projection onto the given nodes.
Return type NetworkX graph or multigraph

## Examples

```
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(4)
>>> G = bipartite.projected_graph(B, [1,3])
>>> list(G)
[1, 3]
>>> list(G.edges())
[(1, 3)]
```

If nodes $a$, and $b$ are connected through both nodes 1 and 2 then building a multigraph results in two edges in the projection onto [a, 'b']:

```
>>> B = nx.Graph()
>>> B.add_edges_from([('a', 1), ('b', 1), ('a', 2), ('b', 2)])
>>> G = bipartite.projected_graph(B, ['a', 'b'], multigraph=True)
>>> print([sorted((u,v)) for u,v in G.edges()])
[['a', 'b'], ['a', 'b']]
```


## Notes

No attempt is made to verify that the input graph $B$ is bipartite. Returns a simple graph that is the projection of the bipartite graph $B$ onto the set of nodes given in list nodes. If multigraph=True then a multigraph is returned with an edge for every shared neighbor.

Directed graphs are allowed as input. The output will also then be a directed graph with edges if there is a directed path between the nodes.
The graph and node properties are (shallow) copied to the projected graph.

## See also:

```
is_bipartite(), is_bipartite_node_set(), sets(), weighted_projected_graph(),
collaboration_weighted_projected_graph(), overlap_weighted_projected_graph(),
generic_weighted_projected_graph()
```


## weighted_projected_graph

weighted_projected_graph ( $B$, nodes, ratio=False)
Returns a weighted projection of $B$ onto one of its node sets.

The weighted projected graph is the projection of the bipartite network B onto the specified nodes with weights representing the number of shared neighbors or the ratio between actual shared neighbors and possible shared neighbors if ratio=True ${ }^{1}$. The nodes retain their attributes and are connected in the resulting graph if they have an edge to a common node in the original graph.

## Parameters

- B (NetworkX graph) - The input graph should be bipartite.
- nodes (list or iterable) - Nodes to project onto (the "bottom" nodes).
- ratio (Bool (default=False)) - If True, edge weight is the ratio between actual shared neighbors and possible shared neighbors. If False, edges weight is the number of shared neighbors.

Returns Graph - A graph that is the projection onto the given nodes.
Return type NetworkX graph

## Examples

```
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(4)
>>> G = bipartite.weighted_projected_graph(B, [1,3])
>>> list(G)
[1, 3]
>>> list(G.edges(data=True))
[(1, 3, {'weight': 1})]
>>> G = bipartite.weighted_projected_graph(B, [1,3], ratio=True)
>>> list(G.edges(data=True))
[(1, 3, {'weight': 0.5})]
```


## Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

## See also:

```
is_bipartite(), is_bipartite_node_set(), sets(), collaboration_weighted_projected_graph(),
overlap_weighted_projected_graph(), generic_weighted_projected_graph(),
projected_graph()
```


## References

```
collaboration_weighted_projected_graph
```

collaboration_weighted_projected_graph ( $B$, nodes)

Newman's weighted projection of $B$ onto one of its node sets.

[^13]The collaboration weighted projection is the projection of the bipartite network $B$ onto the specified nodes with weights assigned using Newman's collaboration model ${ }^{1}$ :

$$
w_{v, u}=\sum_{k} \frac{\delta_{v}^{w} \delta_{w}^{k}}{k_{w}-1}
$$

where $v$ and $u$ are nodes from the same bipartite node set, and $w$ is a node of the opposite node set. The value $k \_w$ is the degree of node $w$ in the bipartite network and delta_\{v $\}^{\wedge}\{w\}$ is 1 if node $v$ is linked to node $w$ in the original bipartite graph or 0 otherwise.

The nodes retain their attributes and are connected in the resulting graph if have an edge to a common node in the original bipartite graph.

## Parameters

- B (NetworkX graph) - The input graph should be bipartite.
- nodes (list or iterable) - Nodes to project onto (the "bottom" nodes).

Returns Graph - A graph that is the projection onto the given nodes.
Return type NetworkX graph

## Examples

```
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(5)
>>> B.add_edge (1,5)
>>> G = bipartite.collaboration_weighted_projected_graph(B, [0, 2, 4, 5])
>>> list(G)
[0, 2, 4, 5]
>>> for edge in G.edges(data=True): print(edge)
(0, 2, {'weight': 0.5})
(0, 5, {'weight': 0.5})
(2, 4, {'weight': 1.0})
(2, 5, {'weight': 0.5})
```


## Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

## See also:

```
is_bipartite(), is_bipartite_node_set(), sets(), weighted_projected_graph(),
overlap_weighted_projected_graph(), generic_weighted_projected_graph(),
projected_graph()
```


## References

overlap_weighted_projected_graph
overlap_weighted_projected_graph ( $B$, nodes, jaccard=True)
Overlap weighted projection of B onto one of its node sets.

[^14]The overlap weighted projection is the projection of the bipartite network B onto the specified nodes with weights representing the Jaccard index between the neighborhoods of the two nodes in the original bipartite network ${ }^{1}$ :

$$
w_{v, u}=\frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|}
$$

or if the parameter 'jaccard' is False, the fraction of common neighbors by minimum of both nodes degree in the original bipartite graph ${ }^{1}$ :

$$
w_{v, u}=\frac{|N(u) \cap N(v)|}{\min (|N(u)|,|N(v)|)}
$$

The nodes retain their attributes and are connected in the resulting graph if have an edge to a common node in the original bipartite graph.

## Parameters

- B (NetworkX graph) - The input graph should be bipartite.
- nodes (list or iterable) - Nodes to project onto (the "bottom" nodes).
- jaccard (Bool (default=True))

Returns Graph - A graph that is the projection onto the given nodes.
Return type NetworkX graph

## Examples

```
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(5)
>>> G = bipartite.overlap_weighted_projected_graph(B, [0, 2, 4])
>>> list(G)
[0, 2, 4]
>>> list(G.edges(data=True))
[(0, 2, {'weight': 0.5}), (2, 4, {'weight': 0.5})]
>>> G = bipartite.overlap_weighted_projected_graph(B, [0, 2, 4], jaccard=False)
>>> list(G.edges(data=True))
[(0, 2, {'weight': 1.0}), (2, 4, {'weight': 1.0})]
```


## Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

## See also:

```
is_bipartite(), is_bipartite_node_set(), sets(), weighted_projected_graph(),
collaboration_weighted_projected_graph(), generic_weighted_projected_graph(),
projected_graph()
```

[^15]
## References

## generic_weighted_projected_graph

generic_weighted_projected_graph ( $B$, nodes, weight_function=None)
Weighted projection of B with a user-specified weight function.
The bipartite network B is projected on to the specified nodes with weights computed by a user-specified function. This function must accept as a parameter the neighborhood sets of two nodes and return an integer or a float.

The nodes retain their attributes and are connected in the resulting graph if they have an edge to a common node in the original graph.

## Parameters

- B (NetworkX graph) - The input graph should be bipartite.
- nodes (list or iterable) - Nodes to project onto (the "bottom" nodes).
- weight_function (function) - This function must accept as parameters the same input graph that this function, and two nodes; and return an integer or a float. The default function computes the number of shared neighbors.

Returns Graph - A graph that is the projection onto the given nodes.
Return type NetworkX graph

## Examples

```
>>> from networkx.algorithms import bipartite
>>> # Define some custom weight functions
>>> def jaccard(G, u, v):
... unbrs = set(G[u])
... vnbrs = set(G[v])
... return float(len(unbrs & vnbrs)) / len(unbrs | vnbrs)
>>> def my_weight(G, u, v, weight='weight'):
... w = 0
... for nbr in set (G[u]) & set(G[v]):
... w += G.edge[u][nbr].get(weight, 1) + G.edge[v][nbr].get(weight, 1)
... return W
>>> # A complete bipartite graph with 4 nodes and 4 edges
>>> B = nx.complete_bipartite_graph (2,2)
>>> # Add some arbitrary weight to the edges
>>> for i,(u,v) in enumerate(B.edges()):
... B.edge[u][v]['weight'] = i + 1
>>> for edge in B.edges(data=True):
... print(edge)
. . .
(0, 2, {'weight': 1})
(0, 3, {'weight': 2})
(1, 2, {'weight': 3})
(1, 3, {'weight': 4})
>>> # Without specifying a function, the weight is equal to # shared partners
>>> G = bipartite.generic_weighted_projected_graph(B, [0, 1])
```

```
>>> print(list(G.edges(data=True)))
[(0, 1, {'weight': 2})]
>>> # To specify a custom weight function use the weight_function parameter
>>> G = bipartite.generic_weighted_projected_graph(B, [0, 1], weight_
\hookrightarrowfunction=jaccard)
>>> print(list(G.edges(data=True)))
[(0, 1, {'weight': 1.0})]
>>> G = bipartite.generic_weighted_projected_graph(B, [0, 1], weight_function=my_
\hookrightarrowweight)
>>> print(list(G.edges(data=True)))
[(0, 1, {'weight': 10})]
```


## Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

## See also:

```
is_bipartite(), is_bipartite_node_set(), sets(), weighted_projected_graph(),
collaboration_weighted_projected_graph(), overlap_weighted_projected_graph(),
projected_graph()
```


### 4.3.5 Spectral

Spectral bipartivity measure.
spectral_bipartivity(G[, nodes, weight]) $\quad$ Returns the spectral bipartivity.

## spectral_bipartivity

```
spectral_bipartivity(G, nodes=None, weight='weight')
```

Returns the spectral bipartivity.

## Parameters

- G (NetworkX graph)
- nodes (list or container optional(default is all nodes)) - Nodes to return value of spectral bipartivity contribution.
- weight (string or None optional (default $=$ 'weight')) - Edge data key to use for edge weights. If None, weights set to 1.

Returns sb-A single number if the keyword nodes is not specified, or a dictionary keyed by node with the spectral bipartivity contribution of that node as the value.
Return type float or dict

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
```

```
>>> bipartite.spectral_bipartivity(G)
1.0
```


## Notes

This implementation uses Numpy (dense) matrices which are not efficient for storing large sparse graphs.
See also:

```
color()
```


## References

### 4.3.6 Clustering

| clustering(G[, nodes, mode $])$ | Compute a bipartite clustering coefficient for nodes. |
| :--- | :--- |
| average_clustering(G[, nodes, mode $])$ | Compute the average bipartite clustering coefficient. |
| latapy_clustering(G[, nodes, mode $)$ | Compute a bipartite clustering coefficient for nodes. |
| robins_alexander_clustering $(\mathrm{G})$ | Compute the bipartite clustering of $\mathbf{G}$. |

## clustering

clustering ( $G$, nodes=None, mode $=$ 'dot')
Compute a bipartite clustering coefficient for nodes.
The bipartie clustering coefficient is a measure of local density of connections defined as ${ }^{1}$ :

$$
c_{u}=\frac{\sum_{v \in N(N(v))} c_{u v}}{|N(N(u))|}
$$

where $N(N(u))$ are the second order neighbors of $u$ in $G$ excluding $u$, and $c_{-}\{u v\}$ is the pairwise clustering coefficient between nodes $u$ and $v$.
The mode selects the function for $c_{-}\{u v\}$ which can be:
dot:

$$
c_{u v}=\frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|}
$$

min:

$$
c_{u v}=\frac{|N(u) \cap N(v)|}{\min (|N(u)|,|N(v)|)}
$$

max:

$$
c_{u v}=\frac{|N(u) \cap N(v)|}{\max (|N(u)|,|N(v)|)}
$$

## Parameters

- G (graph) - A bipartite graph

[^16]- nodes (list or iterable (optional)) - Compute bipartite clustering for these nodes. The default is all nodes in G.
- mode (string) - The pariwise bipartite clustering method to be used in the computation. It must be "dot", "max", or "min".

Returns clustering - A dictionary keyed by node with the clustering coefficient value.
Return type dictionary

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4) # path graphs are bipartite
>>> c = bipartite.clustering(G)
>>> c[0]
0.5
>>> c = bipartite.clustering(G,mode='min')
>>> c[0]
1.0
```


## See also:

```
robins_alexander_clustering(), square_clustering(), average_clustering()
```


## References

average_clustering
average_clustering ( $G$, nodes=None, mode='dot')
Compute the average bipartite clustering coefficient.
A clustering coefficient for the whole graph is the average,

$$
C=\frac{1}{n} \sum_{v \in G} c_{v}
$$

where n is the number of nodes in G .
Similar measures for the two bipartite sets can be defined ${ }^{1}$

$$
C_{X}=\frac{1}{|X|} \sum_{v \in X} c_{v}
$$

where $X$ is a bipartite set of $G$.

## Parameters

- G (graph) - a bipartite graph
- nodes (list or iterable, optional) - A container of nodes to use in computing the average. The nodes should be either the entire graph (the default) or one of the bipartite sets.
- mode (string) - The pariwise bipartite clustering method. It must be "dot", "max", or "min"

[^17]Returns clustering - The average bipartite clustering for the given set of nodes or the entire graph if no nodes are specified.
Return type float

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G=nx.star_graph(3) # star graphs are bipartite
>>> bipartite.average_clustering(G)
0.75
>>> X,Y=bipartite.sets(G)
>>> bipartite.average_clustering(G,X)
0.0
>>> bipartite.average_clustering(G,Y)
1.0
```


## See also:

```
clustering()
```


## Notes

The container of nodes passed to this function must contain all of the nodes in one of the bipartite sets ("top" or "bottom") in order to compute the correct average bipartite clustering coefficients.

## References

latapy_clustering
latapy_clustering ( $G$, nodes=None, mode='dot')
Compute a bipartite clustering coefficient for nodes.
The bipartie clustering coefficient is a measure of local density of connections defined as ${ }^{1}$ :

$$
c_{u}=\frac{\sum_{v \in N(N(v))} c_{u v}}{|N(N(u))|}
$$

where $N(N(u))$ are the second order neighbors of $u$ in $G \operatorname{excluding} u$, and $c_{-}\{u v\}$ is the pairwise clustering coefficient between nodes $u$ and $v$.

The mode selects the function for $c \_\{u v\}$ which can be:
dot:

$$
c_{u v}=\frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|}
$$

min:

$$
c_{u v}=\frac{|N(u) \cap N(v)|}{\min (|N(u)|,|N(v)|)}
$$

[^18]max:
$$
c_{u v}=\frac{|N(u) \cap N(v)|}{\max (|N(u)|,|N(v)|)}
$$

## Parameters

- G (graph) - A bipartite graph
- nodes (list or iterable (optional)) - Compute bipartite clustering for these nodes. The default is all nodes in G .
- mode (string) - The pariwise bipartite clustering method to be used in the computation. It must be "dot", "max", or "min".

Returns clustering - A dictionary keyed by node with the clustering coefficient value.
Return type dictionary

## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4) # path graphs are bipartite
>>> c = bipartite.clustering(G)
>>> c[0]
0.5
>>> c = bipartite.clustering(G,mode='min')
>>> c[0]
1.0
```


## See also:

```
robins_alexander_clustering(), square_clustering(), average_clustering()
```


## References

```
robins_alexander_clustering
```

robins_alexander_clustering ( $G$ )

Compute the bipartite clustering of G .
Robins and Alexander ${ }^{1}$ defined bipartite clustering coefficient as four times the number of four cycles C_4 divided by the number of three paths $L \_3$ in a bipartite graph:

$$
C C_{4}=\frac{4 * C_{4}}{L_{3}}
$$

Parameters G (graph) - a bipartite graph
Returns clustering - The Robins and Alexander bipartite clustering for the input graph.
Return type float

[^19]
## Examples

```
>>> from networkx.algorithms import bipartite
>>> G = nx.davis_southern_women_graph()
>>> print(round(bipartite.robins_alexander_clustering(G), 3))
0.468
```


## See also:

```
latapy_clustering(), square_clustering()
```


## References

### 4.3.7 Redundancy

Node redundancy for bipartite graphs.

```
node_redundancy(G[, nodes])
```

Computes the node redundancy coefficients for the nodes in the bipartite graph G .

## node_redundancy

node_redundancy ( $G$, nodes=None)
Computes the node redundancy coefficients for the nodes in the bipartite graph $G$.
The redundancy coefficient of a node $v$ is the fraction of pairs of neighbors of $v$ that are both linked to other nodes. In a one-mode projection these nodes would be linked together even if v were not there.

More formally, for any vertex $v$, the redundancy coefficient of ' $v$ ' is defined by

$$
r c(v)=\frac{\mid\left\{\{u, w\} \subseteq N(v), \exists v^{\prime} \neq v,\left(v^{\prime}, u\right) \in E \text { and }\left(v^{\prime}, w\right) \in E\right\} \mid}{\frac{|N(v)|(|N(v)|-1)}{2}},
$$

where $N(v)$ is the set of neighbors of $v$ in $G$.

## Parameters

- G (graph) - A bipartite graph
- nodes (list or iterable (optional)) - Compute redundancy for these nodes. The default is all nodes in G.

Returns redundancy - A dictionary keyed by node with the node redundancy value.
Return type dictionary

## Examples

Compute the redundancy coefficient of each node in a graph:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> rc[0]
1.0
```

Compute the average redundancy for the graph:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> sum(rc.values()) / len(G)
1.0
```

Compute the average redundancy for a set of nodes:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> nodes = [0, 2]
>>> sum(rc[n] for n in nodes) / len(nodes)
1.0
```

Raises NetworkXError - If any of the nodes in the graph (or in nodes, if specified) has (out)degree less than two (which would result in division by zero, according to the definition of the redundancy coefficient).

## References

### 4.3.8 Centrality

| closeness_centrality(G, nodes[, normalized]) | Compute the closeness centrality for nodes in a bipartite <br> network. |
| :--- | :--- |
| degree_centrality(G, nodes) | Compute the degree centrality for nodes in a bipartite net- <br> work. |
| betweenness_centrality(G, nodes) | Compute betweenness centrality for nodes in a bipartite <br> network. |

## closeness_centrality

## closeness_centrality ( $G$, nodes, normalized=True)

Compute the closeness centrality for nodes in a bipartite network.
The closeness of a node is the distance to all other nodes in the graph or in the case that the graph is not connected to all other nodes in the connected component containing that node.

## Parameters

- G (graph) - A bipartite network
- nodes (list or container) - Container with all nodes in one bipartite node set.
- normalized (bool, optional) - If True (default) normalize by connected component size.

Returns closeness - Dictionary keyed by node with bipartite closeness centrality as the value.
Return type dictionary

## See also:

```
betweenness_centrality(), degree_centrality(), sets(),is_bipartite()
```


## Notes

The nodes input parameter must conatin all nodes in one bipartite node set, but the dictionary returned contains all nodes from both node sets.

Closeness centrality is normalized by the minimum distance possible. In the bipartite case the minimum distance for a node in one bipartite node set is 1 from all nodes in the other node set and 2 from all other nodes in its own set ${ }^{1}$. Thus the closeness centrality for node $v$ in the two bipartite sets $U$ with $n$ nodes and $V$ with $m$ nodes is

$$
\begin{aligned}
& c_{v}=\frac{m+2(n-1)}{d}, \text { for } v \in U, \\
& c_{v}=\frac{n+2(m-1)}{d}, \text { for } v \in V
\end{aligned}
$$

where d is the sum of the distances from v to all other nodes.
Higher values of closeness indicate higher centrality.
As in the unipartite case, setting normalized=True causes the values to normalized further to $\mathrm{n}-1 / \operatorname{size}(\mathrm{G})-1$ where n is the number of nodes in the connected part of graph containing the node. If the graph is not completely connected, this algorithm computes the closeness centrality for each connected part separately.

## References

## degree_centrality

degree_centrality ( $G$, nodes)
Compute the degree centrality for nodes in a bipartite network.
The degree centrality for a node v is the fraction of nodes connected to it.

## Parameters

- G (graph) - A bipartite network
- nodes (list or container) - Container with all nodes in one bipartite node set.

Returns centrality - Dictionary keyed by node with bipartite degree centrality as the value.
Return type dictionary

## See also:

betweenness_centrality(), closeness_centrality(), sets(), is_bipartite()

## Notes

The nodes input parameter must conatin all nodes in one bipartite node set, but the dictionary returned contains all nodes from both bipartite node sets.

For unipartite networks, the degree centrality values are normalized by dividing by the maximum possible degree (which is $n-1$ where $n$ is the number of nodes in $G$ ).

[^20]In the bipartite case, the maximum possible degree of a node in a bipartite node set is the number of nodes in the opposite node set ${ }^{1}$. The degree centrality for a node $v$ in the bipartite sets $U$ with $n$ nodes and $V$ with $m$ nodes is

$$
\begin{aligned}
& d_{v}=\frac{\operatorname{deg}(v)}{m}, \text { for } v \in U \\
& d_{v}=\frac{d e g(v)}{n}, \text { for } v \in V
\end{aligned}
$$

where $\operatorname{deg}(v)$ is the degree of node $v$.

## References

betweenness_centrality
betweenness_centrality ( $G$, nodes)
Compute betweenness centrality for nodes in a bipartite network.
Betweenness centrality of a node $v$ is the sum of the fraction of all-pairs shortest paths that pass through $v$.
Values of betweenness are normalized by the maximum possible value which for bipartite graphs is limited by the relative size of the two node sets ${ }^{1}$.
Let $n$ be the number of nodes in the node set $U$ and $m$ be the number of nodes in the node set $V$, then nodes in $U$ are normalized by dividing by

$$
\frac{1}{2}\left[m^{2}(s+1)^{2}+m(s+1)(2 t-s-1)-t(2 s-t+3)\right]
$$

where

$$
s=(n-1) \div m, t=(n-1) \quad \bmod m
$$

and nodes in V are normalized by dividing by

$$
\frac{1}{2}\left[n^{2}(p+1)^{2}+n(p+1)(2 r-p-1)-r(2 p-r+3)\right]
$$

where,

$$
p=(m-1) \div n, r=(m-1) \quad \bmod n
$$

## Parameters

- G (graph) - A bipartite graph
- nodes (list or container) - Container with all nodes in one bipartite node set.

Returns betweenness - Dictionary keyed by node with bipartite betweenness centrality as the value.
Return type dictionary

## See also:

degree_centrality(), closeness_centrality(), sets(), is_bipartite()

[^21]
## Notes

The nodes input parameter must contain all nodes in one bipartite node set, but the dictionary returned contains all nodes from both node sets.

## References

### 4.3.9 Generators

Generators and functions for bipartite graphs.

| complete_bipartite_graph(n1, n2[, create_using]) | Return the complete bipartite graph $\mathrm{K} \_\left\{\mathrm{n} \_1, \mathrm{n} \_2\right\}$. |
| :---: | :---: |
| configuration_model(aseq, bseq[, ...]) | Return a random bipartite graph from two given degree sequences. |
| havel_hakimi_graph(aseq, bseq[, create_using]) | Return a bipartite graph from two given degree sequences using a Havel-Hakimi style construction. |
| reverse_havel_hakimi_graph(aseq, bseq[, ..]) | Return a bipartite graph from two given degree sequences using a Havel-Hakimi style construction. |
| alternating_havel_hakimi_graph(aseq, bseq[, ...]) | Return a bipartite graph from two given degree sequences using an alternating Havel-Hakimi style construction. |
| preferential_attachment_graph(aseq, p [, ...]) | Create a bipartite graph with a preferential attachment model from a given single degree sequence. |
| random_graph(n, m, p[, seed, directed]) | Return a bipartite random graph. |
| gnmk_random_graph(n, m, k[, seed, directed]) | Return a random bipartite graph G_\{ $\mathrm{n}, \mathrm{m}, \mathrm{k}\}$. |

## complete_bipartite_graph

complete_bipartite_graph (n1, n2, create_using=None)
Return the complete bipartite graph $K_{-}\left\{n \_1, n \_2\right\}$.
Composed of two partitions with $n \_1$ nodes in the first and $n \_2$ nodes in the second. Each node in the first is connected to each node in the second.

## Parameters

- n1 (integer) - Number of nodes for node set A.
- n2 (integer) - Number of nodes for node set B.
- create_using (NetworkX graph instance, optional) - Return graph of this type.


## Notes

Node labels are the integers 0 to $n \_1+n \_2-1$.
The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.

## configuration_model

configuration_model (aseq, bseq, create_using=None, seed=None)
Return a random bipartite graph from two given degree sequences.

## Parameters

- aseq (list) - Degree sequence for node set A .
- bseq (list) - Degree sequence for node set B.
- create_using (NetworkX graph instance, optional) - Return graph of this type.
- seed (integer, optional) - Seed for random number generator.
- Nodes from the set $\mathbf{A}$ are connected to nodes in the set $\mathbf{B}$ by
- choosing randomly from the possible free stubs, one in $\mathbf{A}$ and
- one in $B$.


## Notes

The sum of the two sequences must be equal: sum(aseq)=sum(bseq) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.

## havel_hakimi_graph

## havel_hakimi_graph (aseq, bseq, create_using=None)

Return a bipartite graph from two given degree sequences using a Havel-Hakimi style construction.
Nodes from the set A are connected to nodes in the set B by connecting the highest degree nodes in set A to the highest degree nodes in set B until all stubs are connected.

## Parameters

- aseq (list) - Degree sequence for node set A.
- bseq (list) - Degree sequence for node set B.
- create_using (NetworkX graph instance, optional) - Return graph of this type.


## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.

The sum of the two sequences must be equal: sum(aseq)=sum(bseq) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.
reverse_havel_hakimi_graph
reverse_havel_hakimi_graph (aseq, bseq, create_using=None)
Return a bipartite graph from two given degree sequences using a Havel-Hakimi style construction.
Nodes from set A are connected to nodes in the set B by connecting the highest degree nodes in set A to the lowest degree nodes in set B until all stubs are connected.

## Parameters

- aseq (list) - Degree sequence for node set A.
- bseq (list) - Degree sequence for node set B.
- create_using (NetworkX graph instance, optional) - Return graph of this type.


## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.
The sum of the two sequences must be equal: sum(aseq)=sum(bseq) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.
The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.

## alternating_havel_hakimi_graph

alternating_havel_hakimi_graph (aseq, bseq, create_using=None)
Return a bipartite graph from two given degree sequences using an alternating Havel-Hakimi style construction.
Nodes from the set A are connected to nodes in the set B by connecting the highest degree nodes in set A to alternatively the highest and the lowest degree nodes in set B until all stubs are connected.

## Parameters

- aseq (list) - Degree sequence for node set A.
- bseq (list) - Degree sequence for node set B.
- create_using (NetworkX graph instance, optional) - Return graph of this type.


## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.
The sum of the two sequences must be equal: sum(aseq)=sum(bseq) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.

## preferential_attachment_graph

preferential_attachment_graph (aseq, p, create_using=None, seed=None)
Create a bipartite graph with a preferential attachment model from a given single degree sequence.

## Parameters

- aseq (list) - Degree sequence for node set A.
- $\mathbf{p}$ (float) - Probability that a new bottom node is added.
- create_using (NetworkX graph instance, optional) - Return graph of this type.
- seed (integer, optional) - Seed for random number generator.


## References

## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.

```
random_graph
```

random_graph ( $n, m, p$, seed $=$ None, directed $=$ False )
Return a bipartite random graph.
This is a bipartite version of the binomial (Erdős-Rényi) graph.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes in the first bipartite set.
- m (int) - The number of nodes in the second bipartite set.
- $\mathbf{p}$ (float) - Probability for edge creation.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True return a directed graph


## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.
The bipartite random graph algorithm chooses each of the $n * m$ (undirected) or $2 * n m$ (directed) possible edges with probability p .

This algorithm is $\mathrm{O}(\mathrm{n}+\mathrm{m})$ where m is the expected number of edges.
The nodes are assigned the attribute 'bipartite' with the value 0 or 1 to indicate which bipartite set the node belongs to.

## See also:

```
gnp_random_graph(), configuration_model()
```


## References

```
gnmk_random_graph
```

gnmk_random_graph ( $n, m, k$, seed=None, directed=False)
Return a random bipartite graph $\mathrm{G}_{-}\{\mathrm{n}, \mathrm{m}, \mathrm{k}\}$.
Produces a bipartite graph chosen randomly out of the set of all graphs with $n$ top nodes, $m$ bottom nodes, and k edges.

## Parameters

- $\mathbf{n}(i n t)$ - The number of nodes in the first bipartite set.
- m (int) - The number of nodes in the second bipartite set.
- $\mathbf{k}$ (int) - The number of edges
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True return a directed graph


## Examples

from networkx.algorithms import bipartite $\mathrm{G}=$ bipartite.gnmk_random_graph(10,20,50)
See also:
gnm_random_graph ()

## Notes

This function is not imported in the main namespace. To use it you have to explicitly import the bipartite package.

If $\mathrm{k}>\mathrm{m} * \mathrm{n}$ then a complete bipartite graph is returned.
This graph is a bipartite version of the $\mathrm{G}_{-}\{\mathrm{nm}\}$ random graph model.

### 4.3.10 Covering

Functions related to graph covers.

| min_edge_cover(G[, matching_algorithm])Returns a set of edges which constitutes the minimum edge <br> cover of the graph. |
| :--- |
| min_edge_cover |
| min_edge_cover (G, matching_algorithm=None) |
| Returns a set of edges which constitutes the minimum edge cover of the graph. |
| The smallest edge cover can be found in polynomial time by finding a maximum matching and extending it |
| greedily so that all nodes are covered. |
| Parameters |
| •G (NetworkX graph $)$ - An undirected bipartite graph. |

- matching_algorithm (function) - A function that returns a maximum cardinality matching in a given bipartite graph. The function must take one input, the graph $G$, and return a dictionary mapping each node to its mate. If not specified, hopcroft_karp_matching () will be used. Other possibilities include eppstein_matching(),

Returns A set of the edges in a minimum edge cover of the graph, given as pairs of nodes. It contains both the edges $(u, v)$ and $(v, u)$ for given nodes $u$ and $v$ among the edges of minimum edge cover.

## Return type set

## Notes

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of the set. A minimum edge cover is an edge covering of smallest cardinality.
Due to its implementation, the worst-case running time of this algorithm is bounded by the worst-case running time of the function matching_algorithm.

### 4.4 Boundary

Routines to find the boundary of a set of nodes.
An edge boundary is a set of edges, each of which has exactly one endpoint in a given set of nodes (or, in the case of directed graphs, the set of edges whose source node is in the set).

A node boundary of a set $S$ of nodes is the set of (out-)neighbors of nodes in $S$ that are outside $S$.

| edge_boundary(G, nbunch1[, nbunch2, data, ...]) | Returns the edge boundary of nbunch1. |
| :--- | :--- |
| node_boundary(G, nbunch1[, nbunch2]) | Returns the node boundary of nbunch1. |

### 4.4.1 edge_boundary

edge_boundary ( $G$, nbunch1, nbunch $2=$ None, data=False, keys=False, default=None)
Returns the edge boundary of nbunch1.
The edge boundary of a set $S$ with respect to a set $T$ is the set of edges $(u, v)$ such that $u$ is in $S$ and $v$ is in $T$. If $T$ is not specified, it is assumed to be the set of all nodes not in $S$.

## Parameters

- G (NetworkX graph)
- nbunch1 (iterable) - Iterable of nodes in the graph representing the set of nodes whose edge boundary will be returned. (This is the set $S$ from the definition above.)
- nbunch2 (iterable) - Iterable of nodes representing the target (or "exterior") set of nodes. (This is the set $T$ from the definition above.) If not specified, this is assumed to be the set of all nodes in G not in nbunch1.
- keys (bool) - This parameter has the same meaning as in MultiGraph.edges ().
- data (bool or object) - This parameter has the same meaning as in MultiGraph.edges().
- default (object) - This parameter has the same meaning as in MultiGraph.edges ().

Returns An iterator over the edges in the boundary of nbunch1 with respect to nbunch2. If keys, data, or default are specified and $G$ is a multigraph, then edges are returned with keys and/or data, as in MultiGraph.edges ().
Return type iterator

## Notes

Any element of nbunch that is not in the graph G will be ignored.
nbunch1 and nbunch2 are usually meant to be disjoint, but in the interest of speed and generality, that is not required here.

### 4.4.2 node_boundary

node_boundary ( $G$, nbunch1, nbunch $2=$ None)
Returns the node boundary of nbunch1.
The node boundary of a set $S$ with respect to a set $T$ is the set of nodes $v$ in $T$ such that for some $u$ in $S$, there is an edge joining $u$ to $v$. If $T$ is not specified, it is assumed to be the set of all nodes not in $S$.

## Parameters

- G (NetworkX graph)
- nbunch1 (iterable) - Iterable of nodes in the graph representing the set of nodes whose node boundary will be returned. (This is the set $S$ from the definition above.)
- nbunch2 (iterable) - Iterable of nodes representing the target (or "exterior") set of nodes. (This is the set $T$ from the definition above.) If not specified, this is assumed to be the set of all nodes in G not in nbunch1.

Returns The node boundary of nbunch1 with respect to nbunch2.
Return type set

## Notes

Any element of nbunch that is not in the graph $G$ will be ignored.
nbunch1 and nbunch2 are usually meant to be disjoint, but in the interest of speed and generality, that is not required here.

### 4.5 Centrality

### 4.5.1 Degree

| degree_centrality $(\mathrm{G})$ | Compute the degree centrality for nodes. |
| :--- | :--- |
| in_degree_centrality $(\mathrm{G})$ | Compute the in-degree centrality for nodes. |
| out_degree_centrality $(\mathrm{G})$ | Compute the out-degree centrality for nodes. |

degree_centrality
degree_centrality ( $G$ )
Compute the degree centrality for nodes.
The degree centrality for a node v is the fraction of nodes it is connected to.
Parameters G (graph) - A networkx graph
Returns nodes - Dictionary of nodes with degree centrality as the value.
Return type dictionary

## See also:

betweenness_centrality(), load_centrality(), eigenvector_centrality()

## Notes

The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph $\mathrm{n}-1$ where n is the number of nodes in G .

For multigraphs or graphs with self loops the maximum degree might be higher than $\mathrm{n}-1$ and values of degree centrality greater than 1 are possible.

```
in_degree_centrality
```

```
in_degree_centrality(G)
```

Compute the in-degree centrality for nodes.
The in-degree centrality for a node v is the fraction of nodes its incoming edges are connected to.
Parameters G (graph) - A NetworkX graph
Returns nodes - Dictionary of nodes with in-degree centrality as values.
Return type dictionary
Raises NetworkXNotImplemented: - If G is undirected.

## See also:

degree_centrality(), out_degree_centrality()

## Notes

The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph $n-1$ where n is the number of nodes in G .

For multigraphs or graphs with self loops the maximum degree might be higher than $\mathrm{n}-1$ and values of degree centrality greater than 1 are possible.

```
out_degree_centrality
```

out_degree_centrality $(G)$
Compute the out-degree centrality for nodes.
The out-degree centrality for a node v is the fraction of nodes its outgoing edges are connected to.

Parameters G (graph) - A NetworkX graph
Returns nodes - Dictionary of nodes with out-degree centrality as values.
Return type dictionary
Raises NetworkXNotImplemented: - If G is undirected.

## See also:

degree_centrality(), in_degree_centrality()

## Notes

The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph $\mathrm{n}-1$ where n is the number of nodes in G .

For multigraphs or graphs with self loops the maximum degree might be higher than $\mathrm{n}-1$ and values of degree centrality greater than 1 are possible.

### 4.5.2 Eigenvector

| eigenvector_centrality $\left(\mathrm{G}\left[, \max \_\right.\right.$iter, tol, ...] $)$ | Compute the eigenvector centrality for the graph G. |
| :--- | :--- |
| eigenvector_centrality_numpy $(\mathrm{G}[$, weight, ...]) | Compute the eigenvector centrality for the graph G. |
| katz_centrality $(\mathrm{G}[$, alpha, beta, max_iter, ...]) | Compute the Katz centrality for the nodes of the graph G. |
| katz_centrality_numpy $(\mathrm{G}[$, alpha, beta, ...]) | Compute the Katz centrality for the graph G. |

## eigenvector_centrality

```
eigenvector_centrality (G, max_iter=100, tol=le-06, nstart=None,weight='weight')
```

Compute the eigenvector centrality for the graph $G$.
Eigenvector centrality computes the centrality for a node based on the centrality of its neighbors. The eigenvector centrality for node $i$ is

$$
\mathbf{A} \mathbf{x}=\lambda \mathbf{x}
$$

where $A$ is the adjacency matrix of the graph $G$ with eigenvalue $\lambda$. By virtue of the Perron-Frobenius theorem, there is a unique and positive solution if $\lambda$ is the largest eigenvalue associated with the eigenvector of the adjacency matrix $A\left({ }^{2}\right)$.

## Parameters

- G (graph) - A networkx graph
- max_iter (integer, optional) - Maximum number of iterations in power method.
- tol (float, optional) - Error tolerance used to check convergence in power method iteration.
- nstart (dictionary, optional) - Starting value of eigenvector iteration for each node.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns nodes - Dictionary of nodes with eigenvector centrality as the value.
Return type dictionary

[^22]
## Examples

```
>>> G = nx.path_graph(4)
>>> centrality = nx.eigenvector_centrality(G)
>>> sorted((v, '{:0.2f}'.format(c)) for v, c in centrality.items())
[(0, '0.37'), (1, '0.60'), (2, '0.60'), (3, '0.37')]
```


## Raises

- NetworkXPointlessConcept - If the graph $G$ is the null graph.
- NetworkXError - If each value in nstart is zero.
- PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.


## See also:

```
eigenvector_centrality_numpy(), pagerank(), hits()
```


## Notes

The measure was introduced by ${ }^{1}$ and is discussed in ${ }^{2}$.
The power iteration method is used to compute the eigenvector and convergence is not guaranteed. Our method stops after max_iter iterations or when the change in the computed vector between two iterations is smaller than an error tolerance of G.number_of_nodes () * tol. This implementation uses $(A+I)$ rather than the adjacency matrix $A$ because it shifts the spectrum to enable discerning the correct eigenvector even for networks with multiple dominant eigenvalues.

For directed graphs this is "left" eigenvector centrality which corresponds to the in-edges in the graph. For out-edges eigenvector centrality first reverse the graph with G.reverse ().

## References

eigenvector_centrality_numpy
eigenvector_centrality_numpy ( $G$, weight='weight', max_iter=50,tol=0)
Compute the eigenvector centrality for the graph G.
Eigenvector centrality computes the centrality for a node based on the centrality of its neighbors. The eigenvector centrality for node $i$ is

$$
\mathbf{A} \mathbf{x}=\lambda \mathbf{x}
$$

where $A$ is the adjacency matrix of the graph $G$ with eigenvalue lambda. By virtue of the Perron-Frobenius theorem, there is a unique and positive solution if lambda is the largest eigenvalue associated with the eigenvector of the adjacency matrix $A\left({ }^{2}\right)$.

## Parameters

> - G (graph) - A networkx graph

[^23]- weight (None or string, optional) - The name of the edge attribute used as weight. If None, all edge weights are considered equal.
- max_iter (integer, optional) - Maximum number of iterations in power method.
- tol (float, optional) - Relative accuracy for eigenvalues (stopping criterion). The default value of 0 implies machine precision.

Returns nodes - Dictionary of nodes with eigenvector centrality as the value.
Return type dictionary

## Examples

```
>>> G = nx.path_graph(4)
>>> centrality = nx.eigenvector_centrality_numpy(G)
>>> print(['%S %0.2f'%(node,centrality[node]) for node in centrality])
['0 0.37','1 0.60',''2 0.60', '3 0.37']
```


## See also:

```
eigenvector_centrality(),pagerank(),hits()
```


## Notes

The measure was introduced by ${ }^{1}$.
This algorithm uses the SciPy sparse eigenvalue solver (ARPACK) to find the largest eigenvalue/eigenvector pair.
For directed graphs this is "left" eigenvector centrality which corresponds to the in-edges in the graph. For out-edges eigenvector centrality first reverse the graph with G.reverse().

Raises NetworkXPointlessConcept - If the graph G is the null graph.

## References

## katz_centrality

katz_centrality ( $G$, alpha=0.1, beta=1.0, max_iter=1000, tol=le-06, nstart=None, normalized=True, weight='weight')
Compute the Katz centrality for the nodes of the graph G.
Katz centrality computes the centrality for a node based on the centrality of its neighbors. It is a generalization of the eigenvector centrality. The Katz centrality for node $i$ is

$$
x_{i}=\alpha \sum_{j} A_{i j} x_{j}+\beta
$$

where $A$ is the adjacency matrix of the graph $G$ with eigenvalues lambda.
The parameter beta controls the initial centrality and

$$
\alpha<\frac{1}{\lambda_{\max }} .
$$

[^24]Katz centrality computes the relative influence of a node within a network by measuring the number of the immediate neighbors (first degree nodes) and also all other nodes in the network that connect to the node under consideration through these immediate neighbors.

Extra weight can be provided to immediate neighbors through the parameter $\beta$. Connections made with distant neighbors are, however, penalized by an attenuation factor alpha which should be strictly less than the inverse largest eigenvalue of the adjacency matrix in order for the Katz centrality to be computed correctly. More information is provided in ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- alpha (float) - Attenuation factor
- beta (scalar or dictionary, optional (default=1.0)) - Weight attributed to the immediate neighborhood. If not a scalar, the dictionary must have an value for every node.
- max_iter (integer, optional (default=1000)) - Maximum number of iterations in power method.
- tol (float, optional (default=1.0e-6)) - Error tolerance used to check convergence in power method iteration.
- nstart (dictionary, optional) - Starting value of Katz iteration for each node.
- normalized (bool, optional (default=True)) - If True normalize the resulting values.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.
Returns nodes - Dictionary of nodes with Katz centrality as the value.
Return type dictionary


## Raises

- NetworkXError - If the parameter beta is not a scalar but lacks a value for at least one node
- PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.


## Examples

```
>>> import math
>>> G = nx.path_graph(4)
>>> phi = (1+math.sqrt(5))/2.0 # largest eigenvalue of adj matrix
>>> centrality = nx.katz_centrality(G,1/phi-0.01)
>>> for n,c in sorted(centrality.items()):
... print("%d %0.2f"%(n,c))
0 0.37
1 0.60
2 0.60
3 0.37
```


## See also:

katz_centrality_numpy(), eigenvector_centrality(), eigenvector_centrality_numpy(), pagerank(), hits()

[^25]
## Notes

Katz centrality was introduced by ${ }^{2}$.
This algorithm it uses the power method to find the eigenvector corresponding to the largest eigenvalue of the adjacency matrix of G. The constant alpha should be strictly less than the inverse of largest eigenvalue of the adjacency matrix for the algorithm to converge. The iteration will stop after max_iter iterations or an error tolerance of number_of_nodes(G)*tol has been reached.

When alpha = $1 /$ lambda_\{max $\}$ and beta=0, Katz centrality is the same as eigenvector centrality.
For directed graphs this finds "left" eigenvectors which corresponds to the in-edges in the graph. For out-edges Katz centrality first reverse the graph with G.reverse().

## References

## katz_centrality_numpy

katz_centrality_numpy ( $G$, alpha=0.1, beta $=1.0$, normalized $=$ True, weight='weight')
Compute the Katz centrality for the graph G.
Katz centrality computes the centrality for a node based on the centrality of its neighbors. It is a generalization of the eigenvector centrality. The Katz centrality for node $i$ is

$$
x_{i}=\alpha \sum_{j} A_{i j} x_{j}+\beta,
$$

where A is the adjacency matrix of the graph G with eigenvalues lambda.
The parameter bet a controls the initial centrality and

$$
\alpha<\frac{1}{\lambda_{\max }} .
$$

Katz centrality computes the relative influence of a node within a network by measuring the number of the immediate neighbors (first degree nodes) and also all other nodes in the network that connect to the node under consideration through these immediate neighbors.
Extra weight can be provided to immediate neighbors through the parameter $\beta$. Connections made with distant neighbors are, however, penalized by an attenuation factor alpha which should be strictly less than the inverse largest eigenvalue of the adjacency matrix in order for the Katz centrality to be computed correctly. More information is provided in ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- alpha (float) - Attenuation factor
- beta (scalar or dictionary, optional (default=1.0)) - Weight attributed to the immediate neighborhood. If not a scalar the dictionary must have an value for every node.
- normalized (bool) - If True normalize the resulting values.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

[^26]Returns nodes - Dictionary of nodes with Katz centrality as the value.
Return type dictionary
Raises NetworkXError - If the parameter beta is not a scalar but lacks a value for at least one node

## Examples

```
>>> import math
>>> G = nx.path_graph(4)
>>> phi = (1+math.sqrt(5))/2.0 # largest eigenvalue of adj matrix
>>> centrality = nx.katz_centrality_numpy(G,1/phi)
>>> for n,c in sorted(centrality.items()):
... print("%d %0.2f"%(n,c))
0 0.37
1 0.60
2 0.60
30.37
```


## See also:

```
katz_centrality(), eigenvector_centrality_numpy(), eigenvector_centrality(),
pagerank(),hits()
```


## Notes

Katz centrality was introduced by ${ }^{2}$.
This algorithm uses a direct linear solver to solve the above equation. The constant alpha should be strictly less than the inverse of largest eigenvalue of the adjacency matrix for there to be a solution. When alpha = $1 /$ lambda_\{max $\}$ and beta=0, Katz centrality is the same as eigenvector centrality.

For directed graphs this finds "left" eigenvectors which corresponds to the in-edges in the graph. For out-edges Katz centrality first reverse the graph with G.reverse().

## References

### 4.5.3 Closeness

closeness_centrality $(\mathrm{G}[, \mathrm{u}$, distance, ...]) Compute closeness centrality for nodes.

```
closeness_centrality
```

closeness_centrality ( $G, u=$ None, distance=None, normalized $=$ True )

Compute closeness centrality for nodes.
Closeness centrality ${ }^{1}$ of a node $u$ is the reciprocal of the sum of the shortest path distances from $u$ to all $n-1$ other nodes. Since the sum of distances depends on the number of nodes in the graph, closeness is normalized

[^27]by the sum of minimum possible distances $n-1$.
$$
C(u)=\frac{n-1}{\sum_{v=1}^{n-1} d(v, u)}
$$
where $d(v, u)$ is the shortest-path distance between $v$ and $u$, and $n$ is the number of nodes in the graph.
Notice that higher values of closeness indicate higher centrality.

## Parameters

- G (graph) - A NetworkX graph
- u (node, optional) - Return only the value for node $u$
- distance (edge attribute key, optional (default=None)) - Use the specified edge attribute as the edge distance in shortest path calculations
- normalized (bool, optional) - If True (default) normalize by the number of nodes in the connected part of the graph.
Returns nodes - Dictionary of nodes with closeness centrality as the value.
Return type dictionary
See also:
betweenness_centrality(), load_centrality(), eigenvector_centrality(), degree_centrality()


## Notes

The closeness centrality is normalized to $(\mathrm{n}-1) /(|\mathrm{G}|-1)$ where n is the number of nodes in the connected part of graph containing the node. If the graph is not completely connected, this algorithm computes the closeness centrality for each connected part separately.
If the 'distance' keyword is set to an edge attribute key then the shortest-path length will be computed using Dijkstra's algorithm with that edge attribute as the edge weight.

## References

### 4.5.4 Current Flow Closeness

current_flow_closeness_centrality(G[, ...]) Compute current-flow closeness centrality for nodes.
current_flow_closeness_centrality
current_flow_closeness_centrality (G, weight='weight', dtype $=<$ type 'float'>, solver='lu')
Compute current-flow closeness centrality for nodes.
Current-flow closeness centrality is variant of closeness centrality based on effective resistance between nodes in a network. This metric is also known as information centrality.

## Parameters

- G (graph) - A NetworkX graph
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float32 for lower
memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).
Returns nodes - Dictionary of nodes with current flow closeness centrality as the value.
Return type dictionary


## See also:

```
closeness_centrality()
```


## Notes

The algorithm is from Brandes ${ }^{1}$.
See also ${ }^{2}$ for the original definition of information centrality.

## References

### 4.5.5 (Shortest Path) Betweenness

| betweenness_centrality $(\mathrm{G}[, \mathrm{k}$, normalized, ...]) | Compute the shortest-path betweenness centrality for <br> nodes. |
| :--- | :--- |
| edge_betweenness_centrality $(\mathrm{G}[, \mathrm{k}, \ldots])$ | Compute betweenness centrality for edges. |
| betweenness_centrality_subset $(\mathrm{G}, \operatorname{sources,~...)~}$ | Compute betweenness centrality for a subset of nodes. |
| edge_betweenness_centrality_subset $(\mathrm{G}, \ldots[$, | Compute betweenness centrality for edges for a subset of <br> nodes. |

## betweenness_centrality

betweenness_centrality ( $G, k=$ None, normalized $=$ True, weight $=$ None, endpoints $=$ False, , eed $=$ None )
Compute the shortest-path betweenness centrality for nodes.
Betweenness centrality of a node v is the sum of the fraction of all-pairs shortest paths that pass through v

$$
c_{B}(v)=\sum_{s, t \in V} \frac{\sigma(s, t \mid v)}{\sigma(s, t)}
$$

where V is the set of nodes, $\sigma(s, t)$ is the number of shortest $(\mathrm{s}, \mathrm{t})$-paths, and $\sigma(s, t \mid v)$ is the number of those paths passing through some node v other than $\mathrm{s}, \mathrm{t}$. If $s=t, \sigma(s, t)=1$, and if $v \in s, t, \sigma(s, t \mid v)=0^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- $\mathbf{k}$ (int, optional (default=None)) - If k is not None use k node samples to estimate betweenness. The value of $\mathrm{k}<=\mathrm{n}$ where n is the number of nodes in the graph. Higher values give better approximation.

[^28]- normalized (bool, optional) - If True the betweenness values are normalized by $2 /((n-1)(n-2))$ for graphs, and $1 /((n-1)(n-2))$ for directed graphs where $n$ is the number of nodes in $G$.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.
- endpoints (bool, optional) - If True include the endpoints in the shortest path counts.

Returns nodes - Dictionary of nodes with betweenness centrality as the value.
Return type dictionary

## See also:

```
edge_betweenness_centrality(),load_centrality()
```


## Notes

The algorithm is from Ulrik Brandes ${ }^{1}$. See ${ }^{4}$ for the original first published version and ${ }^{2}$ for details on algorithms for variations and related metrics.

For approximate betweenness calculations set $\mathrm{k}=\#$ samples to use k nodes ("pivots") to estimate the betweenness values. For an estimate of the number of pivots needed see ${ }^{3}$.

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

## References

edge_betweenness_centrality
edge_betweenness_centrality ( $G, k=$ None, normalized $=$ True, weight $=$ None, seed $=$ None )
Compute betweenness centrality for edges.
Betweenness centrality of an edge $e$ is the sum of the fraction of all-pairs shortest paths that pass through e

$$
c_{B}(e)=\sum_{s, t \in V} \frac{\sigma(s, t \mid e)}{\sigma(s, t)}
$$

where V is the set of nodes, $\sigma(s, t)$ is the number of shortest ( $\mathrm{s}, \mathrm{t}$ ) -paths, and $\sigma(s, t \mid e)$ is the number of those paths passing through edge $e^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- $\mathbf{k}$ (int, optional (default=None)) - If k is not None use k node samples to estimate betweenness. The value of $\mathrm{k}<=\mathrm{n}$ where n is the number of nodes in the graph. Higher values give better approximation.

[^29]- normalized (bool, optional) - If True the betweenness values are normalized by $2 /(n(n-1))$ for graphs, and $1 /(n(n-1))$ for directed graphs where $n$ is the number of nodes in $G$.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns edges - Dictionary of edges with betweenness centrality as the value.
Return type dictionary

## See also:

betweenness_centrality(), edge_load()

## Notes

The algorithm is from Ulrik Brandes ${ }^{1}$.
For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

## References

## betweenness_centrality_subset

betweenness_centrality_subset ( $G$, sources, targets, normalized=False, weight=None)
Compute betweenness centrality for a subset of nodes.

$$
c_{B}(v)=\sum_{s \in S, t \in T} \frac{\sigma(s, t \mid v)}{\sigma(s, t)}
$$

where S is the set of sources, T is the set of targets, $\sigma(s, t)$ is the number of shortest ( $\mathrm{s}, \mathrm{t}$ ) -paths, and $\sigma(s, t \mid v)$ is the number of those paths passing through some node $v$ other than $s, t$. If $s=t, \sigma(s, t)=1$, and if $v \in s, t, \sigma(s, t \mid v)=0^{2}$.

## Parameters

- G (graph)
- sources (list of nodes) - Nodes to use as sources for shortest paths in betweenness
- targets (list of nodes) - Nodes to use as targets for shortest paths in betweenness
- normalized (bool, optional) - If True the betweenness values are normalized by $2 /((n-1)(n-2))$ for graphs, and $1 /((n-1)(n-2))$ for directed graphs where $n$ is the number of nodes in $G$.
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns nodes - Dictionary of nodes with betweenness centrality as the value.
Return type dictionary

[^30]
## See also:

```
edge_betweenness_centrality(), load_centrality()
```


## Notes

The basic algorithm is from ${ }^{1}$.
For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

The normalization might seem a little strange but it is the same as in betweenness_centrality() and is designed to make betweenness_centrality(G) be the same as betweenness_centrality_subset(G,sources=G.nodes(),targets=G.nodes()).

## References

edge_betweenness_centrality_subset
edge_betweenness_centrality_subset ( $G$, sources, targets, normalized=False, weight=None)
Compute betweenness centrality for edges for a subset of nodes.

$$
c_{B}(v)=\sum_{s \in S, t \in T} \frac{\sigma(s, t \mid e)}{\sigma(s, t)}
$$

where S is the set of sources, T is the set of targets, $\sigma(s, t)$ is the number of shortest ( $\mathrm{s}, \mathrm{t}$ ) -paths, and $\sigma(s, t \mid e)$ is the number of those paths passing through edge $\mathrm{e}^{2}$.

## Parameters

- G (graph) - A networkx graph
- sources (list of nodes) - Nodes to use as sources for shortest paths in betweenness
- targets (list of nodes) - Nodes to use as targets for shortest paths in betweenness
- normalized (bool, optional) - If True the betweenness values are normalized by $2 /(n(n-1))$ for graphs, and $1 /(n(n-1))$ for directed graphs where $n$ is the number of nodes in G .
- weight (None or string, optional) - If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns edges - Dictionary of edges with Betweenness centrality as the value.
Return type dictionary

## See also:

betweenness_centrality(), edge_load()

[^31]
## Notes

The basic algorithm is from ${ }^{1}$.
For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

The normalization might seem a little strange but it is the same as in edge_betweenness_centrality() and is designed to make edge_betweenness_centrality( $G$ ) be the same as edge_betweenness_centrality_subset(G,sources=G.nodes(),targets=G.nodes()).

## References

### 4.5.6 Current Flow Betweenness

| ```current_flow_betweenness_centrality(G[, ...])``` | Compute current-flow betweenness centrality for nodes |
| :---: | :---: |
| edge_current_flow_betweenness_centrality(G)ompute current-flow betweenness centrality for edges. |  |
| approximate_current_flow_betweenness_cen | Comptate( (Th) approximate current-flow betweenness centrality for nodes. |
| current_flow_betweenness_centrality_subse $\ldots)$ | (ompute current-flow betweenness centrality for subsets of odes. |
| edge_current_flow_betweenness_centrality_ ...) | Compate (Current-flow betweenness centrality for edges using subsets of nodes. |

current_flow_betweenness_centrality
current_flow_betweenness_centrality ( $G$, normalized=True, weight='weight', dtype=<type 'float'>, solver $=$ 'full')
Compute current-flow betweenness centrality for nodes.
Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.
Current-flow betweenness centrality is also known as random-walk betweenness centrality ${ }^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- normalized (bool, optional (default=True)) - If True the betweenness values are normalized by $2 /[(n-1)(n-2)]$ where $n$ is the number of nodes in $G$.
- weight (string or None, optional (default='weight')) - Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).
Returns nodes - Dictionary of nodes with betweenness centrality as the value.

[^32]Return type dictionary

## See also:

approximate_current_flow_betweenness_centrality(), betweenness_centrality(), edge_betweenness_centrality(), edge_current_flow_betweenness_centrality()

## Notes

Current-flow betweenness can be computed in $O(I(n-1)+m n \log n)$ time ${ }^{1}$, where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $O\left(n^{\wedge} 3\right)$ but using sparse methods you can achieve $O$ ( $n m\{$ sqrt $k\}$ ) where $k$ is the Laplacian matrix condition number.

The space required is $O(n w)$ where $w$ is the width of the sparse Laplacian matrix. Worse case is $w=n$ for O ( $\mathrm{n}^{\wedge} 2$ ).

If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1 .

## References

edge_current_flow_betweenness_centrality
edge_current_flow_betweenness_centrality( $G$, normalized=True, weight='weight', dtype=<type 'float'>, solver='full')
Compute current-flow betweenness centrality for edges.
Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality ${ }^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- normalized (bool, optional (default=True)) - If True the betweenness values are normalized by $2 /[(n-1)(n-2)]$ where $n$ is the number of nodes in $G$.
- weight (string or None, optional (default='weight')) - Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).

Returns nodes - Dictionary of edge tuples with betweenness centrality as the value.
Return type dictionary
Raises NetworkXError - The algorithm does not support DiGraphs. If the input graph is an instance of DiGraph class, NetworkXError is raised.

[^33]
## See also:

```
betweenness_centrality(),
edge_betweenness_centrality(),
```

current_flow_betweenness_centrality()

## Notes

Current-flow betweenness can be computed in $O(I(n-1)+m n \log n)$ time ${ }^{1}$, where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $O\left(n^{\wedge} 3\right)$ but using sparse methods you can achieve $O$ ( $\mathrm{nm}\{$ sqrt $k\}$ ) where $k$ is the Laplacian matrix condition number.

The space required is $O$ ( nw ) where ' w is the width of the sparse Laplacian matrix. Worse case is $\mathrm{w}=\mathrm{n}$ for $O\left(n^{\wedge} 2\right)$.
If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1 .

## References

```
approximate_current_flow_betweenness_centrality
```

approximate_current_flow_betweenness_centrality (G, normalized=True, weight='weight', dtype $=$ <type 'float'>, solver='full', epsilon $=0.5, k m a x=10000$ )
Compute the approximate current-flow betweenness centrality for nodes.
Approximates the current-flow betweenness centrality within absolute error of epsilon with high probability ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- normalized (bool, optional (default=True)) - If True the betweenness values are normalized by $2 /[(n-1)(n-2)]$ where $n$ is the number of nodes in $G$.
- weight (string or None, optional (default='weight')) - Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).
- epsilon (float) - Absolute error tolerance.
- kmax (int) - Maximum number of sample node pairs to use for approximation.

Returns nodes - Dictionary of nodes with betweenness centrality as the value.
Return type dictionary

## See also:

current_flow_betweenness_centrality()

[^34]
## Notes

The running time is $O\left(\left(1 / e p s i l o n^{\wedge} 2\right) m\{s q r t k\} \log n\right)$ and the space required $O(m)$ for $n$ nodes and $m$ edges.

If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1 .

## References

```
current_flow_betweenness_centrality_subset
```

current_flow_betweenness_centrality_subset ( $G$, sources, targets, normalized=True,
weight='weight', dtype=<type 'float'>,
solver='lu')

Compute current-flow betweenness centrality for subsets of nodes.
Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.
Current-flow betweenness centrality is also known as random-walk betweenness centrality ${ }^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- sources (list of nodes) - Nodes to use as sources for current
- targets (list of nodes) - Nodes to use as sinks for current
- normalized (bool, optional (default=True)) - If True the betweenness values are normalized by $\mathrm{b}=\mathrm{b} /(\mathrm{n}-1)(\mathrm{n}-2)$ where n is the number of nodes in G .
- weight (string or None, optional (default='weight')) - Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float 32 for lower memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).

Returns nodes - Dictionary of nodes with betweenness centrality as the value.
Return type dictionary

## See also:

approximate_current_flow_betweenness_centrality(), betweenness_centrality(), edge_betweenness_centrality(), edge_current_flow_betweenness_centrality()

## Notes

Current-flow betweenness can be computed in $O(I(n-1)+m n \log n)$ time ${ }^{1}$, where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $0\left(n^{\wedge} 3\right)$ but using sparse methods you can achieve $O$ ( $\mathrm{nm}\{$ sqrt $k$ ) where $k$ is the Laplacian matrix condition number.

[^35]The space required is $O(n w)$ where ' w is the width of the sparse Laplacian matrix. Worse case is $\mathrm{w}=\mathrm{n}$ for $O\left(n^{\wedge} 2\right)$.

If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1 .

## References

```
edge_current_flow_betweenness_centrality_subset
edge_current_flow_betweenness_centrality_subset (G, sources, targets, normalized=True,
                                    weight='weight', dtype=<type
                            'float'>, solver='lu')
```

Compute current-flow betweenness centrality for edges using subsets of nodes.
Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality ${ }^{2}$.

## Parameters

- G (graph) - A NetworkX graph
- sources (list of nodes) - Nodes to use as sources for current
- targets (list of nodes) - Nodes to use as sinks for current
- normalized (bool, optional (default=True)) - If True the betweenness values are normalized by $\mathrm{b}=\mathrm{b} /(\mathrm{n}-1)(\mathrm{n}-2)$ where n is the number of nodes in G .
- weight (string or None, optional (default='weight')) - Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) - Default data type for internal matrices. Set to np.float 32 for lower memory consumption.
- solver (string (default='lu')) - Type of linear solver to use for computing the flow matrix. Options are "full" (uses most memory), "lu" (recommended), and "cg" (uses least memory).
Returns nodes - Dictionary of edge tuples with betweenness centrality as the value.
Return type dictionary


## See also:

betweenness_centrality(), edge_betweenness_centrality(),
current_flow_betweenness_centrality()

## Notes

Current-flow betweenness can be computed in $O(I(n-1)+m n \log n)$ time ${ }^{1}$, where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $O\left(n^{\wedge} 3\right)$ but using sparse methods you can achieve $O$ ( $\mathrm{nm}\{$ sqrt k$\}$ ) where k is the Laplacian matrix condition number.
The space required is $O(n w)$ where ` $w$ is the width of the sparse Laplacian matrix. Worse case is $w=n$ for $O\left(n^{\wedge} 2\right)$.

[^36]If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1 .

## References

### 4.5.7 Communicability Betweenness

communicability_betweenness_centrality (G[, Return subgraph communicability for all pairs of nodes in

| ...]) | G. |
| :--- | :--- |

communicability_betweenness_centrality
communicability_betweenness_centrality (G, normalized=True)
Return subgraph communicability for all pairs of nodes in $G$.
Communicability betweenness measure makes use of the number of walks connecting every pair of nodes as the basis of a betweenness centrality measure.

Parameters G (graph)
Returns nodes - Dictionary of nodes with communicability betweenness as the value.
Return type dictionary
Raises NetworkXError - If the graph is not undirected and simple.

## Notes

Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be a simple undirected graph with n nodes and m edges, and $A$ denote the adjacency matrix of G .
Let $G(r)=(V, E(r))$ be the graph resulting from removing all edges connected to node $r$ but not the node itself.

The adjacency matrix for $G(r)$ is $A+E(r)$, where $E(r)$ has nonzeros only in row and column $r$.
The subraph betweenness of a node $r$ is ${ }^{1}$

$$
\omega_{r}=\frac{1}{C} \sum_{p} \sum_{q} \frac{G_{p r q}}{G_{p q}}, p \neq q, q \neq r
$$

where $G \_\{p r q\}=\left(e^{\wedge}\{A\} \_\{p q\}-\left(e^{\wedge}\{A+E(r)\}\right) \_\{p q\}\right.$ is the number of walks involving node $r$, $G_{\_}\{p q\}=\left(e^{\wedge}\{A\}\right) \_\{p q\}$ is the number of closed walks starting at node $p$ and ending at node $q$, and $C=(n-1)^{\wedge}\{2\}-(n-1)$ is a normalization factor equal to the number of terms in the sum.

The resulting omega_\{r\} takes values between zero and one. The lower bound cannot be attained for a connected graph, and the upper bound is attained in the star graph.

[^37]
## References

## Examples

```
>>> G = nx.Graph ([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> cbc = nx.communicability_betweenness_centrality(G)
```


### 4.5.8 Load

| load_centrality $(\mathrm{G}[, \mathrm{v}$, cutoff, normalized, ...]) | Compute load centrality for nodes. |
| :--- | :--- |
| edge_load_centrality $(\mathrm{G}[$, cutoff $])$ | Compute edge load. |

load_centrality
load_centrality ( $G, v=$ None, cutoff=None, normalized=True, weight=None)
Compute load centrality for nodes.
The load centrality of a node is the fraction of all shortest paths that pass through that node.

## Parameters

- G (graph) - A networkx graph
- normalized (bool, optional) - If True the betweenness values are normalized by $\mathrm{b}=\mathrm{b} /(\mathrm{n}-$ 1)( $\mathrm{n}-2$ ) where n is the number of nodes in $G$.
- weight (None or string, optional) - If None, edge weights are ignored. Otherwise holds the name of the edge attribute used as weight.
- cutoff (bool, optional) - If specified, only consider paths of length <= cutoff.

Returns nodes - Dictionary of nodes with centrality as the value.
Return type dictionary

## See also:

betweenness_centrality()

## Notes

Load centrality is slightly different than betweenness. It was originally introduced by ${ }^{2}$. For this load algorithm see ${ }^{1}$.

## References

```
edge_load_centrality
```

edge_load_centrality (G, cutoff=False)

Compute edge load.

[^38]WARNING: This concept of edge load has not been analysed or discussed outside of NetworkX that we know of. It is based loosely on load_centrality in the sense that it counts the number of shortest paths which cross each edge. This function is for demonstration and testing purposes.

## Parameters

- G (graph) - A networkx graph
- cutoff (bool, optional) - If specified, only consider paths of length <= cutoff.


## Returns

- A dict keyed by edge 2-tuple to the number of shortest paths
- which use that edge. Where more than one path is shortest
- the count is divided equally among paths.


### 4.5.9 Subgraph

| subgraph_centrality $(\mathbf{G})$ | Return subgraph centrality for each node in G. |
| :--- | :--- |
| subgraph_centrality_exp $(\mathbf{G})$ | Return the subgraph centrality for each node of G. |
| estrada_index $(\mathbf{G})$ | Return the Estrada index of a the graph G. |

subgraph_centrality
subgraph_centrality ( $G$ )
Return subgraph centrality for each node in G.
Subgraph centrality of a node $n$ is the sum of weighted closed walks of all lengths starting and ending at node n. The weights decrease with path length. Each closed walk is associated with a connected subgraph $\left({ }^{1}\right)$.

## Parameters G (graph)

Returns nodes - Dictionary of nodes with subgraph centrality as the value.
Return type dictionary
Raises NetworkXError - If the graph is not undirected and simple.

## See also:

subgraph_centrality_exp () Alternative algorithm of the subgraph centrality for each node of G.

## Notes

This version of the algorithm computes eigenvalues and eigenvectors of the adjacency matrix.
Subgraph centrality of a node $u$ in $G$ can be found using a spectral decomposition of the adjacency matrix ${ }^{1}$,

$$
S C(u)=\sum_{j=1}^{N}\left(v_{j}^{u}\right)^{2} e^{\lambda_{j}}
$$

where $v_{-} j$ is an eigenvector of the adjacency matrix $A$ of $G$ corresponding corresponding to the eigenvalue lambda_j.

[^39]
## Examples

```
>>> G = nx.Graph([(1, 2),(1,5),(1,8),(2,3),(2,8),(3,4),(3,6),(4,5),(4,7),(5,6),(6,
\hookrightarrow7),(7, 8)])
>>> sc = nx.subgraph_centrality(G)
>>> print(['%S %0.2f'%(node,sc[node]) for node in sc])
['1 3.90', '2 3.90', '3 3.64', '4 3.71', '5 3.64', '6 3.71', '7 3.64', '8 3.90']
```


## References

## subgraph_centrality_exp

subgraph_centrality_exp ( $G$ )
Return the subgraph centrality for each node of G.
Subgraph centrality of a node n is the sum of weighted closed walks of all lengths starting and ending at node n. The weights decrease with path length. Each closed walk is associated with a connected subgraph $\left({ }^{1}\right)$.

## Parameters G (graph)

Returns nodes - Dictionary of nodes with subgraph centrality as the value.
Return type dictionary
Raises NetworkXError - If the graph is not undirected and simple.

## See also:

subgraph_centrality () Alternative algorithm of the subgraph centrality for each node of G.

## Notes

This version of the algorithm exponentiates the adjacency matrix.
The subgraph centrality of a node $u$ in $G$ can be found using the matrix exponential of the adjacency matrix of $G^{1}$,

$$
S C(u)=\left(e^{A}\right)_{u u} .
$$

## References

## Examples

$\left(\right.$ from $\left.{ }^{1}\right) \ggg \mathrm{G}=\mathrm{nx} . \operatorname{Graph}([(1,2),(1,5),(1,8),(2,3),(2,8),(3,4),(3,6),(4,5),(4,7),(5,6),(6,7),(7,8)]) \ggg \mathrm{sc}=$ nx.subgraph_centrality_exp(G) >>> print([' $\% \mathrm{~s} \% 0.2 \mathrm{f}$ ' $\%$ (node,sc[node]) for node in sc]) ['1 3.90', '2 3.90', '3 3.64', ‘4 3.71', ‘5 3.64', ‘6 3.71', ‘7 3.64', ‘8 3.90']

[^40]
## estrada_index

```
estrada_index(G)
```

Return the Estrada index of a the graph G.
The Estrada Index is a topological index of folding or 3D "compactness" $\left({ }^{1}\right)$.
Parameters G (graph)

## Returns estrada index

Return type float
Raises NetworkXError - If the graph is not undirected and simple.

## Notes

Let $G=(V, E)$ be a simple undirected graph with $n$ nodes and let lambda_\{1\}leqlambda_\{2\}leqcdotslambda_\{n\} be a non-increasing ordering of the eigenvalues of its adjacency matrix A. The Estrada index is $\left({ }^{1},{ }^{2}\right)$

$$
E E(G)=\sum_{j=1}^{n} e^{\lambda_{j}} .
$$

## References

## Examples

$\ggg G=n x . \operatorname{Graph}([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])$
>>> ei=nx.estrada_index (G)

### 4.5.10 Harmonic Centrality

harmonic_centrality(G[, nbunch, distance]) Compute harmonic centrality for nodes.

## harmonic_centrality

harmonic_centrality ( $G$, nbunch=None, distance=None)
Compute harmonic centrality for nodes.
Harmonic centrality ${ }^{1}$ of a node $u$ is the sum of the reciprocal of the shortest path distances from all other nodes to $u$

$$
C(u)=\sum_{v \neq u} \frac{1}{d(v, u)}
$$

where $d(v, u)$ is the shortest-path distance between $v$ and $u$.

[^41]Notice that higher values indicate higher centrality.

## Parameters

- G (graph) - A NetworkX graph
- nbunch (container) - Container of nodes. If provided harmonic centrality will be computed only over the nodes in nbunch.
- distance (edge attribute key, optional (default=None)) - Use the specified edge attribute as the edge distance in shortest path calculations. If None, then each edge will have distance equal to 1 .

Returns nodes - Dictionary of nodes with harmonic centrality as the value.
Return type dictionary

## See also:

betweenness_centrality(), load_centrality(), eigenvector_centrality(), degree_centrality(), closeness_centrality()

## Notes

If the 'distance' keyword is set to an edge attribute key then the shortest-path length will be computed using Dijkstra's algorithm with that edge attribute as the edge weight.

## References

### 4.5.11 Reaching

| local_reaching_centrality $(\mathrm{G}, \mathrm{v}[$, paths, ...]) | Returns the local reaching centrality of a node in a directed <br> graph. |
| :--- | :--- |
| global_reaching_centrality $(\mathrm{G}[$, weight, ...]) | Returns the global reaching centrality of a directed graph. |

local_reaching_centrality
local_reaching_centrality (G, v, paths=None, weight=None, normalized=True)
Returns the local reaching centrality of a node in a directed graph.
The local reaching centrality of a node in a directed graph is the proportion of other nodes reachable from that node ${ }^{1}$.

## Parameters

- G (DiGraph) - A NetworkX graph.
- $\mathbf{v}$ (node) - A node in the directed graph G .
- paths (dictionary) - If this is not None it must be a dictionary representation of singlesource shortest paths, as computed by, for example, networkx.shortest_path() with source node v . Use this keyword argument if you intend to invoke this function many times but don't want the paths to be recomputed each time.

[^42]- weight (object) - Attribute to use for edge weights. If None, each edge weight is assumed to be one. A higher weight implies a stronger connection between nodes and a shorter path length.
- normalized (bool) - Whether to normalize the edge weights by the total sum of edge weights.

Returns $\mathbf{h}$ - The local reaching centrality of the node v in the graph G .
Return type float

```
Examples
```

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge(1, 2)
>>> G.add_edge(1, 3)
>>> nx.local_reaching_centrality(G, 3)
0.0
>>> G.add_edge(3, 2)
>>> nx.local_reaching_centrality(G, 3)
0.5
```


## See also:

```
global_reaching_centrality()
```


## References

global_reaching_centrality
global_reaching_centrality ( $G$, weight=None, normalized=True)
Returns the global reaching centrality of a directed graph.
The global reaching centrality of a weighted directed graph is the average over all nodes of the difference between the local reaching centrality of the node and the greatest local reaching centrality of any node in the graph ${ }^{1}$. For more information on the local reaching centrality, see local_reaching_centrality(). Informally, the local reaching centrality is the proportion of the graph that is reachable from the neighbors of the node.

## Parameters

- G (DiGraph)
- weight (object) - Attribute to use for edge weights. If None, each edge weight is assumed to be one. A higher weight implies a stronger connection between nodes and a shorter path length.
- normalized (bool) - Whether to normalize the edge weights by the total sum of edge weights.

Returns $\mathbf{h}$ - The global reaching centrality of the graph.
Return type float

[^43]
## Examples

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge (1, 2)
>>> G.add_edge (1, 3)
>>> nx.global_reaching_centrality(G)
1.0
>>> G.add_edge (3, 2)
>>> nx.global_reaching_centrality(G)
0.75
```


## See also:

```
local_reaching_centrality()
```


## References

### 4.6 Chains

Functions for finding chains in a graph.
chain_decomposition(G[, root]) Return the chain decomposition of a graph.

### 4.6.1 chain_decomposition

chain_decomposition ( $G$, root=None)
Return the chain decomposition of a graph.
The chain decomposition of a graph with respect a depth-first search tree is a set of cycles or paths derived from the set of fundamental cycles of the tree in the following manner. Consider each fundamental cycle with respect to the given tree, represented as a list of edges beginning with the nontree edge oriented away from the root of the tree. For each fundamental cycle, if it overlaps with any previous fundamental cycle, just take the initial non-overlapping segment, which is a path instead of a cycle. Each cycle or path is called a chain. For more information, see ${ }^{1}$.

## Parameters

- G (undirected graph)
- root (node (optional)) - A node in the graph G. If specified, only the chain decomposition for the connected component containing this node will be returned. This node indicates the root of the depth-first search tree.

Yields chain (list) - A list of edges representing a chain. There is no guarantee on the orientation of the edges in each chain (for example, if a chain includes the edge joining nodes 1 and 2, the chain may include either $(1,2)$ or $(2,1))$.
Raises NodeNotFound - If root is not in the graph G.

[^44]
## Notes

The worst-case running time of this implementation is linear in the number of nodes and number of edges ${ }^{1}$.

## References

### 4.7 Chordal

Algorithms for chordal graphs.
A graph is chordal if every cycle of length at least 4 has a chord (an edge joining two nodes not adjacent in the cycle). http://en.wikipedia.org/wiki/Chordal_graph

| is_chordal $(\mathbf{G})$ | Checks whether G is a chordal graph. |
| :--- | :--- |
| chordal_graph_cliques $(\mathbf{G})$ | Returns the set of maximal cliques of a chordal graph. |
| chordal_graph_treewidt $h(\mathbf{G})$ | Returns the treewidth of the chordal graph G. |
| find_induced_nodes $(\mathrm{G}, \mathrm{s}, \mathrm{t}[$, treewidth_bound $])$ | Returns the set of induced nodes in the path from s to t. |

### 4.7.1 is_chordal

is_chordal ( $G$ )
Checks whether G is a chordal graph.
A graph is chordal if every cycle of length at least 4 has a chord (an edge joining two nodes not adjacent in the cycle).

Parameters G (graph) - A NetworkX graph.
Returns chordal - True if G is a chordal graph and False otherwise.
Return type bool
Raises NetworkXError - The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised.

## Examples

```
>>> import networkx as nx
>>> e=[(1,2),(1,3),(2,3),(2,4),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6)]
>>> G=nx.Graph(e)
>>> nx.is_chordal(G)
True
```


## Notes

The routine tries to go through every node following maximum cardinality search. It returns False when it finds that the separator for any node is not a clique. Based on the algorithms in ${ }^{1}$.

[^45]
## References

### 4.7.2 chordal_graph_cliques

## chordal_graph_cliques ( $G$ )

Returns the set of maximal cliques of a chordal graph.
The algorithm breaks the graph in connected components and performs a maximum cardinality search in each component to get the cliques.

Parameters G (graph) - A NetworkX graph

## Returns cliques

Return type A set containing the maximal cliques in $G$.
Raises NetworkXError - The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be nonchordal, a NetworkXError is raised.

## Examples

```
>>> import networkx as nx
>>> e=[(1,2),(1,3),(2,3),(2,4),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6),(7,8)]
>>> G = nx.Graph(e)
>>> G.add_node (9)
>>> setlist = nx.chordal_graph_cliques(G)
```


### 4.7.3 chordal_graph_treewidth

## chordal_graph_treewidth ( $G$ )

Returns the treewidth of the chordal graph G.
Parameters G (graph) - A NetworkX graph
Returns treewidth - The size of the largest clique in the graph minus one.
Return type int
Raises NetworkXError - The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be nonchordal, a NetworkXError is raised.

## Examples

```
>>> import networkx as nx
>>> e = [(1,2),(1,3),(2,3),(2,4),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6),(7,8)]
>>> G = nx.Graph(e)
>>> G.add_node(9)
>>> nx.chordal_graph_treewidth(G)
3
```


## References

### 4.7.4 find_induced_nodes

find_induced_nodes $(G, s, t$, treewidth_bound $=9223372036854775807$ )
Returns the set of induced nodes in the path from s to $t$.

## Parameters

- G (graph) - A chordal NetworkX graph
- $\mathbf{s}($ node $)$ - Source node to look for induced nodes
- $\mathbf{t}$ (node) - Destination node to look for induced nodes
- treewith_bound (float) - Maximum treewidth acceptable for the graph H. The search for induced nodes will end as soon as the treewidth_bound is exceeded.

Returns I - The set of induced nodes in the path from $s$ to $t$ in $G$
Return type Set of nodes
Raises NetworkXError - The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be nonchordal, a NetworkXError is raised.

Examples

```
>>> import networkx as nx
>>> G=nx.Graph()
>>> G = nx.generators.classic.path_graph(10)
>>> I = nx.find_induced_nodes(G,1,9,2)
>>> list(I)
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```


## Notes

G must be a chordal graph and ( $\mathrm{s}, \mathrm{t}$ ) an edge that is not in G .
If a treewidth_bound is provided, the search for induced nodes will end as soon as the treewidth_bound is exceeded.

The algorithm is inspired by Algorithm 4 in ${ }^{1}$. A formal definition of induced node can also be found on that reference.

## References

### 4.8 Clique

Functions for finding and manipulating cliques.

[^46]Finding the largest clique in a graph is NP-complete problem, so most of these algorithms have an exponential running time; for more information, see the Wikipedia article on the clique problem ${ }^{1}$.

| enumerate_all_cliques(G) | Returns all cliques in an undirected graph. |
| :--- | :--- |
| find_cliques(G) | Returns all maximal cliques in an undirected graph. |
| make_max_clique_graph(G[, create_using]) | Returns the maximal clique graph of the given graph. |
| make_clique_bipartite(G[, fpos, ...]) | Returns the bipartite clique graph corresponding to G. |
| graph_clique_number(G[, cliques]) | Returns the clique number of the graph. |
| graph_number_of_cliques(G[, cliques]) | Returns the number of maximal cliques in the graph. |
| node_clique_number(G[, nodes, cliques]) | Returns the size of the largest maximal clique containing <br> each given node. |
| number_of_cliques(G[, nodes, cliques]) | Returns the number of maximal cliques for each node. |
| cliques_containing_node(G[, nodes, cliques]) | Returns a list of cliques containing the given node. |

### 4.8.1 enumerate_all_cliques

```
enumerate_all_cliques (G)
```

Returns all cliques in an undirected graph.
This function returns an iterator over cliques, each of which is a list of nodes. The iteration is ordered by cardinality of the cliques: first all cliques of size one, then all cliques of size two, etc.

Parameters G (NetworkX graph) - An undirected graph.
Returns An iterator over cliques, each of which is a list of nodes in $G$. The cliques are ordered according to size.

## Return type iterator

## Notes

To obtain a list of all cliques, use list (enumerate_all_cliques (G)). However, be aware that in the worst-case, the length of this list can be exponential in the number of nodes in the graph (for example, when the graph is the complete graph). This function avoids storing all cliques in memory by only keeping current candidate node lists in memory during its search.
The implementation is adapted from the algorithm by Zhang, et al. (2005) ${ }^{1}$ to output all cliques discovered.
This algorithm ignores self-loops and parallel edges, since cliques are not conventionally defined with such edges.

## References

### 4.8.2 find_cliques

find_cliques $(G)$
Returns all maximal cliques in an undirected graph.

[^47]For each node $v$, a maximal clique for $v$ is a largest complete subgraph containing $v$. The largest maximal clique is sometimes called the maximum clique.

This function returns an iterator over cliques, each of which is a list of nodes. It is an iterative implementation, so should not suffer from recursion depth issues.

Parameters G (NetworkX graph) - An undirected graph.
Returns An iterator over maximal cliques, each of which is a list of nodes in G . The order of cliques is arbitrary.

Return type iterator

## See also:

find_cliques_recursive() A recursive version of the same algorithm.

## Notes

To obtain a list of all maximal cliques, use list (find_cliques (G)). However, be aware that in the worstcase, the length of this list can be exponential in the number of nodes in the graph (for example, when the graph is the complete graph). This function avoids storing all cliques in memory by only keeping current candidate node lists in memory during its search.

This implementation is based on the algorithm published by Bron and Kerbosch (1973) ${ }^{1}$, as adapted by Tomita, Tanaka and Takahashi (2006) ${ }^{2}$ and discussed in Cazals and Karande (2008) ${ }^{3}$. It essentially unrolls the recursion used in the references to avoid issues of recursion stack depth (for a recursive implementation, see find_cliques_recursive()).
This algorithm ignores self-loops and parallel edges, since cliques are not conventionally defined with such edges.

## References

### 4.8.3 make_max_clique_graph

make_max_clique_graph (G, create_using=None)
Returns the maximal clique graph of the given graph.
The nodes of the maximal clique graph of $G$ are the cliques of $G$ and an edge joins two cliques if the cliques are not disjoint.

## Parameters

- G (NetworkX graph)
- create_using (NetworkX graph) - If provided, this graph will be cleared and the nodes and edges of the maximal clique graph will be added to this graph.

Returns A graph whose nodes are the cliques of $G$ and whose edges join two cliques if they are not disjoint.

[^48]
## Return type NetworkX graph

## Notes

This function behaves like the following code:

```
import networkx as nx
G = nx.make_clique_bipartite(G)
cliques = [v for v in G.nodes() if G.node[v]['bipartite'] == 0]
G = nx.bipartite.project(G, cliques)
G = nx.relabel_nodes(G, {-v: v - 1 for v in G})
```

It should be faster, though, since it skips all the intermediate steps.

### 4.8.4 make_clique_bipartite

make_clique_bipartite (G,fpos=None, create_using=None, name=None)
Returns the bipartite clique graph corresponding to $G$.
In the returned bipartite graph, the "bottom" nodes are the nodes of G and the "top" nodes represent the maximal cliques of $G$. There is an edge from node $v$ to clique $C$ in the returned graph if and only if $v$ is an element of $C$.

## Parameters

- G (NetworkX graph) - An undirected graph.
- fpos (bool) - If True or not None, the returned graph will have an additional attribute, pos, a dictionary mapping node to position in the Euclidean plane.
- create_using (NetworkX graph) - If provided, this graph will be cleared and the nodes and edges of the bipartite graph will be added to this graph.


## Returns

A bipartite graph whose "bottom" set is the nodes of the graph G, whose "top" set is the cliques of G , and whose edges join nodes of G to the cliques that contain them.
The nodes of the graph $G$ have the node attribute 'bipartite' set to 1 and the nodes representing cliques have the node attribute 'bipartite' set to 0 , as is the convention for bipartite graphs in NetworkX.

## Return type NetworkX graph

### 4.8.5 graph_clique_number

graph_clique_number $(G$, cliques=None $)$
Returns the clique number of the graph.
The clique number of a graph is the size of the largest clique in the graph.

## Parameters

- G (NetworkX graph) - An undirected graph.
- cliques (list) - A list of cliques, each of which is itself a list of nodes. If not specified, the list of all cliques will be computed, as by find_cliques().

Returns The size of the largest clique in G .

## Return type int

## Notes

You should provide cliques if you have already computed the list of maximal cliques, in order to avoid an exponential time search for maximal cliques.

### 4.8.6 graph_number_of_cliques

graph_number_of_cliques ( $G$, cliques=None)
Returns the number of maximal cliques in the graph.

## Parameters

- G (NetworkX graph) - An undirected graph.
- cliques (list) - A list of cliques, each of which is itself a list of nodes. If not specified, the list of all cliques will be computed, as by find_cliques ().

Returns The number of maximal cliques in G .
Return type int

## Notes

You should provide cliques if you have already computed the list of maximal cliques, in order to avoid an exponential time search for maximal cliques.

### 4.8.7 node_clique_number

```
node_clique_number ( }G\mathrm{ , nodes=None, cliques=None)
```

Returns the size of the largest maximal clique containing each given node.
Returns a single or list depending on input nodes. Optional list of cliques can be input if already computed.

### 4.8.8 number_of_cliques

number_of_cliques ( $G$, nodes=None, cliques $=$ None )
Returns the number of maximal cliques for each node.
Returns a single or list depending on input nodes. Optional list of cliques can be input if already computed.

### 4.8.9 cliques_containing_node

cliques_containing_node ( $G$, nodes=None, cliques=None)
Returns a list of cliques containing the given node.
Returns a single list or list of lists depending on input nodes. Optional list of cliques can be input if already computed.

### 4.9 Clustering

Algorithms to characterize the number of triangles in a graph.

| triangles(G[, nodes]) | Compute the number of triangles. |
| :--- | :--- |
| transitivity $(\mathrm{G})$ | Compute graph transitivity, the fraction of all possible tri- <br> angles present in G. |
| clustering(G[, nodes, weight $])$ | Compute the clustering coefficient for nodes. |
| average_clustering(G[, nodes, weight, ...]) | Compute the average clustering coefficient for the graph G. |
| square_clustering(G[, nodes]) | Compute the squares clustering coefficient for nodes. |
| generalized_degree(G[, nodes $])$ | Compute the generalized degree for nodes. |

### 4.9.1 triangles

triangles ( $G$, nodes $=$ None)
Compute the number of triangles.
Finds the number of triangles that include a node as one vertex.

## Parameters

- G (graph) - A networkx graph
- nodes (container of nodes, optional (default=all nodes in $G$ ) ) - Compute triangles for nodes in this container.

Returns out - Number of triangles keyed by node label.
Return type dictionary

## Examples

```
>>> G=nx.complete_graph (5)
>>> print(nx.triangles(G,0))
6
>>> print(nx.triangles(G))
{0: 6, 1: 6, 2: 6, 3: 6, 4: 6}
>>> print(list(nx.triangles(G, (0,1)).values()))
[6, 6]
```


## Notes

When computing triangles for the entire graph each triangle is counted three times, once at each node. Self loops are ignored.

### 4.9.2 transitivity

transitivity ( $G$ )
Compute graph transitivity, the fraction of all possible triangles present in G.
Possible triangles are identified by the number of "triads" (two edges with a shared vertex).

The transitivity is

$$
T=3 \frac{\# \text { triangles }}{\# \text { triads }}
$$

Parameters G (graph)
Returns out - Transitivity
Return type float

## Examples

```
>>> G = nx.complete_graph(5)
>>> print(nx.transitivity(G))
1.0
```


### 4.9.3 clustering

clustering (G, nodes=None, weight=None)
Compute the clustering coefficient for nodes.
For unweighted graphs, the clustering of a node $u$ is the fraction of possible triangles through that node that exist,

$$
c_{u}=\frac{2 T(u)}{\operatorname{deg}(u)(\operatorname{deg}(u)-1)}
$$

where $T(u)$ is the number of triangles through node $u$ and deg (u) is the degree of $u$.
For weighted graphs, the clustering is defined as the geometric average of the subgraph edge weights ${ }^{1}$,

$$
c_{u}=\frac{1}{\operatorname{deg}(u)(\operatorname{deg}(u)-1))} \sum_{u v}\left(\hat{w}_{u v} \hat{w}_{u w} \hat{w}_{v w}\right)^{1 / 3}
$$

The edge weights hat $\{w\} \_\{u v\}$ are normalized by the maximum weight in the network hat $\{w\} \_\{u v\}=$ w_\{uv\}/max (w).

The value of c_u is assigned to 0 if deg (u) < 2 .

## Parameters

- G (graph)
- nodes (container of nodes, optional (default=all nodes in $G$ )) - Compute clustering for nodes in this container.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns out - Clustering coefficient at specified nodes
Return type float, or dictionary

[^49]
## Examples

```
>>> G=nx.complete_graph(5)
>>> print(nx.clustering(G,0))
1.0
>>> print(nx.clustering(G))
{0: 1.0, 1: 1.0, 2: 1.0, 3: 1.0, 4: 1.0}
```


## Notes

Self loops are ignored.

## References

### 4.9.4 average_clustering

average_clustering ( $G$, nodes=None, weight=None, count_zeros=True)
Compute the average clustering coefficient for the graph $G$.
The clustering coefficient for the graph is the average,

$$
C=\frac{1}{n} \sum_{v \in G} c_{v},
$$

where n is the number of nodes in G .

## Parameters

- G (graph)
- nodes (container of nodes, optional (default=all nodes in $G$ )) - Compute average clustering for nodes in this container.
- weight (string or None, optional (default=None)) - The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.
- count_zeros (bool) - If False include only the nodes with nonzero clustering in the average.

Returns avg - Average clustering
Return type float

## Examples

```
>>> G=nx.complete_graph(5)
>>> print(nx.average_clustering(G))
1.0
```


## Notes

This is a space saving routine; it might be faster to use the clustering function to get a list and then take the average.

Self loops are ignored.

## References

### 4.9.5 square_clustering

square_clustering ( $G$, nodes=None)
Compute the squares clustering coefficient for nodes.
For each node return the fraction of possible squares that exist at the node ${ }^{1}$

$$
C_{4}(v)=\frac{\sum_{u=1}^{k_{v}} \sum_{w=u+1}^{k_{v}} q_{v}(u, w)}{\sum_{u=1}^{k_{v}} \sum_{w=u+1}^{k_{v}}\left[a_{v}(u, w)+q_{v}(u, w)\right]}
$$

where $q \_v(u, w)$ are the number of common neighbors of $u$ and $w$ other than $v$ (ie squares), and $a \_v(u, w)=\left(k \_u-\left(1+q \_v(u, w)+t h e t a \_\{u v\}\right)\right)\left(k \_w-\left(1+q \_v(u, w)+t h e t a \_\{u w\}\right)\right)$, where theta_\{uw\} $=1$ if $u$ and $w$ are connected and 0 otherwise.

## Parameters

- G (graph)
- nodes (container of nodes, optional (default=all nodes in $G$ )) - Compute clustering for nodes in this container.

Returns c4-A dictionary keyed by node with the square clustering coefficient value.
Return type dictionary

## Examples

```
>>> G=nx.complete_graph(5)
>>> print(nx.square_clustering(G,0))
1.0
>>> print(nx.square_clustering(G))
{0: 1.0, 1: 1.0, 2: 1.0, 3: 1.0, 4: 1.0}
```


## Notes

While C_3 (v) (triangle clustering) gives the probability that two neighbors of node v are connected with each other, C_4 (v) is the probability that two neighbors of node v share a common neighbor different from v . This algorithm can be applied to both bipartite and unipartite networks.

## References

### 4.9.6 generalized_degree

generalized_degree ( $G$, nodes=None)
Compute the generalized degree for nodes.
For each node, the generalized degree shows how many edges of given triangle multiplicity the node is connected to. The triangle multiplicity of an edge is the number of triangles an edge participates in. The generalized
 $k^{\prime} i^{\wedge}\{(j)\}$ is the number of edges attached to node $i$ that participate in $j$ triangles.

[^50]
## Parameters

- G (graph)
- nodes (container of nodes, optional (default=all nodes in $G$ )) - Compute the generalized degree for nodes in this container.

Returns out - Generalized degree of specified nodes. The Counter is keyed by edge triangle multiplicity.

Return type Counter, or dictionary of Counters

## Examples

```
>>> G=nx.complete_graph(5)
>>> print(nx.generalized_degree(G,0))
Counter({3: 4})
>>> print(nx.generalized_degree(G))
{0: Counter({3:4}), 1: Counter({3:4}), 2: Counter({3:4}), 3: Counter({3: 4}), - 
\hookrightarrow4: Counter({3: 4})}
```

To recover the number of triangles attached to a node:

```
>>> kl = nx.generalized_degree(G,0)
>>> sum([k*v for k,v in kl.items()])/2 == nx.triangles(G,0)
True
```


## Notes

In a network of N nodes, the highest triangle multiplicty an edge can have is $\mathrm{N}-2$.
The return value does not include a zero entry if no edges of a particular triangle multiplicity are present.
The number of triangles node $i$ is attached to can be recovered from the generalized de-
 $\left.+(\mathrm{N}-2) \mathrm{k} \mathrm{B}^{\mathrm{i}}\{(\mathrm{N}-2)\}\right) / 2$.

## References

### 4.10 Coloring

greedy_color(G[, strategy, interchange]) Color a graph using various strategies of greedy graph coloring.

### 4.10.1 greedy_color

greedy_color (G, strategy='largest_first', interchange=False)
Color a graph using various strategies of greedy graph coloring.
Attempts to color a graph using as few colors as possible, where no neighbours of a node can have same color as the node itself. The given strategy determines the order in which nodes are colored.

The strategies are described in ${ }^{1}$, and smallest-last is based on ${ }^{2}$.

## Parameters

- G (NetworkX graph)
- strategy (string or function(G, colors)) - A function (or a string representing a function) that provides the coloring strategy, by returning nodes in the ordering they should be colored. G is the graph, and colors is a dictionary of the currently assigned colors, keyed by nodes. The function must return an iterable over all the nodes in G .

If the strategy function is an iterator generator (that is, a function with yield statements), keep in mind that the colors dictionary will be updated after each yield, since this function chooses colors greedily.

If strategy is a string, it must be one of the following, each of which represents one of the built-in strategy functions.

```
- 'largest_first'
- 'random_sequential'
_ 'smallest_last'
- 'independent_set'
- 'connected_sequential_bfs'
- 'connected_sequential_dfs'
- 'connected_sequential' (alias for the previous strategy)
- 'strategy_saturation_largest_first'
- 'DSATUR' (alias for the previous strategy)
```

- interchange (bool) - Will use the color interchange algorithm described by ${ }^{3}$ if set to True. Note that strategy_saturation_largest_first and strategy_independent_set do not work with interchange. Furthermore, if you use interchange with your own strategy function, you cannot rely on the values in the colors argument.


## Returns

- A dictionary with keys representing nodes and values representing
- corresponding coloring.


## Examples

```
>>> G = nx.cycle_graph(4)
>>> d = nx.coloring.greedy_color(G, strategy='largest_first')
>> d in [{0: 0, 1: 1, 2: 0, 3: 1}, {0: 1, 1:0, 2: 1, 3: 0}]
True
```

[^51]Raises NetworkXPointlessConcept $\quad-\quad$ If $\quad$ strategy $\quad$| strategy_saturation_largest_first |
| :---: |
| and interchange is True. |

## References

Some node ordering strategies are provided for use with greedy_color().

| strategy_connected_sequential(G, colors[, <br> _..]) | Returns an iterable over nodes in G in the order given by a <br> breadth-first or depth-first traversal. |
| :--- | :--- |
| strategy_connected_sequential_dfs(G, col- <br> ors) | Returns an iterable over nodes in $G$ in the order given by a <br> depth-first traversal. |
| strategy_connected_sequential_bfs(G, col- <br> ors) | Returns an iterable over nodes in G in the order given by a <br> breadth-first traversal. |
| strategy_independent_set(G, colors) | Uses a greedy independent set removal strategy to deter- <br> mine the colors. |
| strategy_largest_first(G, colors) | Returns a list of the nodes of G in decreasing order by de- <br> gree. |
| strategy_random_sequential(G, colors) | Returns a random permutation of the nodes of G as a list. |
| strategy_saturation_largest_first(G, col- <br> ors) | Iterates over all the nodes of G in "saturation order" (also <br> known as "DSATUR"). |
| strategy_smallest_last(G, colors) | Returns a deque of the nodes of G, "smallest" last. |

### 4.10.2 strategy_connected_sequential

strategy_connected_sequential ( $G$, colors, traversal='bfs')
Returns an iterable over nodes in G in the order given by a breadth-first or depth-first traversal.
traversal must be one of the strings 'dfs' or 'bfs', representing depth-first traversal or breadth-first traversal, respectively.
The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
$G$ is a NetworkX graph. colors is ignored.

### 4.10.3 strategy_connected_sequential_dfs

strategy_connected_sequential_dfs ( $G$, colors)
Returns an iterable over nodes in G in the order given by a depth-first traversal.
The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
$G$ is a NetworkX graph. colors is ignored.

### 4.10.4 strategy_connected_sequential_bfs

strategy_connected_sequential_bfs ( $G$, colors)
Returns an iterable over nodes in G in the order given by a breadth-first traversal.
The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
$G$ is a NetworkX graph. colors is ignored.

### 4.10.5 strategy_independent_set

## strategy_independent_set ( $G$, colors)

Uses a greedy independent set removal strategy to determine the colors.
This function updates colors in-place and return None, unlike the other strategy functions in this module.
This algorithm repeatedly finds and removes a maximal independent set, assigning each node in the set an unused color.

G is a NetworkX graph.
This strategy is related to strategy_smallest_last (): in that strategy, an independent set of size one is chosen at each step instead of a maximal independent set.

### 4.10.6 strategy_largest_first

strategy_largest_first (G, colors)
Returns a list of the nodes of G in decreasing order by degree.
G is a NetworkX graph. colors is ignored.

### 4.10.7 strategy_random_sequential

strategy_random_sequential ( $G$, colors)
Returns a random permutation of the nodes of G as a list.
G is a NetworkX graph. colors is ignored.

### 4.10.8 strategy_saturation_largest_first

strategy_saturation_largest_first ( $G$, colors)
Iterates over all the nodes of G in "saturation order" (also known as "DSATUR").
G is a NetworkX graph. colors is a dictionary mapping nodes of G to colors, for those nodes that have already been colored.

### 4.10.9 strategy_smallest_last

strategy_smallest_last ( $G$, colors)
Returns a deque of the nodes of G, "smallest" last.
Specifically, the degrees of each node are tracked in a bucket queue. From this, the node of minimum degree is repeatedly popped from the graph, updating its neighbors' degrees.
G is a NetworkX graph. colors is ignored.
This implementation of the strategy runs in $O(n+m)$ time (ignoring polylogarithmic factors), where $n$ is the number of nodes and $m$ is the number of edges.

This strategy is related to strategy_independent_set (): if we interpret each node removed as an independent set of size one, then this strategy chooses an independent set of size one instead of a maximal independent set.

### 4.11 Communicability

Communicability.

| communicability $(\mathrm{G})$ | Return communicability between all pairs of nodes in G. |
| :--- | :--- |
| communicability_ $\exp (\mathrm{G})$ | Return communicability between all pairs of nodes in G. |

### 4.11.1 communicability

## communicability ( $G$ )

Return communicability between all pairs of nodes in G.
The communicability between pairs of nodes in G is the sum of closed walks of different lengths starting at node u and ending at node v .

## Parameters G (graph)

Returns comm - Dictionary of dictionaries keyed by nodes with communicability as the value.
Return type dictionary of dictionaries
Raises NetworkXError - If the graph is not undirected and simple.

## See also:

communicability_exp () Communicability between all pairs of nodes in $G$ using spectral decomposition.
communicability_betweenness_centrality() Communicability betweeness centrality for each node in G.

## Notes

This algorithm uses a spectral decomposition of the adjacency matrix. Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be a simple undirected graph. Using the connection between the powers of the adjacency matrix and the number of walks in the graph, the communicability between nodes $u$ and $v$ based on the graph spectrum is ${ }^{1}$

$$
C(u, v)=\sum_{j=1}^{n} \phi_{j}(u) \phi_{j}(v) e^{\lambda_{j}}
$$

where phi_\{j\}(u) is the $\operatorname{urm}\{t h\}$ element of the $\operatorname{jrm}\{t h\}$ orthonormal eigenvector of the adjacency matrix associated with the eigenvalue lambda_ $\{j\}$.

## References

## Examples

```
>>>G=nx.Graph([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> c = nx.communicability(G)
```

[^52]
### 4.11.2 communicability_exp

communicability_exp (G)
Return communicability between all pairs of nodes in $G$.
Communicability between pair of node ( $u, v$ ) of node in $G$ is the sum of closed walks of different lengths starting at node u and ending at node v .

## Parameters G (graph)

Returns comm - Dictionary of dictionaries keyed by nodes with communicability as the value.
Return type dictionary of dictionaries
Raises NetworkXError - If the graph is not undirected and simple.

## See also:

communicability () Communicability between pairs of nodes in $G$.
communicability_betweenness_centrality() Communicability betweeness centrality for each node in $G$.

## Notes

This algorithm uses matrix exponentiation of the adjacency matrix.
Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be a simple undirected graph. Using the connection between the powers of the adjacency matrix and the number of walks in the graph, the communicability between nodes $u$ and $v$ is ${ }^{1}$,

$$
C(u, v)=\left(e^{A}\right)_{u v}
$$

where $A$ is the adjacency matrix of G.

## References

## Examples

```
>>>G=nx.Graph([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> c = nx.communicability_exp(G)
```


### 4.12 Communities

### 4.12.1 Bipartitions

Functions for computing the Kernighan-Lin bipartition algorithm.

| kernighan_lin_bisection(G[, partition, ...]) | Partition a graph into two blocks using the Kernighan-Lin <br> algorithm. |
| :--- | :--- |

[^53]
## kernighan_lin_bisection

kernighan_lin_bisection (G, partition=None, max_iter=10, weight='weight')
Partition a graph into two blocks using the Kernighan-Lin algorithm.
This algorithm paritions a network into two sets by iteratively swapping pairs of nodes to reduce the edge cut between the two sets.

## Parameters

- G (graph)
- partition (tuple) - Pair of iterables containing an intial partition. If not specified, a random balanced partition is used.
- max_iter (int) - Maximum number of times to attempt swaps to find an improvemement before giving up.
- weight (key) - Edge data key to use as weight. If None, the weights are all set to one.

Returns partition - A pair of sets of nodes representing the bipartition.
Return type tuple
Raises NetworkXError - If partition is not a valid partition of the nodes of the graph.

References

### 4.12.2 Generators

## LFR_benchmark_graph

### 4.12.3 K-Clique

| k_clique_communities(G, k[, cliques]) | Find k-clique communities in graph using the percolation <br> method. |
| :--- | :--- |

## k_clique_communities

```
k_clique_communities( }G,k\mathrm{ , cliques=None)
```

Find k-clique communities in graph using the percolation method.
A k -clique community is the union of all cliques of size k that can be reached through adjacent (sharing $\mathrm{k}-1$ nodes) k-cliques.

## Parameters

- G (NetworkX graph)
- $\mathbf{k}($ int $)$ - Size of smallest clique
- cliques (list or generator) - Precomputed cliques (use networkx.find_cliques(G))


## Returns

Return type Yields sets of nodes, one for each k-clique community.

## Examples

```
>>> G = nx.complete_graph(5)
>>> K5 = nx.convert_node_labels_to_integers(G,first_label=2)
>>> G.add_edges_from(K5.edges())
>>> c = list(nx.k_clique_communities(G, 4))
>>> list(c[0])
[0, 1, 2, 3, 4, 5, 6]
>>> list(nx.k_clique_communities(G, 6))
[ ]
```


## References

### 4.12.4 Label propagation

Asynchronous label propagation algorithms for community detection.

```
asyn_lpa_communities(G[,weight])
```

Returns communities in G as detected by asynchronous label propagation.

## asyn_Ipa_communities

asyn_lpa_communities ( $G$, weight=None)
Returns communities in G as detected by asynchronous label propagation.
The asynchronous label propagation algorithm is described in ${ }^{1}$. The algorithm is probabilistic and the found communities may vary on different executions.

The algorithm proceeds as follows. After initializing each node with a unique label, the algorithm repeatedly sets the label of a node to be the label that appears most frequently among that nodes neighbors. The algorithm halts when each node has the label that appears most frequently among its neighbors. The algorithm is asynchronous because each node is updated without waiting for updates on the remaining nodes.

This generalized version of the algorithm in ${ }^{1}$ accepts edge weights.

## Parameters

## - G (Graph)

- weight (string) - The edge attribute representing the weight of an edge. If None, each edge is assumed to have weight one. In this algorithm, the weight of an edge is used in determining the frequency with which a label appears among the neighbors of a node: a higher weight means the label appears more often.

Returns communities - Iterable of communities given as sets of nodes.

## Return type iterable

## Notes

Edge weight attributes must be numerical.

[^54]
## References

### 4.12.5 Measuring partitions

Functions for measuring the quality of a partition (into communities).

| coverage $(\backslash$ args, $\backslash * \backslash \mathrm{kw})$ | Returns the coverage of a partition. |
| :--- | :--- |
| performance $(\backslash \operatorname{args}, \backslash * \backslash \mathrm{kw})$ | Returns the performance of a partition. |

## coverage

coverage (*args, **kw)
Returns the coverage of a partition.
The coverage of a partition is the ratio of the number of intra-community edges to the total number of edges in the graph.

## Parameters

- G (NetworkX graph)
- partition (sequence) - Partition of the nodes of $G$, represented as a sequence of sets of nodes. Each block of the partition represents a community.

Returns The coverage of the partition, as defined above.
Return type float
Raises NetworkXError - If partition is not a valid partition of the nodes of G.

## Notes

If G is a multigraph, the multiplicity of edges is counted.

## References

## performance

```
performance (*args, **kw)
```

Returns the performance of a partition.
The performance of a partition is the ratio of the number of intra-community edges plus inter-community nonedges with the total number of potential edges.

## Parameters

- G (NetworkX graph) - A simple graph (directed or undirected).
- partition (sequence) - Partition of the nodes of G , represented as a sequence of sets of nodes. Each block of the partition represents a community.

Returns The performance of the partition, as defined above.
Return type float
Raises NetworkXError - If partition is not a valid partition of the nodes of G.

## References

### 4.12.6 Partitions via centrality measures

Functions for computing communities based on centrality notions.

> girvan_newman(G[, most_valuable_edge])

Finds communities in a graph using the Girvan-Newman method.

## girvan_newman

girvan_newman (G, most_valuable_edge=None)
Finds communities in a graph using the Girvan-Newman method.

## Parameters

- G (NetworkX graph)
- most_valuable_edge (function) - Function that takes a graph as input and outputs an edge. The edge returned by this function will be recomputed and removed at each iteration of the algorithm.

If not specified, the edge with the highest networkx.edge_betweenness_centrality () will be used.

Returns Iterator over tuples of sets of nodes in G. Each set of node is a community, each tuple is a sequence of communities at a particular level of the algorithm.
Return type iterator

## Examples

To get the first pair of communities:
$\ggg G=n x \cdot p a t h \_g r a p h(10)$
$\ggg$ comp $=$ girvan_newman (G)
>>> tuple(sorted(c) for $c$ in next (comp))
$([0,1,2,3,4],[5,6,7,8,9])$
To get only the first $k$ tuples of communities, use itertools.islice():

```
>>> import itertools
>>> G = nx.path_graph(8)
>>> k = 2
>>> comp = girvan_newman(G)
>>> for communities in itertools.islice(comp, k):
... print(tuple(sorted(c) for c in communities))
...
([0, 1, 2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5, 6, 7])
```

To stop getting tuples of communities once the number of communities is greater than $k$, use itertools.takewhile():

```
>>> import itertools
>>> G = nx.path_graph(8)
>>> k = 4
>>> comp = girvan_newman(G)
>>> limited = itertools.takewhile(lambda c: len(c) <= k, comp)
>>> for communities in limited:
... print(tuple(sorted(c) for c in communities))
. . -
([0, 1, 2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5], [6, 7])
```

To just choose an edge to remove based on the weight:

```
>>> from operator import itemgetter
>>> G = nx.path_graph(10)
>>> edges = G.edges()
>>> nx.set_edge_attributes(G, 'weight', {(u, v): v for u, v in edges})
>>> def heaviest(G):
... u, v, w = max(G.edges(data='weight'), key=itemgetter(2))
... return (u, v)
...
>>> comp = girvan_newman(G, most_valuable_edge=heaviest)
>>> tuple(sorted(c) for c in next(comp))
([0, 1, 2, 3, 4, 5, 6, 7, 8], [9])
```

To utilize edge weights when choosing an edge with, for example, the highest betweenness centrality:

```
>>> from networkx import edge_betweenness_centrality as betweenness
>>> def most_central_edge(G):
... centrality = betweenness(G, weight='weight')
... return max(centrality, key=centrality.get)
...
>>> G = nx.path_graph(10)
>>> comp = girvan_newman(G, most_valuable_edge=most_central_edge)
>>> tuple(sorted(c) for c in next(comp))
([0, 1, 2, 3, 4], [5, 6, 7, 8, 9])
```

To specify a different ranking algorithm for edges, use the most_valuable_edge keyword argument:

```
>>> from networkx import edge_betweenness_centrality
>>> from random import random
>>> def most_central_edge(G):
... centrality = edge_betweenness_centrality(G)
... max_cent = max(centrality.values())
... # Scale the centrality values so they are between 0 and 1,
... # and add some random noise.
... centrality = {e: c / max_cent for e, c in centrality.items()}
... # Add some random noise.
... centrality = {e: c + random() for e, c in centrality.items()}
... return max(centrality, key=centrality.get)
* .
>>> G = nx.path_graph(10)
>>> comp = girvan_newman(G, most_valuable_edge=most_central_edge)
```


## Notes

The Girvan-Newman algorithm detects communities by progressively removing edges from the original graph. The algorithm removes the "most valuable" edge, traditionally the edge with the highest betweenness centrality, at each step. As the graph breaks down into pieces, the tightly knit community structure is exposed and the result can be depicted as a dendrogram.

### 4.13 Components

### 4.13.1 Connectivity

| is_connected $(\mathbf{G})$ | Return True if the graph is connected, false otherwise. |
| :--- | :--- |
| number_connected_components $(\mathrm{G})$ | Return the number of connected components. |
| connected_components $(\mathrm{G})$ | Generate connected components. |
| connected_component_subgraphs $(\mathrm{G}[$, copy $])$ | Generate connected components as subgraphs. |
| node_connected_component $(\mathrm{G}, \mathrm{n})$ | Return the nodes in the component of graph containing <br> node n. |

## is_connected

## is_connected ( $G$ )

Return True if the graph is connected, false otherwise.
Parameters G (NetworkX Graph) - An undirected graph.
Returns connected - True if the graph is connected, false otherwise.
Return type bool
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

```
>>> G = nx.path_graph(4)
>>> print(nx.is_connected(G))
True
```


## See also:

```
    is_strongly_connected(), is_weakly_connected(), is_semiconnected(),
    is_biconnected(), connected_components()
```


## Notes

For undirected graphs only.
number_connected_components
number_connected_components ( $G$ )
Return the number of connected components.

Parameters G (NetworkX graph) - An undirected graph.
Returns $\mathbf{n}$ - Number of connected components
Return type integer
See also:
connected_components(), number_weakly_connected_components (),
number_strongly_connected_components ()

## Notes

For undirected graphs only.

## connected_components

connected_components ( $G$ )
Generate connected components.
Parameters G (NetworkX graph) - An undirected graph
Returns comp - A generator of sets of nodes, one for each component of G.
Return type generator of sets
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of connected components, largest first.

```
>>> G = nx.path_graph(4)
>>> nx.add_path(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.connected_components(G), key=len, reverse=True)]
[4, 3]
```

If you only want the largest connected component, it's more efficient to use max instead of sort.
$\ggg$ largest_cc $=\max \left(n x . c o n n e c t e d \_c o m p o n e n t s(G)\right.$, key=len)

## See also:

```
strongly_connected_components(),weakly_connected_components()
```


## Notes

For undirected graphs only.

## connected_component_subgraphs

connected_component_subgraphs ( $G$, copy=True)
Generate connected components as subgraphs.

## Parameters

- G (NetworkX graph) - An undirected graph.
- copy (bool (default=True)) - If True make a copy of the graph attributes

Returns comp - A generator of graphs, one for each connected component of G.
Return type generator
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

```
>>> G = nx.path_graph(4)
>>> G.add_edge (5,6)
>>> graphs = list(nx.connected_component_subgraphs(G))
```

If you only want the largest connected component, it's more efficient to use max instead of sort:

```
>>> Gc = max(nx.connected_component_subgraphs(G), key=len)
```


## See also:

```
connected_components(), strongly_connected_component_subgraphs(),
weakly_connected_component_subgraphs()
```


## Notes

For undirected graphs only. Graph, node, and edge attributes are copied to the subgraphs by default.

```
node_connected_component
```

node_connected_component ( $G, n$ )

Return the nodes in the component of graph containing node $n$.

## Parameters

- G (NetworkX Graph) - An undirected graph.
- $\mathbf{n}$ (node label) - A node in G

Returns comp - A set of nodes in the component of G containing node n .
Return type set
Raises NetworkXNotImplemented: - If G is directed.

## See also:

```
connected_components()
```


## Notes

For undirected graphs only.

### 4.13.2 Strong connectivity

| is_ |  |
| :---: | :---: |
| $n$ | Return number of strongly connected components in graph. |
| strongly_connected_components(G) | Generate nodes in strongly connected components of graph. |
| strongly_connected_component_subgraphs(G[, Generate strongly connected components as subgraphs. copy]) |  |
| strongly_connected_components_recursive(G)Generate nodes in strongly connected components of graph. |  |
| kosaraju_strongly_connected_components(G[, Generate nodes in strongly connected components of ...]) graph. |  |
| condensation(G[, scc]) | Returns the condensation of G. |

is_strongly_connected
is_strongly_connected ( $G$ )
Test directed graph for strong connectivity.
Parameters G (NetworkX Graph) - A directed graph.
Returns connected - True if the graph is strongly connected, False otherwise.
Return type bool
Raises NetworkXNotImplemented: - If G is undirected.
See also:
is_weakly_connected(), is_semiconnected(), is_connected(), is_biconnected(), strongly_connected_components()

## Notes

For directed graphs only.

## number_strongly_connected_components

number_strongly_connected_components $(G)$
Return number of strongly connected components in graph.
Parameters G (NetworkX graph) - A directed graph.
Returns $\mathbf{n}$ - Number of strongly connected components
Return type integer
Raises NetworkXNotImplemented: - If G is undirected.

## See also:

```
    strongly_connected_components(),
number_connected_components(),
number_weakly_connected_components()
```

Notes

For directed graphs only.
strongly_connected_components
strongly_connected_components ( $G$ )
Generate nodes in strongly connected components of graph.
Parameters G (NetworkX Graph) - An directed graph.
Returns comp - A generator of sets of nodes, one for each strongly connected component of G.
Return type generator of sets
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of strongly connected components, largest first.

```
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.strongly_connected_components(G),
... key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort.

```
>>> largest = max(nx.strongly_connected_components(G), key=len)
```


## See also:

```
connected_components(), weakly_connected_components(),
kosaraju_strongly_connected_components()
```


## Notes

Uses Tarjan's algorithm[1]_ with Nuutila's modifications[2]_. Nonrecursive version of algorithm.

## References

```
strongly_connected_component_subgraphs
```

strongly_connected_component_subgraphs ( $G$, copy=True)

Generate strongly connected components as subgraphs.

## Parameters

- G (NetworkX Graph) - A directed graph.
- copy (boolean, optional) - if copy is True, Graph, node, and edge attributes are copied to the subgraphs.

Returns comp - A generator of graphs, one for each strongly connected component of G.
Return type generator of graphs
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of strongly connected components, largest first.

```
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(Gc) for Gc in sorted(nx.strongly_connected_component_subgraphs(G),
... key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort.

```
>>> Gc = max(nx.strongly_connected_component_subgraphs(G), key=len)
```


## See also:

```
strongly_connected_components(), connected_component_subgraphs(),
weakly_connected_component_subgraphs()
```

strongly_connected_components_recursive
strongly_connected_components_recursive ( $G$ )

Generate nodes in strongly connected components of graph.
Recursive version of algorithm.
Parameters G (NetworkX Graph) - An directed graph.
Returns comp - A generator of sets of nodes, one for each strongly connected component of G.
Return type generator of sets
Raises NetworkXNotImplemented : - If G is undirected.

## Examples

Generate a sorted list of strongly connected components, largest first.

```
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.strongly_connected_components_recursive(G),
... key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort.

```
>>> largest = max(nx.strongly_connected_components_recursive(G), key=len)
```


## See also:

```
connected_components()
```


## Notes

Uses Tarjan's algorithm[1]_ with Nuutila's modifications[2]_.

## References

## kosaraju_strongly_connected_components

kosaraju_strongly_connected_components $(G$, source=None)
Generate nodes in strongly connected components of graph.
Parameters G (NetworkX Graph) - An directed graph.
Returns comp - A genrator of sets of nodes, one for each strongly connected component of G.
Return type generator of sets
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of strongly connected components, largest first.

```
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.kosaraju_strongly_connected_components(G),
.. key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort.

```
>>> largest = max(nx.kosaraju_strongly_connected_components(G), key=len)
```


## See also:

strongly_connected_components ()

## Notes

Uses Kosaraju’s algorithm.

## condensation

condensation ( $G, s c c=$ None)
Returns the condensation of G .
The condensation of $G$ is the graph with each of the strongly connected components contracted into a single node.

## Parameters

- G (NetworkX DiGraph) - A directed graph.
- scc (list or generator (optional, default=None)) - Strongly connected components. If provided, the elements in scc must partition the nodes in G . If not provided, it will be calculated as $\operatorname{scc}=\mathrm{nx}$.strongly_connected_components(G).

Returns $\mathbf{C}$ - The condensation graph C of G . The node labels are integers corresponding to the index of the component in the list of strongly connected components of G . C has a graph attribute named 'mapping' with a dictionary mapping the original nodes to the nodes in C to which they
belong. Each node in C also has a node attribute 'members' with the set of original nodes in G that form the SCC that the node in C represents.

## Return type NetworkX DiGraph

Raises NetworkXNotImplemented: - If G is undirected.

## Notes

After contracting all strongly connected components to a single node, the resulting graph is a directed acyclic graph.

### 4.13.3 Weak connectivity

| is_weakly_connected $(\mathbf{G})$ | Test directed graph for weak connectivity. |
| :--- | :--- |
| number_weakly_connected_components $(\mathbf{G})$ | Return the number of weakly connected components in G. |
| weakly_connected_components $(\mathrm{G})$ | Generate weakly connected components of G. |
| weakly_connected_component_subgraphs $(\mathrm{G}[$, <br> copy $])$ | Generate weakly connected components as subgraphs. |

is_weakly_connected
is_weakly_connected ( $G$ )
Test directed graph for weak connectivity.
A directed graph is weakly connected if, and only if, the graph is connected when the direction of the edge between nodes is ignored.

Parameters G (NetworkX Graph) - A directed graph.
Returns connected - True if the graph is weakly connected, False otherwise.
Return type bool
Raises NetworkXNotImplemented: - If G is undirected.

## See also:

is_strongly_connected(), is_semiconnected(), is_connected(), is_biconnected(), weakly_connected_components()

## Notes

For directed graphs only.

```
number_weakly_connected_components
number_weakly_connected_components (G)
```

Return the number of weakly connected components in G.
Parameters G (NetworkX graph) - A directed graph.
Returns $\mathbf{n}$ - Number of weakly connected components
Return type integer

Raises NetworkXNotImplemented: - If G is undirected.
See also:

```
weakly_connected_components(),
number_connected_components(),
number_strongly_connected_components()
```


## Notes

For directed graphs only.

## weakly_connected_components

weakly_connected_components $(G)$
Generate weakly connected components of $G$.
Parameters G (NetworkX graph) - A directed graph
Returns comp - A generator of sets of nodes, one for each weakly connected component of G.
Return type generator of sets
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of weakly connected components, largest first.

```
>>> G = nx.path_graph(4, create_using=nx.DiGraph())
>>> nx.add_path(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.weakly_connected_components(G),
key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort:

```
>>> largest_cc = max(nx.weakly_connected_components(G), key=len)
```


## See also:

```
connected_components(),strongly_connected_components()
```


## Notes

For directed graphs only.
weakly_connected_component_subgraphs
weakly_connected_component_subgraphs ( $G$, copy=True)
Generate weakly connected components as subgraphs.

## Parameters

- G (NetworkX graph) - A directed graph.
- copy (bool (default=True)) - If True make a copy of the graph attributes

Returns comp - A generator of graphs, one for each weakly connected component of G.
Return type generator
Raises NetworkXNotImplemented: - If G is undirected.

## Examples

Generate a sorted list of weakly connected components, largest first.

```
>>> G = nx.path_graph(4, create_using=nx.DiGraph())
>>> nx.add_path(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.weakly_connected_component_subgraphs(G),
... key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use max instead of sort:

```
>>> Gc = max(nx.weakly_connected_component_subgraphs(G), key=len)
```


## See also:

```
weakly_connected_components(), strongly_connected_component_subgraphs(),
connected_component_subgraphs()
```


## Notes

For directed graphs only. Graph, node, and edge attributes are copied to the subgraphs by default.

### 4.13.4 Attracting components

| is_attracting_component $(\mathrm{G})$ | Returns True if G consists of a single attracting component. |
| :--- | :--- |
| number_attracting_components $(\mathrm{G})$ | Returns the number of attracting components in G. |
| attracting_components $(\mathrm{G})$ | Generates a list of attracting components in G. |
| attracting_component_subgraphs $(\mathrm{G}[$, copy $])$ | Generates a list of attracting component subgraphs from G. |

is_attracting_component
is_attracting_component ( $G$ )
Returns True if G consists of a single attracting component.
Parameters G (DiGraph, MultiDiGraph) - The graph to be analyzed.
Returns attracting - True if G has a single attracting component. Otherwise, False.
Return type bool
Raises NetworkXNotImplemented : - If the input graph is undirected.

## See also:

```
attracting_components(),
number_attracting_components(),
    attracting_component_subgraphs()
```


## number_attracting_components

number_attracting_components ( $G$ )
Returns the number of attracting components in G .
Parameters G (DiGraph, MultiDiGraph) - The graph to be analyzed.
Returns $\mathbf{n}$ - The number of attracting components in $G$.
Return type int
Raises NetworkXNotImplemented : - If the input graph is undirected.

## See also:

attracting_components(), is_attracting_component(), attracting_component_subgraphs()

## attracting_components

attracting_components ( $G$ )
Generates a list of attracting components in $G$.
An attracting component in a directed graph $G$ is a strongly connected component with the property that a random walker on the graph will never leave the component, once it enters the component.

The nodes in attracting components can also be thought of as recurrent nodes. If a random walker enters the attractor containing the node, then the node will be visited infinitely often.

Parameters G (DiGraph, MultiDiGraph) - The graph to be analyzed.
Returns attractors - A generator of sets of nodes, one for each attracting component of G.
Return type generator of sets
Raises NetworkXNotImplemented :- If the input graph is undirected.

## See also:

```
number_attracting_components(), is_attracting_component(),
attracting_component_subgraphs()
```


## attracting_component_subgraphs

attracting_component_subgraphs ( $G$, copy=True)
Generates a list of attracting component subgraphs from $G$.
Parameters G (DiGraph, MultiDiGraph) - The graph to be analyzed.

## Returns

- subgraphs (list) - A list of node-induced subgraphs of the attracting components of G.
- copy (bool) - If copy is True, graph, node, and edge attributes are copied to the subgraphs.

Raises NetworkXNotImplemented: - If the input graph is undirected.

## See also:

```
attracting_components(),
number_attracting_components(),
is_attracting_component()
```


### 4.13.5 Biconnected components

| is_biconnected $(\mathbf{G})$ | Return True if the graph is biconnected, False otherwise. |
| :--- | :--- |
| biconnected_components $(\mathbf{G})$ | Return a generator of sets of nodes, one set for each bicon- <br> nected |
| biconnected_component_edges $(\mathrm{G})$ | Return a generator of lists of edges, one list for each bicon- <br> nected component of the input graph. |
| biconnected_component_subgraphs $(\mathrm{G}[$, copy $])$ | Return a generator of graphs, one graph for each bicon- <br> nected component of the input graph. |
| articulation_points $(\mathrm{G})$ | Return a generator of articulation points, or cut vertices, of <br> a graph. |

## is_biconnected

## is_biconnected ( $G$ )

Return True if the graph is biconnected, False otherwise.
A graph is biconnected if, and only if, it cannot be disconnected by removing only one node (and all edges incident on that node). If removing a node increases the number of disconnected components in the graph, that node is called an articulation point, or cut vertex. A biconnected graph has no articulation points.

Parameters G (NetworkX Graph) - An undirected graph.
Returns biconnected - True if the graph is biconnected, False otherwise.
Return type bool
Raises NetworkXNotImplemented : - If the input graph is not undirected.

## Examples

```
>>> G = nx.path_graph(4)
>>> print(nx.is_biconnected(G))
False
>>> G.add_edge(0, 3)
>>> print(nx.is_biconnected(G))
True
```


## See also:

```
biconnected_components(), articulation_points(), biconnected_component_edges(),
biconnected_component_subgraphs(), is_strongly_connected(),
is_weakly_connected(), is_connected(), is_semiconnected()
```


## Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node $n$ is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

## References

## biconnected_components

## biconnected_components ( $G$ )

Return a generator of sets of nodes, one set for each biconnected component of the graph
Biconnected components are maximal subgraphs such that the removal of a node (and all edges incident on that node) will not disconnect the subgraph. Note that nodes may be part of more than one biconnected component. Those nodes are articulation points, or cut vertices. The removal of articulation points will increase the number of connected components of the graph.
Notice that by convention a dyad is considered a biconnected component.
Parameters G (NetworkX Graph) - An undirected graph.
Returns nodes - Generator of sets of nodes, one set for each biconnected component.
Return type generator
Raises NetworkXNotImplemented:- If the input graph is not undirected.

## Examples

```
>>> G = nx.lollipop_graph(5, 1)
>>> print(nx.is_biconnected(G))
False
>>> bicomponents = list(nx.biconnected_components(G))
>>> len(bicomponents)
2
>>> G.add_edge(0, 5)
>>> print(nx.is_biconnected(G))
True
>>> bicomponents = list(nx.biconnected_components(G))
>>> len(bicomponents)
1
```

You can generate a sorted list of biconnected components, largest first, using sort.

```
>>> G.remove_edge(0, 5)
>>> [len(c) for c in sorted(nx.biconnected_components(G), key=len, reverse=True)]
[5, 2]
```

If you only want the largest connected component, it's more efficient to use max instead of sort.

```
>>> Gc = max(nx.biconnected_components(G), key=len)
```


## See also:

```
is_biconnected(), articulation_points(), biconnected_component_edges(),
biconnected_component_subgraphs()
```


## Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node n
is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

## References

## biconnected_component_edges

## biconnected_component_edges $(G)$

Return a generator of lists of edges, one list for each biconnected component of the input graph.
Biconnected components are maximal subgraphs such that the removal of a node (and all edges incident on that node) will not disconnect the subgraph. Note that nodes may be part of more than one biconnected component. Those nodes are articulation points, or cut vertices. However, each edge belongs to one, and only one, biconnected component.
Notice that by convention a dyad is considered a biconnected component.
Parameters G (NetworkX Graph) - An undirected graph.
Returns edges - Generator of lists of edges, one list for each bicomponent.
Return type generator of lists
Raises NetworkXNotImplemented : - If the input graph is not undirected.

## Examples

```
>>>G G=nx.barbell_graph(4, 2)
>>> print(nx.is_biconnected(G))
False
>>> bicomponents_edges = list(nx.biconnected_component_edges (G))
>>> len(bicomponents_edges)
5
>>> G.add_edge(2, 8)
>>> print(nx.is_biconnected(G))
True
>>> bicomponents_edges = list(nx.biconnected_component_edges(G))
>>> len(bicomponents_edges)
1
```


## See also:

```
is_biconnected(), biconnected_components(), articulation_points(),
biconnected_component_subgraphs()
```


## Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node n is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

## References

## biconnected_component_subgraphs

biconnected_component_subgraphs ( $G$, copy=True)
Return a generator of graphs, one graph for each biconnected component of the input graph.
Biconnected components are maximal subgraphs such that the removal of a node (and all edges incident on that node) will not disconnect the subgraph. Note that nodes may be part of more than one biconnected component. Those nodes are articulation points, or cut vertices. The removal of articulation points will increase the number of connected components of the graph.

Notice that by convention a dyad is considered a biconnected component.
Parameters G (NetworkX Graph) - An undirected graph.
Returns graphs - Generator of graphs, one graph for each biconnected component.

## Return type generator

Raises NetworkXNotImplemented : - If the input graph is not undirected.

## Examples

```
>>> G = nx.lollipop_graph(5, 1)
>>> print(nx.is_biconnected(G))
False
>>> bicomponents = list(nx.biconnected_component_subgraphs(G))
>>> len(bicomponents)
2
>>> G.add_edge (0, 5)
>>> print(nx.is_biconnected(G))
True
>>> bicomponents = list(nx.biconnected_component_subgraphs(G))
>>> len(bicomponents)
1
```

You can generate a sorted list of biconnected components, largest first, using sort.

```
>>> G.remove_edge (0, 5)
>>> [len(c) for c in sorted(nx.biconnected_component_subgraphs(G),
... key=len, reverse=True)]
[5, 2]
```

If you only want the largest connected component, it's more efficient to use max instead of sort.

```
>>> Gc = max(nx.biconnected_component_subgraphs(G), key=len)
```


## See also:

```
is_biconnected(), articulation_points(), biconnected_component_edges(),
```

biconnected_components ()

## Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node n
is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.
Graph, node, and edge attributes are copied to the subgraphs.

## References

## articulation_points

```
articulation_points(G)
```

Return a generator of articulation points, or cut vertices, of a graph.
An articulation point or cut vertex is any node whose removal (along with all its incident edges) increases the number of connected components of a graph. An undirected connected graph without articulation points is biconnected. Articulation points belong to more than one biconnected component of a graph.

Notice that by convention a dyad is considered a biconnected component.
Parameters G (NetworkX Graph) - An undirected graph.
Returns articulation points - generator of nodes
Return type generator
Raises NetworkXNotImplemented : - If the input graph is not undirected.

## Examples

```
>>>G G nx.barbell_graph(4, 2)
>>> print(nx.is_biconnected(G))
False
>>> len(list(nx.articulation_points(G)))
4
>>> G.add_edge(2, 8)
>>> print(nx.is_biconnected(G))
True
>>> len(list(nx.articulation_points(G)))
0
```


## See also:

```
is_biconnected(), biconnected_components(), biconnected_component_edges(),
biconnected_component_subgraphs()
```


## Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node n is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

## References

### 4.13.6 Semiconnectedness

```
is__semiconnected(G)
```

Return True if the graph is semiconnected, False otherwise.
is_semiconnected
is_semiconnected ( $G$ )
Return True if the graph is semiconnected, False otherwise.
A graph is semiconnected if, and only if, for any pair of nodes, either one is reachable from the other, or they are mutually reachable.

Parameters G (NetworkX graph) - A directed graph.
Returns semiconnected - True if the graph is semiconnected, False otherwise.

## Return type bool

## Raises

- NetworkXNotImplemented : - If the input graph is undirected.
- NetworkXPointlessConcept : - If the graph is empty.


## Examples

```
>>> G=nx.path_graph(4,create_using=nx.DiGraph())
>>> print(nx.is_semiconnected(G))
True
>>> G=nx.DiGraph([(1, 2), (3, 2)])
>>> print(nx.is_semiconnected(G))
False
```


## See also:

```
    is_strongly_connected(), is_weakly_connected(), is_connected(),
    is_biconnected()
```


### 4.14 Connectivity

Connectivity and cut algorithms

### 4.14.1 K-node-components

Moody and White algorithm for k-components
k_components(G[, flow_func]) Returns the k-component structure of a graph G.

## k_components

## k_components ( $G$, flow_func=None)

Returns the k-component structure of a graph G.
A k -component is a maximal subgraph of a graph G that has, at least, node connectivity k : we need to remove at least $k$ nodes to break it into more components. $k$-components have an inherent hierarchical structure because they are nested in terms of connectivity: a connected graph can contain several 2-components, each of which can contain one or more 3-components, and so forth.

## Parameters

- G (NetworkX graph)
- flow_func (function) - Function to perform the underlying flow computations. Default value edmonds_karp (). This function performs better in sparse graphs with right tailed degree distributions. shortest_augmenting_path() will perform better in denser graphs.
Returns k_components - Dictionary with all connectivity levels k in the input Graph as keys and a list of sets of nodes that form a k-component of level k as values.

Return type dict
Raises NetworkXNotImplemented: - If the input graph is directed.

## Examples

```
>>> # Petersen graph has 10 nodes and it is triconnected, thus all
>>> # nodes are in a single component on all three connectivity levels
>>> G = nx.petersen_graph()
>>> k_components = nx.k_components(G)
```


## Notes

Moody and White ${ }^{1}$ (appendix A) provide an algorithm for identifying k-components in a graph, which is based on Kanevsky's algorithm ${ }^{2}$ for finding all minimum-size node cut-sets of a graph (implemented in all_node_cuts () function):
1.Compute node connectivity, k , of the input graph G .
2.Identify all k-cutsets at the current level of connectivity using Kanevsky's algorithm.
3.Generate new graph components based on the removal of these cutsets. Nodes in a cutset belong to both sides of the induced cut.
4.If the graph is neither complete nor trivial, return to 1 ; else end.

This implementation also uses some heuristics (see ${ }^{3}$ for details) to speed up the computation.

## See also:

node_connectivity(), all_node_cuts()

[^55]
## References

### 4.14.2 K-node-cutsets

Kanevsky all minimum node k cutsets algorithm.
all_node_cuts(G[, k, flow_func]) $\quad$ Returns all minimum k cutsets of an undirected graph G.
all_node_cuts
all_node_cuts ( $G, k=$ None, flow_func=None)
Returns all minimum k cutsets of an undirected graph G .
This implementation is based on Kanevsky's algorithm ${ }^{1}$ for finding all minimum-size node cut-sets of an undirected graph G; ie the set (or sets) of nodes of cardinality equal to the node connectivity of G. Thus if removed, would break $G$ into two or more connected components.

## Parameters

- G (NetworkX graph) - Undirected graph
- $\mathbf{k}$ (Integer) - Node connectivity of the input graph. If $k$ is None, then it is computed. Default value: None.
- flow_func (function) - Function to perform the underlying flow computations. Default value edmonds_karp. This function performs better in sparse graphs with right tailed degree distributions. shortest_augmenting_path will perform better in denser graphs.
Returns cuts - Each node cutset has cardinality equal to the node connectivity of the input graph.
Return type a generator of node cutsets


## Examples

```
>>> # A two-dimensional grid graph has 4 cutsets of cardinality 2
>>> G = nx.grid_2d_graph(5, 5)
>>> cutsets = list(nx.all_node_cuts(G))
>>> len(cutsets)
4
>>> all(2 == len(cutset) for cutset in cutsets)
True
>>> nx.node_connectivity(G)
2
```


## Notes

This implementation is based on the sequential algorithm for finding all minimum-size separating vertex sets in a graph ${ }^{1}$. The main idea is to compute minimum cuts using local maximum flow computations among a set of nodes of highest degree and all other non-adjacent nodes in the Graph. Once we find a minimum cut, we add an edge between the high degree node and the target node of the local maximum flow computation to make sure that we will not find that minimum cut again.

[^56]
## See also:

```
node_connectivity(), edmonds_karp(), shortest_augmenting_path()
```


## References

### 4.14.3 Flow-based Connectivity

Flow based connectivity algorithms

| average_node_connectivity(G[, flow_func]) | Returns the average connectivity of a graph G. |
| :---: | :---: |
| all_pairs_node_connectivity(G[, nbunch, ...]) | Compute node connectivity between all pairs of nodes of G. |
| edge_connectivity(G[, s, t, flow_func]) | Returns the edge connectivity of the graph or digraph G. |
| local_edge_connectivity(G, u, v[, ...]) | Returns local edge connectivity for nodes s and tin G. |
| local_node_connectivity(G, s, t[, ...]) | Computes local node connectivity for nodes s and t. |
| node_connectivity(G[, s, t, flow_func]) | Returns node connectivity for a graph or digraph G. |

```
average_node_connectivity
```

average_node_connectivity ( $G$, flow_func=None)
Returns the average connectivity of a graph G.
The average connectivity bar \{kappa \} of a graph G is the average of local node connectivity over all pairs of nodes of $\mathrm{G}^{1}$.

$$
\bar{\kappa}(G)=\frac{\sum_{u, v} \kappa_{G}(u, v)}{\binom{n}{2}}
$$

## Parameters

- G (NetworkX graph) - Undirected graph
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See local_node_connectivity () for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns K - Average node connectivity
Return type float

## See also:

local_node_connectivity(), node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

[^57]
## References

all_pairs_node_connectivity
all_pairs_node_connectivity (G, nbunch=None, flow_func=None)
Compute node connectivity between all pairs of nodes of G .

## Parameters

- G (NetworkX graph) - Undirected graph
- nbunch (container) - Container of nodes. If provided node connectivity will be computed only over pairs of nodes in nbunch.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns all_pairs - A dictionary with node connectivity between all pairs of nodes in G, or in nbunch if provided.

## Return type dict

## See also:

local_node_connectivity(), edge_connectivity(), local_edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

```
edge_connectivity
```

edge_connectivity ( $G$, $s=$ None, $t=$ None, flow_func=None)

Returns the edge connectivity of the graph or digraph G.
The edge connectivity is equal to the minimum number of edges that must be removed to disconnect G or render it trivial. If source and target nodes are provided, this function returns the local edge connectivity: the minimum number of edges that must be removed to break all paths from source to target in G.

## Parameters

- G (NetworkX graph) - Undirected or directed graph
- s (node) - Source node. Optional. Default value: None.
- $\mathbf{t}$ (node) - Target node. Optional. Default value: None.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns $\mathbf{K}$ - Edge connectivity for G, or local edge connectivity if source and target were provided
Return type integer

## Examples

```
>>> # Platonic icosahedral graph is 5-edge-connected
>>> G = nx.icosahedral_graph()
>>> nx.edge_connectivity(G)
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp (), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> nx.edge_connectivity(G, flow_func=shortest_augmenting_path)
5
```

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local edge connectivity.

```
>>> nx.edge_connectivity(G, 3, 7)
5
```

If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See local_edge_connectivity() for details.

## Notes

This is a flow based implementation of global edge connectivity. For undirected graphs the algorithm works by finding a 'small' dominating set of nodes of $G$ (see algorithm 7 in ${ }^{1}$ ) and computing local maximum flow (see local_edge_connectivity()) between an arbitrary node in the dominating set and the rest of nodes in it. This is an implementation of algorithm 6 in ${ }^{1}$. For directed graphs, the algorithm does $n$ calls to the maximum flow function. This is an implementation of algorithm 8 in ${ }^{1}$.

## See also:

```
local_edge_connectivity(), local_node_connectivity(), node_connectivity(),
maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()
```


## References

## local_edge_connectivity

local_edge_connectivity ( $G, u, v$, flow_func $=$ None, auxiliary=None, residual $=$ None, cutoff $=$ None )
Returns local edge connectivity for nodes $s$ and $t$ in $G$.
Local edge connectivity for two nodes $s$ and $t$ is the minimum number of edges that must be removed to disconnect them.

This is a flow based implementation of edge connectivity. We compute the maximum flow on an auxiliary digraph build from the original network (see below for details). This is equal to the local edge connectivity

[^58]because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut (Ford and Fulkerson theorem) ${ }^{1}$.

## Parameters

- G (NetworkX graph) - Undirected or directed graph
- s (node) - Source node
- t (node) - Target node
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- auxiliary (NetworkX DiGraph) - Auxiliary digraph for computing flow based edge connectivity. If provided it will be reused instead of recreated. Default value: None.
- residual (NetworkX DiGraph) - Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.
- cutoff (integer, float) - If specified, the maximum flow algorithm will terminate when the flow value reaches or exceeds the cutoff. This is only for the algorithms that support the cutoff parameter: edmonds_karp() and shortest_augmenting_path(). Other algorithms will ignore this parameter. Default value: None.

Returns K - local edge connectivity for nodes s and t .
Return type integer

## Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```
>>> from networkx.algorithms.connectivity import local_edge_connectivity
```

We use in this example the platonic icosahedral graph, which has edge connectivity 5.

```
>>> G = nx.icosahedral_graph()
>>> local_edge_connectivity(G, 0, 6)
5
```

If you need to compute local connectivity on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for edge connectivity, and the residual network for the underlying maximum flow computation.
Example of how to compute local edge connectivity among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```
>>> import itertools
>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (
... build_auxiliary_edge_connectivity)
```

[^59]```
>>> H = build_auxiliary_edge_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> # Note that the auxiliary digraph has an edge attribute named capacity
>>> R = build_residual_network(H, 'capacity')
>>> result = dict.fromkeys(G, dict())
>>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>>> for u, v in itertools.combinations(G, 2):
... k = local_edge_connectivity(G, u, v, auxiliary=H, residual=R)
... result[u][v] = k
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing edge connectivity. For instance, in dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp () which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> local_edge_connectivity(G, 0, 6, flow_func=shortest_augmenting_path)
5
```


## Notes

This is a flow based implementation of edge connectivity. We compute the maximum flow using, by default, the edmonds_karp () algorithm on an auxiliary digraph build from the original input graph:

If the input graph is undirected, we replace each edge ( $u,{ }^{\prime} v^{`}$ ) with two reciprocal arcs ( $u, v$ ) and ( $v, u$ ) and then we set the attribute 'capacity' for each arc to 1 . If the input graph is directed we simply add the 'capacity' attribute. This is an implementation of algorithm 1 in ${ }^{1}$.

The maximum flow in the auxiliary network is equal to the local edge connectivity because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut (Ford and Fulkerson theorem).

## See also:

```
edge_connectivity(), local_node_connectivity(), node_connectivity(),
maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()
```


## References

## local_node_connectivity

local_node_connectivity ( $G, s, t$, flow_func=None, auxiliary=None, residual=None, cutoff=None)
Computes local node connectivity for nodes $s$ and $t$.
Local node connectivity for two non adjacent nodes $s$ and $t$ is the minimum number of nodes that must be removed (along with their incident edges) to disconnect them.
This is a flow based implementation of node connectivity. We compute the maximum flow on an auxiliary digraph build from the original input graph (see below for details).

## Parameters

- G (NetworkX graph) - Undirected graph
- s (node) - Source node
- t (node) - Target node
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- auxiliary (NetworkX DiGraph) - Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in $G$ and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- residual (NetworkX DiGraph) - Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.
- cutoff (integer, float) - If specified, the maximum flow algorithm will terminate when the flow value reaches or exceeds the cutoff. This is only for the algorithms that support the cutoff parameter: edmonds_karp() and shortest_augmenting_path(). Other algorithms will ignore this parameter. Default value: None.

Returns K - local node connectivity for nodes s and t
Return type integer

## Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```
>>> from networkx.algorithms.connectivity import local_node_connectivity
```

We use in this example the platonic icosahedral graph, which has node connectivity 5.

```
>>> G = nx.icosahedral_graph()
>>> local_node_connectivity(G, 0, 6)
5
```

If you need to compute local connectivity on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for node connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute local node connectivity among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```
>>> import itertools
>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (
... build_auxiliary_node_connectivity)
\cdots .
>>> H = build_auxiliary_node_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
```

```
>>> from networkx.algorithms.flow import build_residual_network
>> # Note that the auxiliary digraph has an edge attribute named capacity
>>> R = build_residual_network(H, 'capacity')
>>> result = dict.fromkeys(G, dict())
>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>>> for u, v in itertools.combinations(G, 2):
... k = local_node_connectivity(G, u, v, auxiliary=H, residual=R)
... result[u][v] = k
•••
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing node connectivity. For instance, in dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp () which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> local_node_connectivity(G, 0, 6, flow_func=shortest_augmenting_path)
5
```


## Notes

This is a flow based implementation of node connectivity. We compute the maximum flow using, by default, the edmonds_karp () algorithm (see: maximum_flow()) on an auxiliary digraph build from the original input graph:

For an undirected graph $G$ having $n$ nodes and $m$ edges we derive a directed graph H with 2 n nodes and $2 \mathrm{~m}+\mathrm{n}$ arcs by replacing each original node $v$ with two nodes $v \_A, v \_B$ linked by an (internal) arc in $H$. Then for each edge $(u, v)$ in $G$ we add two arcs ( $u \_B, v \_A$ ) and ( $\left.v \_B, u \_A\right)$ in $H$. Finally we set the attribute capacity $=1$ for each arc in $\mathrm{H}^{1}$.

For a directed graph $G$ having $n$ nodes and $m$ arcs we derive a directed graph $H$ with $2 n$ nodes and $m+n$ arcs by replacing each original node $v$ with two nodes $v \_A, v \_B$ linked by an (internal) arc ( $v \_A, v \_B$ ) in $H$. Then for each $\operatorname{arc}(u, v)$ in $G$ we add one $\operatorname{arc}\left(u \_B, v \_A\right)$ in $H$. Finally we set the attribute capacity $=1$ for each arc in $H$.

This is equal to the local node connectivity because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut.

## See also:

```
local_edge_connectivity(), node_connectivity(), minimum_node_cut(),
maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()
```


## References

```
node_connectivity
```

node_connectivity ( $G, s=$ None, $t=$ None, flow_func=None)

Returns node connectivity for a graph or digraph $G$.

[^60]Node connectivity is equal to the minimum number of nodes that must be removed to disconnect G or render it trivial. If source and target nodes are provided, this function returns the local node connectivity: the minimum number of nodes that must be removed to break all paths from source to target in G.

## Parameters

- G (NetworkX graph) - Undirected graph
- s (node) - Source node. Optional. Default value: None.
- t (node) - Target node. Optional. Default value: None.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp ()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
Returns K - Node connectivity of G, or local node connectivity if source and target are provided.
Return type integer


## Examples

```
>>> # Platonic icosahedral graph is 5-node-connected
>>> G = nx.icosahedral_graph()
>>> nx.node_connectivity(G)
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp (), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> nx.node_connectivity(G, flow_func=shortest_augmenting_path)
5
```

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local node connectivity.

```
>>> nx.node_connectivity(G, 3, 7)
5
```

If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See local_node_connectivity() for details.

## Notes

This is a flow based implementation of node connectivity. The algorithm works by solving $O((\mathrm{n}-\mathrm{delta}-1+$ delta (delta-1)/2)) maximum flow problems on an auxiliary digraph. Where delta is the minimum degree of G. For details about the auxiliary digraph and the computation of local node connectivity see local_node_connectivity(). This implementation is based on algorithm 11 in ${ }^{1}$.

[^61]
## See also:

```
local_node_connectivity(), edge_connectivity(), maximum_flow(),
edmonds_karp(), preflow_push(), shortest_augmenting_path()
```


## References

### 4.14.4 Flow-based Minimum Cuts

Flow based cut algorithms

| minimum_edge_cut(G[, s, t, flow_func $])$ | Returns a set of edges of minimum cardinality that discon- <br> nects $G$. |
| :--- | :--- |
| minimum_node_cut(G[, s, t, flow_func $])$ | Returns a set of nodes of minimum cardinality that discon- <br> nects $G$. |
| minimum_st_edge_cut(G, s, t[, flow_func, ...]) | Returns the edges of the cut-set of a minimum (s, t)-cut. |
| minimum_st_node_cut(G, s, t[, flow_func, ...]) | Returns a set of nodes of minimum cardinality that discon- <br> nect source from target in G. |

## minimum_edge_cut

minimum_edge_cut ( $G, s=$ None, $t=$ None, flow_func=None)
Returns a set of edges of minimum cardinality that disconnects G.
If source and target nodes are provided, this function returns the set of edges of minimum cardinality that, if removed, would break all paths among source and target in G. If not, it returns a set of edges of minimum cardinality that disconnects G.

## Parameters

- G (NetworkX graph)
- s (node) - Source node. Optional. Default value: None.
- t (node) - Target node. Optional. Default value: None.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns cutset - Set of edges that, if removed, would disconnect G. If source and target nodes are provided, the set contians the edges that if removed, would destroy all paths between source and target.

Return type set

## Examples

```
>>> # Platonic icosahedral graph has edge connectivity 5
>>> G = nx.icosahedral_graph()
>>> len(nx.minimum_edge_cut(G))
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp (), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(nx.minimum_edge_cut(G, flow_func=shortest_augmenting_path))
5
```

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local edge connectivity.

```
>>> nx.edge_connectivity(G, 3, 7)
5
```

If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See local_edge_connectivity() for details.

## Notes

This is a flow based implementation of minimum edge cut. For undirected graphs the algorithm works by finding a 'small' dominating set of nodes of G (see algorithm 7 in ${ }^{1}$ ) and computing the maximum flow between an arbitrary node in the dominating set and the rest of nodes in it. This is an implementation of algorithm 6 in ${ }^{1}$. For directed graphs, the algorithm does $n$ calls to the max flow function. The function raises an error if the directed graph is not weakly connected and returns an empty set if it is weakly connected. It is an implementation of algorithm 8 in ${ }^{1}$.

## See also:

```
minimum_st_edge_cut(), minimum_node_cut(), stoer_wagner(),
node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(),
preflow_push(), shortest_augmenting_path()
```


## References

```
minimum_node_cut
```

minimum_node_cut ( $G, s=$ None, $t=$ None, flow_func $=$ None )

Returns a set of nodes of minimum cardinality that disconnects G.
If source and target nodes are provided, this function returns the set of nodes of minimum cardinality that, if removed, would destroy all paths among source and target in G. If not, it returns a set of nodes of minimum cardinality that disconnects G.

## Parameters

- G (NetworkX graph)
- s (node) - Source node. Optional. Default value: None.
- t (node) - Target node. Optional. Default value: None.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node,

[^62]and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns cutset - Set of nodes that, if removed, would disconnect G. If source and target nodes are provided, the set contians the nodes that if removed, would destroy all paths between source and target.

## Return type set

## Examples

```
>>> # Platonic icosahedral graph has node connectivity 5
>>> G = nx.icosahedral_graph()
>>> node_cut = nx.minimum_node_cut(G)
>>> len(node_cut)
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp (), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> node_cut == nx.minimum_node_cut(G, flow_func=shortest_augmenting_path)
True
```

If you specify a pair of nodes (source and target) as parameters, this function returns a local st node cut.

```
>>> len(nx.minimum_node_cut(G, 3, 7))
5
```

If you need to perform several local st cuts among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See minimum_st_node_cut () for details.

## Notes

This is a flow based implementation of minimum node cut. The algorithm is based in solving a number of maximum flow computations to determine the capacity of the minimum cut on an auxiliary directed network that corresponds to the minimum node cut of G. It handles both directed and undirected graphs. This implementation is based on algorithm 11 in ${ }^{1}$.

## See also:

```
minimum_st_node_cut(), minimum_cut(), minimum_edge_cut(), stoer_wagner(),
node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(),
preflow_push(), shortest_augmenting_path()
```

[^63]
## References

## minimum_st_edge_cut

minimum_st_edge_cut $(G, s, t$, flow_func $=$ None, auxiliary=None, residual=None)
Returns the edges of the cut-set of a minimum ( $\mathrm{s}, \mathrm{t}$ )-cut.
This function returns the set of edges of minimum cardinality that, if removed, would destroy all paths among source and target in G. Edge weights are not considered. See minimum_cut () for computing minimum cuts considering edge weights.

## Parameters

- G (NetworkX graph)
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- auxiliary (NetworkX DiGraph) - Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in $G$ and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See node_connectivity() for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- residual (NetworkX DiGraph) - Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

Returns cutset - Set of edges that, if removed from the graph, will disconnect it.
Return type set

## See also:

minimum_cut(), minimum_node_cut(), minimum_edge_cut(), stoer_wagner(), node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

## Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:
>>> from networkx.algorithms.connectivity import minimum_st_edge_cut

We use in this example the platonic icosahedral graph, which has edge connectivity 5.

```
>>> G = nx.icosahedral_graph()
>>> len(minimum_st_edge_cut(G, 0, 6))
5
```

If you need to compute local edge cuts on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for edge connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute local edge cuts among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```
>>> import itertools
>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (
... build_auxiliary_edge_connectivity)
>>> H = build_auxiliary_edge_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> # Note that the auxiliary digraph has an edge attribute named capacity
>>> R = build_residual_network(H, 'capacity')
>>> result = dict.fromkeys(G, dict())
>>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>>> for u, v in itertools.combinations(G, 2):
... k = len(minimum_st_edge_cut(G, u, v, auxiliary=H, residual=R))
... result[u][v] = k
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing edge cuts. For instance, in dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp() which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(minimum_st_edge_cut(G, 0, 6, flow_func=shortest_augmenting_path))
5
```


## minimum_st_node_cut

minimum_st_node_cut $(G, s, t$, flow_func $=$ None, auxiliary=None, residual=None)
Returns a set of nodes of minimum cardinality that disconnect source from target in G.
This function returns the set of nodes of minimum cardinality that, if removed, would destroy all paths among source and target in G.

## Parameters

- G (NetworkX graph)
- $\mathbf{s}$ (node) - Source node.
- t (node) - Target node.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow () for details). If flow_func is None, the default maximum flow function (edmonds_karp ()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- auxiliary (NetworkX DiGraph) - Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in $G$ and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- residual (NetworkX DiGraph) - Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

Returns cutset - Set of nodes that, if removed, would destroy all paths between source and target in G.

Return type set

## Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```
>>> from networkx.algorithms.connectivity import minimum_st_node_cut
```

We use in this example the platonic icosahedral graph, which has node connectivity 5.

```
>>> G = nx.icosahedral_graph()
>>> len(minimum_st_node_cut(G, 0, 6))
5
```

If you need to compute local st cuts between several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for node connectivity and node cuts, and the residual network for the underlying maximum flow computation.

Example of how to compute local st node cuts reusing the data structures:

```
>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (
... build_auxiliary_node_connectivity)
>>> H = build_auxiliary__node_connectivity(G)
>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>> # Note that the auxiliary digraph has an edge attribute named capacity
>>> R = build_residual_network(H, 'capacity')
>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>> len(minimum_st_node_cut(G, 0, 6, auxiliary=H, residual=R))
5
```

You can also use alternative flow algorithms for computing minimum st node cuts. For instance, in dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp () which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(minimum_st_node_cut(G, 0, 6, flow_func=shortest_augmenting_path))
5
```


## Notes

This is a flow based implementation of minimum node cut. The algorithm is based in solving a number of maximum flow computations to determine the capacity of the minimum cut on an auxiliary directed network that corresponds to the minimum node cut of G. It handles both directed and undirected graphs. This implementation is based on algorithm 11 in ${ }^{1}$.

## See also:

```
minimum_node_cut(), minimum_edge_cut(), stoer_wagner(), node_connectivity(),
edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(),
shortest_augmenting_path()
```


## References

### 4.14.5 Stoer-Wagner minimum cut

Stoer-Wagner minimum cut algorithm.

$$
\begin{array}{ll}
\hline \text { stoer_wagner }(\mathrm{G}[\text {, weight, heap }]) & \begin{array}{l}
\text { Returns the weighted minimum edge cut using the Stoer- } \\
\\
\text { Wagner algorithm. }
\end{array}
\end{array}
$$

## stoer_wagner

stoer_wagner ( $G$, weight $=$ 'weight', heap $=<$ class 'networkx.utils.heaps.BinaryHeap'> )
Returns the weighted minimum edge cut using the Stoer-Wagner algorithm.
Determine the minimum edge cut of a connected graph using the Stoer-Wagner algorithm. In weighted cases, all weights must be nonnegative.

The running time of the algorithm depends on the type of heaps used:

| Type of heap | Running time |
| :--- | :--- |
| Binary heap | $O(n(m+n) \log n)$ |
| Fibonacci heap | $O\left(n m+n^{\wedge} 2 \log n\right)$ |
| Pairing heap | $O\left(2^{\wedge}\{2 \operatorname{sqrt}\{\log \log n\}\} n m+n^{\wedge} 2 \log n\right)$ |

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute named by the weight parameter below. If this attribute is not present, the edge is considered to have unit weight.
- weight (string) - Name of the weight attribute of the edges. If the attribute is not present, unit weight is assumed. Default value: 'weight'.
- heap (class) - Type of heap to be used in the algorithm. It should be a subclass of MinHeap or implement a compatible interface.
If a stock heap implementation is to be used, BinaryHeap is recommeded over PairingHeap for Python implementations without optimized attribute accesses (e.g., CPython) despite a slower asymptotic running time. For Python implementations with optimized attribute accesses (e.g., PyPy), PairingHeap provides better performance. Default value: BinaryHeap.

[^64]
## Returns

- cut_value (integer or float) - The sum of weights of edges in a minimum cut.
- partition (pair of node lists) - A partitioning of the nodes that defines a minimum cut.


## Raises

- NetworkXNot Implemented - If the graph is directed or a multigraph.
- NetworkXError - If the graph has less than two nodes, is not connected or has a negativeweighted edge.


## Examples

```
>>> G = nx.Graph()
>>> G.add_edge('x','a', weight=3)
>>> G.add_edge('x','b', weight=1)
>>> G.add_edge('a','c', weight=3)
>>> G.add_edge('b','c', weight=5)
>>> G.add_edge('b','d', weight=4)
>>> G.add_edge('d','e', weight=2)
>>> G.add_edge('C','y', weight=2)
>>> G.add_edge('e','y', weight=3)
>>> cut_value, partition = nx.stoer_wagner(G)
>>> cut_value
4
```


### 4.14.6 Utils for flow-based connectivity

Utilities for connectivity package

| build_auxiliary_edge_connectivity(G) | Auxiliary digraph for computing flow based edge connec- <br> tivity |
| :--- | :--- |
| build_auxiliary_node_connectivity $(\mathrm{G})$ | Creates a directed graph D from an undirected graph G to <br> compute flow based node connectivity. |

build_auxiliary_edge_connectivity
build_auxiliary_edge_connectivity ( $G$ )
Auxiliary digraph for computing flow based edge connectivity
If the input graph is undirected, we replace each edge ( $u,{ }^{\prime} v^{\prime}$ ) with two reciprocal arcs $(u, v)$ and $(v, u)$ and then we set the attribute 'capacity' for each arc to 1 . If the input graph is directed we simply add the 'capacity' attribute. Part of algorithm 1 in ${ }^{1}$.

[^65]
## References

## build_auxiliary_node_connectivity

build_auxiliary_node_connectivity ( $G$ )
Creates a directed graph D from an undirected graph $G$ to compute flow based node connectivity.
For an undirected graph $G$ having $n$ nodes and $m$ edges we derive a directed graph $D$ with $2 n$ nodes and $2 m+n$ arcs by replacing each original node $v$ with two nodes $v A$, vB linked by an (internal) arc in D . Then for each edge $(u, v)$ in $G$ we add two arcs ( $u B, v A$ ) and (vB, $u A$ ) in D. Finally we set the attribute capacity $=1$ for each arc in $\mathrm{D}^{1}$.

For a directed graph having $n$ nodes and $m$ arcs we derive a directed graph $D$ with $2 n$ nodes and $m+n$ arcs by replacing each original node v with two nodes $\mathrm{vA}, \mathrm{vB}$ linked by an (internal) arc ( $\mathrm{vA}, \mathrm{vB}$ ) in D . Then for each $\operatorname{arc}(u, v)$ in $G$ we add one $\operatorname{arc}(u B, v A)$ in D. Finally we set the attribute capacity $=1$ for each arc in D.

A dictionary with a mapping between nodes in the original graph and the auxiliary digraph is stored as a graph attribute: H.graph['mapping'].

## References

### 4.15 Cores

Find the k-cores of a graph.
The k-core is found by recursively pruning nodes with degrees less than k .
See the following references for details:
An O(m) Algorithm for Cores Decomposition of Networks Vladimir Batagelj and Matjaz Zaversnik, 2003. http: //arxiv.org/abs/cs.DS/0310049

Generalized Cores Vladimir Batagelj and Matjaz Zaversnik, 2002. http://arxiv.org/pdf/cs/0202039
For directed graphs a more general notion is that of D-cores which looks at ( $k, 1$ ) restrictions on (in, out) degree. The $(\mathrm{k}, \mathrm{k}) \mathrm{D}$-core is the k -core.

D-cores: Measuring Collaboration of Directed Graphs Based on Degeneracy Christos Giatsidis, Dimitrios M. Thilikos, Michalis Vazirgiannis, ICDM 2011. http://www.graphdegeneracy.org/dcores_ICDM_2011.pdf

| core_number $(\mathrm{G})$ | Return the core number for each vertex. |
| :--- | :--- |
| $k \_$core $(\mathrm{G}[, \mathrm{k}$, core_number $])$ | Return the k-core of G. |
| $k \_\operatorname{shell}(\mathrm{G}[, \mathrm{k}$, core_number $])$ | Return the k-shell of G. |
| $k$ _crust $(\mathrm{G}[, \mathrm{k}$, core_number $])$ | Return the k-crust of G. |
| k_corona $(\mathrm{G}, \mathrm{k}[$, core_number $])$ | Return the k-corona of G. |

### 4.15.1 core_number

## core_number ( $G$ )

Return the core number for each vertex.

[^66]A k-core is a maximal subgraph that contains nodes of degree k or more.
The core number of a node is the largest value k of a k -core containing that node.
Parameters G (NetworkX graph) - A graph or directed graph
Returns core_number - A dictionary keyed by node to the core number.
Return type dictionary
Raises NetworkXError - The k-core is not implemented for graphs with self loops or parallel edges.

## Notes

Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.

## References

### 4.15.2 k_core

$\mathbf{k}$ _core ( $G, k=$ None, core_number=None)
Return the k-core of G.
A k-core is a maximal subgraph that contains nodes of degree k or more.

## Parameters

- G (NetworkX graph) - A graph or directed graph
- k (int, optional) - The order of the core. If not specified return the main core.
- core_number (dictionary, optional) - Precomputed core numbers for the graph G.

Returns G - The k-core subgraph
Return type NetworkX graph
Raises NetworkXError - The k-core is not defined for graphs with self loops or parallel edges.

## Notes

The main core is the core with the largest degree.
Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

## See also:

core_number()

## References

### 4.15.3 k_shell

$\mathbf{k}$ _shell $(G, k=N o n e$, core_number $=$ None $)$
Return the k -shell of G .
The k -shell is the subgraph induced by nodes with core number k . That is, nodes in the k-core that are not in the ( $k+1$ )-core.

## Parameters

- G (NetworkX graph) - A graph or directed graph.
- $\mathbf{k}$ (int, optional) - The order of the shell. If not specified return the outer shell.
- core_number (dictionary, optional) - Precomputed core numbers for the graph G.

Returns G - The k-shell subgraph
Return type NetworkX graph
Raises NetworkXError - The k-shell is not implemented for graphs with self loops or parallel edges.

## Notes

This is similar to $\mathrm{k} \_$corona but in that case only neighbors in the k -core are considered.
Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

## See also:

```
core_number(), k_corona()
```


## References

### 4.15.4 k_crust

$\mathbf{k}$ _crust $(G, k=$ None, core_number=None)
Return the k-crust of $G$.
The k-crust is the graph G with the k -core removed.

## Parameters

- G (NetworkX graph) - A graph or directed graph.
- $\mathbf{k}$ (int, optional) - The order of the shell. If not specified return the main crust.
- core_number (dictionary, optional) - Precomputed core numbers for the graph G.

Returns G - The k-crust subgraph
Return type NetworkX graph
Raises NetworkXError - The k-crust is not implemented for graphs with self loops or parallel edges.

## Notes

This definition of k -crust is different than the definition in ${ }^{1}$. The k -crust in ${ }^{1}$ is equivalent to the $\mathrm{k}+1$ crust of this algorithm.

Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

## See also:

```
core_number()
```


## References

### 4.15.5 k_corona

$\mathbf{k}$ _corona ( $G, k$, core_number=None)
Return the k -corona of G .
The k-corona is the subgraph of nodes in the k-core which have exactly k neighbours in the k -core.

## Parameters

- G (NetworkX graph) - A graph or directed graph
- $\mathbf{k}($ int $)$ - The order of the corona.
- core_number (dictionary, optional) - Precomputed core numbers for the graph G.

Returns G - The k-corona subgraph
Return type NetworkX graph
Raises NetworkXError - The k-cornoa is not defined for graphs with self loops or parallel edges.

## Notes

Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

## See also:

core_number()

## References

### 4.16 Covering

Functions related to graph covers.

[^67]| min_edge_cover(G[, matching_algorithm] | Returns a set of edges which constitutes the minimum edge <br> cover of the graph. |
| :--- | :--- |
| is_edge_cover $(\mathbf{G}$, cover $)$ | Decides whether a set of edges is a valid edge cover of the <br> graph. |

### 4.16.1 min_edge_cover

min_edge_cover (G, matching_algorithm=None)
Returns a set of edges which constitutes the minimum edge cover of the graph.
A smallest edge cover can be found in polynomial time by finding a maximum matching and extending it greedily so that all nodes are covered.

## Parameters

- G (NetworkX graph) - An undirected bipartite graph.
- matching_algorithm (function) - A function that returns a maximum cardinality matching in a given bipartite graph. The function must take one input, the graph $G$, and return a dictionary mapping each node to its mate. If not specified, hopcroft_karp_matching() will be used. Other possibilities include eppstein_matching(), or matching algorithms in the networkx.algorithms.matching module.
Returns min_cover - It contains all the edges of minimum edge cover in form of tuples. It contains both the edges $(u, v)$ and $(v, u)$ for given nodes $u$ and $v$ among the edges of minimum edge cover.
Return type set


## Notes

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of the set. The minimum edge cover is an edge covering of smallest cardinality.
Due to its implementation, the worst-case running time of this algorithm is bounded by the worst-case running time of the function matching_algorithm.
Minimum edge cover for bipartite graph can also be found using the function present in networkx.algorithms.bipartite.covering

### 4.16.2 is_edge_cover

is_edge_cover ( $G$, cover)
Decides whether a set of edges is a valid edge cover of the graph.
Given a set of edges, whether it is an edge covering can be decided if we just check whether all nodes of the graph has an edge from the set, incident on it.

## Parameters

- G (NetworkX graph) - An undirected bipartite graph.
- cover (set) - Set of edges to be checked.

Returns Whether the set of edges is a valid edge cover of the graph.
Return type bool

## Notes

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of the set.

### 4.17 Cycles

### 4.17.1 Cycle finding algorithms

| cycle_basis(G[, root $])$ | Returns a list of cycles which form a basis for cycles of G. |
| :--- | :--- |
| simple_cycles(G) | Find simple cycles (elementary circuits) of a directed <br> graph. |
| find_cycle(G[, source, orientation]) | Returns the edges of a cycle found via a directed, depth- <br> first traversal. |

### 4.17.2 cycle_basis

cycle_basis (G, root=None)
Returns a list of cycles which form a basis for cycles of $G$.
A basis for cycles of a network is a minimal collection of cycles such that any cycle in the network can be written as a sum of cycles in the basis. Here summation of cycles is defined as "exclusive or" of the edges. Cycle bases are useful, e.g. when deriving equations for electric circuits using Kirchhoff's Laws.

## Parameters

- G (NetworkX Graph)
- root (node, optional) - Specify starting node for basis.


## Returns

- A list of cycle lists. Each cycle list is a list of nodes
- which forms a cycle (loop) in $G$.


## Examples

```
>>> G=nx.Graph()
>>> nx.add_cycle(G, [0, 1, 2, 3])
>>> nx.add_cycle(G, [0, 3, 4, 5])
>>> print(nx.cycle_basis(G,0))
[[3, 4, 5, 0], [1, 2, 3, 0]]
```


## Notes

This is adapted from algorithm CACM $491{ }^{1}$.

[^68]
## References

## See also:

simple_cycles()

### 4.17.3 simple_cycles

## simple_cycles $(G)$

Find simple cycles (elementary circuits) of a directed graph.
A simple cycle, or elementary circuit, is a closed path where no node appears twice. Two elementary circuits are distinct if they are not cyclic permutations of each other.

This is a nonrecursive, iterator/generator version of Johnson's algorithm ${ }^{1}$. There may be better algorithms for some cases ${ }^{23}$.

Parameters G (NetworkX DiGraph) - A directed graph
Returns cycle_generator - A generator that produces elementary cycles of the graph. Each cycle is represented by a list of nodes along the cycle.

## Return type generator

## Examples

```
>>> G = nx.DiGraph([(0, 0), (0, 1), (0, 2), (1, 2), (2, 0), (2, 1), (2, 2)])
>>> len(list(nx.simple_cycles(G)))
5
```

To filter the cycles so that they don't include certain nodes or edges, copy your graph and eliminate those nodes or edges before calling

```
>>> copyG = G.copy()
>>> copyG.remove_nodes_from([1])
>>> copyG.remove_edges_from([(0, 1)])
>>> len(list(nx.simple_cycles(copyG)))
3
```


## Notes

The implementation follows pp. 79-80 in ${ }^{1}$.
The time complexity is $O((n+e)(c+1))$ for $n$ nodes, e edges and $c$ elementary circuits.

## References

## See also:

[^69]
### 4.17.4 find_cycle

find_cycle ( $G$, source $=$ None, orientation='original')
Returns the edges of a cycle found via a directed, depth-first traversal.

## Parameters

- G (graph) - A directed/undirected graph/multigraph.
- source (node, list of nodes) - The node from which the traversal begins. If None, then a source is chosen arbitrarily and repeatedly until all edges from each node in the graph are searched.
- orientation ('original'। 'reverse'। 'ignore') - For directed graphs and directed multigraphs, edge traversals need not respect the original orientation of the edges. When set to 'reverse', then every edge will be traversed in the reverse direction. When set to 'ignore', then each directed edge is treated as a single undirected edge that can be traversed in either direction. For undirected graphs and undirected multigraphs, this parameter is meaningless and is not consulted by the algorithm.

Returns edges - A list of directed edges indicating the path taken for the loop. If no cycle is found, then an exception is raised. For graphs, an edge is of the form $(u, v)$ where $u$ and $v$ are the tail and head of the edge as determined by the traversal. For multigraphs, an edge is of the form ( $u, v, k e y$ ), where key is the key of the edge. When the graph is directed, then $u$ and $v$ are always in the order of the actual directed edge. If orientation is 'ignore', then an edge takes the form ( $\mathrm{u}, \mathrm{v}, \mathrm{key}$, direction) where direction indicates if the edge was followed in the forward (tail to head) or reverse (head to tail) direction. When the direction is forward, the value of direction is 'forward'. When the direction is reverse, the value of direction is 'reverse'.

## Return type directed edges

Raises NetworkXNoCycle - If no cycle was found.

## Examples

In this example, we construct a DAG and find, in the first call, that there are no directed cycles, and so an exception is raised. In the second call, we ignore edge orientations and find that there is an undirected cycle. Note that the second call finds a directed cycle while effectively traversing an undirected graph, and so, we found an "undirected cycle". This means that this DAG structure does not form a directed tree (which is also known as a polytree).

```
>>> import networkx as nx
>>> G = nx.DiGraph([(0,1), (0,2), (1,2)])
>>> try:
    find_cycle(G, orientation='original')
... except:
... pass
>>> list(find_cycle(G, orientation='ignore'))
[(0, 1, 'forward'), (1, 2, 'forward'), (0, 2, 'reverse')]
```


### 4.18 Cuts

Functions for finding and evaluating cuts in a graph.

| boundary_expansion $(\mathbf{G}, \mathrm{S})$ | Returns the boundary expansion of the set S. |
| :--- | :--- |
| conductance(G, S[, T, weight $])$ | Returns the conductance of two sets of nodes. |
| cut_size(G, S[, T, weight $])$ | Returns the size of the cut between two sets of nodes. |
| edge_expansion(G, S[, T, weight $])$ | Returns the edge expansion between two node sets. |
| mixing_expansion $(\mathrm{G}, \mathrm{S}[, \mathrm{T}$, weight $])$ | Returns the mixing expansion between two node sets. |
| node_expansion(G, S) | Returns the node expansion of the set S. |
| normalized_cut_size(G, S[, T, weight $])$ | Returns the normalized size of the cut between two sets of <br> nodes. |
| volume(G, S[, weight $])$ | Returns the volume of a set of nodes. |

### 4.18.1 boundary_expansion

## boundary_expansion ( $G, S$ )

Returns the boundary expansion of the set $S$.
The boundary expansion is the quotient of the size of the edge boundary and the cardinality of S. [1]

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in G.

Returns The boundary expansion of the set $S$.
Return type number

## See also:

edge_expansion(), mixing_expansion(), node_expansion()

## References

### 4.18.2 conductance

```
conductance (G,S,T=None, weight=None)
```

Returns the conductance of two sets of nodes.
The conductance is the quotient of the cut size and the smaller of the volumes of the two sets. [1]

## Parameters

## - G (NetworkX graph)

- $\mathbf{S}$ (sequence) - A sequence of nodes in G.
- T (sequence) - A sequence of nodes in G.
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.

Returns The conductance between the two sets $S$ and $T$.
Return type number

## See also:

```
cut_size(), edge_expansion(), normalized_cut_size(),volume()
```


## References

### 4.18.3 cut_size

## cut_size ( $G, S, T=$ None, weight=None)

Returns the size of the cut between two sets of nodes.
A cut is a partition of the nodes of a graph into two sets. The cut size is the sum of the weights of the edges "between" the two sets of nodes.

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in $G$.
- T (sequence) - A sequence of nodes in G. If not specified, this is taken to be the set complement of $S$.
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.

Returns Total weight of all edges from nodes in set $S$ to nodes in set $T$ (and, in the case of directed graphs, all edges from nodes in $T$ to nodes in $S$ ).

Return type number

## Examples

In the graph with two cliques joined by a single edges, the natural bipartition of the graph into two blocks, one for each clique, yields a cut of weight one:

```
>>> G = nx.barbell_graph(3, 0)
>>> S = {0, 1, 2}
>>> T = {3, 4, 5}
>>> nx.cut_size(G, S, T)
1
```

Each parallel edge in a multigraph is counted when determining the cut size:

```
>>> G = nx.MultiGraph(['ab', 'ab'])
>>> S = {'a'}
>>> T = {'b'}
>>> nx.cut_size(G, S, T)
2
```


## Notes

In a multigraph, the cut size is the total weight of edges including multiplicity.

### 4.18.4 edge_expansion

## edge_expansion ( $G, S, T=$ None, weight=None)

Returns the edge expansion between two node sets.
The edge expansion is the quotient of the cut size and the smaller of the cardinalities of the two sets. [1]

## Parameters

- G (NetworkX graph)
- S (sequence) - A sequence of nodes in G.
- T (sequence) - A sequence of nodes in G.
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.

Returns The edge expansion between the two sets S and T .
Return type number
See also:
boundary_expansion(), mixing_expansion(), node_expansion()

## References

### 4.18.5 mixing_expansion

mixing_expansion ( $G, S, T=$ None, weight=None)
Returns the mixing expansion between two node sets.
The mixing expansion is the quotient of the cut size and twice the number of edges in the graph. [1]

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in G.
- $\mathbf{T}$ (sequence) - A sequence of nodes in G .
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.

Returns The mixing expansion between the two sets S and T .
Return type number

## See also:

boundary_expansion (), edge_expansion(), node_expansion ()

## References

### 4.18.6 node_expansion

## node_expansion ( $G, S$ )

Returns the node expansion of the set $S$.
The node expansion is the quotient of the size of the node boundary of $S$ and the cardinality of $S$. [1]

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in G.

Returns The node expansion of the set $S$.
Return type number

## See also:

boundary_expansion(), edge_expansion(), mixing_expansion()

## References

### 4.18.7 normalized_cut_size

normalized_cut_size ( $G, S, T=$ None, weight=None)
Returns the normalized size of the cut between two sets of nodes.
The normalized cut size is the cut size times the sum of the reciprocal sizes of the volumes of the two sets. [1]

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in G.
- T (sequence) - A sequence of nodes in G.
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.

Returns The normalized cut size between the two sets $S$ and $T$.
Return type number

## Notes

In a multigraph, the cut size is the total weight of edges including multiplicity.
See also:

```
conductance(), cut_size(), edge_expansion(),volume()
```


## References

### 4.18.8 volume

volume ( $G$, $S$, weight=None)
Returns the volume of a set of nodes.
The volume of a set $S$ is the sum of the (out-)degrees of nodes in $S$ (taking into account parallel edges in multigraphs). [1]

## Parameters

- G (NetworkX graph)
- $\mathbf{S}$ (sequence) - A sequence of nodes in G.
- weight (object) - Edge attribute key to use as weight. If not specified, edges have weight one.
Returns The volume of the set of nodes represented by $S$ in the graph $G$.
Return type number


## See also:

conductance(), cut_size(), edge_expansion(), edge_boundary(),
normalized_cut_size()

## References

### 4.19 Directed Acyclic Graphs

Algorithms for directed acyclic graphs (DAGs).

| ancestors(G, source) | Return all nodes having a path to source in G. |
| :--- | :--- |
| descendants(G, source) | Return all nodes reachable from source in G. |
| topological_sort(G) | Return a generator of nodes in topologically sorted order. |
| lexicographical_topological_sort(G[, key]) | Return a generator of nodes in lexicographically topologi- <br> cally sorted order. |
| is_directed_acyclic_graph(G) | Return True if the graph G is a directed acyclic graph <br> (DAG) or False if not. |
| is_aperiodic(G) | Return True if G is aperiodic. |
| transitive_closure(G) | Returns transitive closure of a directed graph |
| transitive_reduction(G) | Returns transitive reduction of a directed graph |
| antichains(G) | Generates antichains from a DAG. |
| dag_longest_path(G[,weight, default_weight]) | Returns the longest path in a DAG If G has edges with <br> 'weight' attribute the edge data are used as weight values. |
| dag_longest_path_length(G[, weight, ...]) | Returns the longest path length in a DAG |

### 4.19.1 ancestors

## ancestors ( $G$, source)

Return all nodes having a path to source in G.

## Parameters

- G (NetworkX DiGraph)
- source (node in G)

Returns ancestors - The ancestors of source in G
Return type set()

### 4.19.2 descendants

## descendants ( $G$, source)

Return all nodes reachable from source in G.

## Parameters

- G (NetworkX DiGraph)
- source (node in G)

Returns des - The descendants of source in G
Return type set()

### 4.19.3 topological_sort

## topological_sort (G)

Return a generator of nodes in topologically sorted order.
A topological sort is a nonunique permutation of the nodes such that an edge from $u$ to $v$ implies that $u$ appears before v in the topological sort order.

Parameters G (NetworkX digraph) - A directed graph
Returns topologically_sorted_nodes - An iterable of node names in topological sorted order.
Return type iterable

## Raises

- NetworkXError - Topological sort is defined for directed graphs only. If the graph G is undirected, a NetworkXError is raised.
- NetworkXUnfeasible - If G is not a directed acyclic graph (DAG) no topological sort exists and a NetworkXUnfeasible exception is raised. This can also be raised if $G$ is changed while the returned iterator is being processed.
- RuntimeError - If G is changed while the returned iterator is being processed.


## Examples

To get the reverse order of the topological sort:

```
>>> DG = nx.DiGraph([(1, 2), (2, 3)])
>>> list(reversed(list(nx.topological_sort(DG))))
[3, 2, 1]
```


## Notes

This algorithm is based on a description and proof in Introduction to algorithms - a creative approach ${ }^{1}$.
See also:

```
is_directed_acyclic_graph(), lexicographical_topological_sort()
```

[^70]
## References

### 4.19.4 lexicographical_topological_sort

## lexicographical_topological_sort ( $G$, key=None)

Return a generator of nodes in lexicographically topologically sorted order.
A topological sort is a nonunique permutation of the nodes such that an edge from $u$ to $v$ implies that $u$ appears before v in the topological sort order.

## Parameters

- G (NetworkX digraph) - A directed graph
- key (function, optional) - This function maps nodes to keys with which to resolve ambiguities in the sort order. Defaults to the identity function.

Returns lexicographically_topologically_sorted_nodes - An iterable of node names in lexicographical topological sort order.

Return type iterable

## Raises

- NetworkXError - Topological sort is defined for directed graphs only. If the graph G is undirected, a NetworkXError is raised.
- NetworkXUnfeasible - If G is not a directed acyclic graph (DAG) no topological sort exists and a NetworkXUnfeasible exception is raised. This can also be raised if $G$ is changed while the returned iterator is being processed.
- RuntimeError - If G is changed while the returned iterator is being processed.


## Notes

This algorithm is based on a description and proof in Introduction to algorithms - a creative approach ${ }^{1}$.
See also:
topological_sort()

## References

### 4.19.5 is_directed_acyclic_graph

is_directed_acyclic_graph ( $G$ )
Return True if the graph $G$ is a directed acyclic graph (DAG) or False if not.
Parameters G (NetworkX graph) - A graph
Returns is_dag - True if G is a DAG, false otherwise
Return type bool

[^71]
### 4.19.6 is_aperiodic

## is_aperiodic ( $G$ )

Return True if G is aperiodic.
A directed graph is aperiodic if there is no integer $\mathrm{k}>1$ that divides the length of every cycle in the graph.
Parameters G (NetworkX DiGraph) - Graph
Returns aperiodic - True if the graph is aperiodic False otherwise
Return type boolean
Raises NetworkXError - If G is not directed

## Notes

This uses the method outlined in ${ }^{1}$, which runs in $\mathrm{O}(\mathrm{m})$ time given $m$ edges in $G$. Note that a graph is not aperiodic if it is acyclic as every integer trivial divides length 0 cycles.

## References

### 4.19.7 transitive_closure

transitive_closure ( $G$ )
Returns transitive closure of a directed graph
The transitive closure of $G=(V, E)$ is a graph $G+=(V, E+)$ such that for all $v, w$ in $V$ there is an edge $(v, w)$ in $E+$ if and only if there is a non-null path from $v$ to $w$ in $G$.

Parameters G (NetworkX DiGraph) - Graph
Returns TC - Graph
Return type NetworkX DiGraph
Raises NetworkXNot Implemented - If G is not directed

## References

### 4.19.8 transitive_reduction

transitive_reduction ( $G$ )
Returns transitive reduction of a directed graph
The transitive reduction of $G=(V, E)$ is a graph $G-=(V, E-)$ such that for all $v, w$ in $V$ there is an edge $(v, w)$ in $E$ - if and only if $(\mathrm{v}, \mathrm{w})$ is in E and there is no path from v to w in G with length greater than 1.

Parameters G (NetworkX DiGraph) - Graph
Returns TR - Graph

## Return type NetworkX DiGraph

Raises NetworkXError - If G is not a directed acyclic graph (DAG) transitive reduction is not uniquely defined and a NetworkXError exception is raised.

[^72]
## References

https://en.wikipedia.org/wiki/Transitive_reduction

### 4.19.9 antichains

## antichains ( $G$ )

Generates antichains from a DAG.
An antichain is a subset of a partially ordered set such that any two elements in the subset are incomparable.
Parameters G (NetworkX DiGraph) - Graph
Returns antichain
Return type generator object

## Raises

- NetworkXNot Implemented - If G is not directed
- NetworkXUnfeasible - If G contains a cycle


## Notes

This function was originally developed by Peter Jipsen and Franco Saliola for the SAGE project. It's included in NetworkX with permission from the authors. Original SAGE code at:
https://sage.informatik.uni-goettingen.de/src/combinat/posets/hasse_diagram.py

## References

### 4.19.10 dag_longest_path

dag_longest_path ( $G$, weight='weight', default_weight=1)
Returns the longest path in a DAG If G has edges with 'weight' attribute the edge data are used as weight values.

## Parameters

- G (NetworkX DiGraph) - Graph
- weight (string (default 'weight')) - Edge data key to use for weight
- default_weight (integer (default 1)) - The weight of edges that do not have a weight attribute

Returns path - Longest path
Return type list
Raises NetworkXNotImplemented - If G is not directed
See also:
dag_longest_path_length()

### 4.19.11 dag_longest_path_length

dag_longest_path_length ( $G$, weight='weight', default_weight=1)
Returns the longest path length in a DAG

## Parameters

- G (NetworkX DiGraph) - Graph
- weight (string (default 'weight')) - Edge data key to use for weight
- default_weight (integer (default 1)) - The weight of edges that do not have a weight attribute

Returns path_length - Longest path length
Return type int
Raises NetworkXNot Implemented - If G is not directed
See also:
dag_longest_path()

### 4.20 Dispersion

dispersion

### 4.21 Distance Measures

Graph diameter, radius, eccentricity and other properties.

| center $(\mathrm{G}[, \mathrm{e}])$ | Return the center of the graph G. |
| :--- | :--- |
| diameter $(\mathrm{G}[, \mathrm{e}])$ | Return the diameter of the graph G. |
| eccentricity $(\mathrm{G}[, \mathrm{v}, \mathrm{sp}])$ | Return the eccentricity of nodes in G. |
| periphery $(\mathrm{G}[, \mathrm{e}])$ | Return the periphery of the graph G. |
| radius $(\mathrm{G}[, \mathrm{e}])$ | Return the radius of the graph G. |

### 4.21.1 center

center ( $G, e=$ None)
Return the center of the graph G.
The center is the set of nodes with eccentricity equal to radius.

## Parameters

- G (NetworkX graph) - A graph
- e (eccentricity dictionary, optional) - A precomputed dictionary of eccentricities.

Returns c-List of nodes in center
Return type list

### 4.21.2 diameter

diameter ( $G, e=$ None)
Return the diameter of the graph G.
The diameter is the maximum eccentricity.

## Parameters

- G (NetworkX graph) - A graph
- e (eccentricity dictionary, optional) - A precomputed dictionary of eccentricities.

Returns d - Diameter of graph
Return type integer
See also:

```
eccentricity()
```


### 4.21.3 eccentricity

eccentricity ( $G, v=$ None, $s p=$ None)
Return the eccentricity of nodes in G.
The eccentricity of a node $v$ is the maximum distance from $v$ to all other nodes in $G$.

## Parameters

- G (NetworkX graph) - A graph
- $\mathbf{v}$ (node, optional) - Return value of specified node
- $\mathbf{s p}$ (dict of dicts, optional) - All pairs shortest path lengths as a dictionary of dictionaries

Returns ecc - A dictionary of eccentricity values keyed by node.
Return type dictionary

### 4.21.4 periphery

periphery ( $G, e=$ None)
Return the periphery of the graph G.
The periphery is the set of nodes with eccentricity equal to the diameter.

## Parameters

- G (NetworkX graph) - A graph
- e (eccentricity dictionary, optional) - A precomputed dictionary of eccentricities.

Returns $\mathbf{p}$ - List of nodes in periphery
Return type list

### 4.21 .5 radius

radius ( $G, e=$ None)
Return the radius of the graph G .
The radius is the minimum eccentricity.

## Parameters

- G (NetworkX graph) - A graph
- e (eccentricity dictionary, optional) - A precomputed dictionary of eccentricities.

Returns $\mathbf{r}$ - Radius of graph
Return type integer

### 4.22 Distance-Regular Graphs

### 4.22.1 Distance-regular graphs

| is_distance_regular $(\mathrm{G})$ | Returns True if the graph is distance regular, False other- <br> wise. |
| :--- | :--- |
| is_strongly_regular $(\mathrm{G})$ | Returns True if and only if the given graph is strongly reg- <br> ular. |
| intersection_array $(\mathrm{G})$ | Returns the intersection array of a distance-regular graph. |
| global_parameters $(\mathrm{b}, \mathrm{c})$ | Return global parameters for a given intersection array. |

### 4.22.2 is_distance_regular

## is_distance_regular $(G)$

Returns True if the graph is distance regular, False otherwise.
A connected graph $G$ is distance-regular if for any nodes $x, y$ and any integers $i, j=0,1, \ldots, d$ (where $d$ is the graph diameter), the number of vertices at distance $i$ from $x$ and distance $j$ from $y$ depends only on $i, j$ and the graph distance between $x$ and $y$, independently of the choice of $x$ and $y$.

Parameters G (Networkx graph (undirected))
Returns True if the graph is Distance Regular, False otherwise
Return type bool

## Examples

```
>>> G=nx.hypercube_graph (6)
>>> nx.is_distance_regular(G)
True
```


## See also:

```
intersection_array(), global_parameters()
```


## Notes

For undirected and simple graphs only

## References

### 4.22.3 is_strongly_regular

## is_strongly_regular ( $G$ )

Returns True if and only if the given graph is strongly regular.
An undirected graph is strongly regular if
-it is regular,

- each pair of adjacent vertices has the same number of neighbors in common,
-each pair of nonadjacent vertices has the same number of neighbors in common.
Each strongly regular graph is a distance-regular graph. Conversely, if a distance-regular graph has diameter two, then it is a strongly regular graph. For more information on distance-regular graphs, see is_distance_regular().

Parameters G (NetworkX graph) - An undirected graph.
Returns Whether G is strongly regular.
Return type bool

## Examples

The cycle graph on five vertices is strongly regular. It is two-regular, each pair of adjacent vertices has no shared neighbors, and each pair of nonadjacent vertices has one shared neighbor:

```
>>> import networkx as nx
>>> G = nx.cycle_graph(5)
>>> nx.is_strongly_regular(G)
True
```


### 4.22.4 intersection_array

intersection_array ( $G$ )
Returns the intersection array of a distance-regular graph.
Given a distance-regular graph $G$ with integers $b \_i, c_{-} i, i=0, \ldots ., d$ such that for any 2 vertices $x, y$ in $G$ at a distance $\mathrm{i}=\mathrm{d}(\mathrm{x}, \mathrm{y})$, there are exactly $\mathrm{c} \mathrm{\_i}$ neighbors of y at a distance of $\mathrm{i}-1$ from x and $\mathrm{b} \_i$ neighbors of y at a distance of $\mathrm{i}+1$ from x .

A distance regular graph's intersection array is given by, $\left[\mathrm{b} \_0, \mathrm{~b} \_1, \ldots . . \mathrm{b} \_\{\mathrm{d}-1\} ; \mathrm{c} \_1, \mathrm{c} \_2, \ldots . . \mathrm{c} \_\mathrm{d}\right]$
Parameters G (Networkx graph (undirected))
Returns b,c
Return type tuple of lists

## Examples

```
>>> G=nx.icosahedral_graph()
>>> nx.intersection_array(G)
([5, 2, 1], [1, 2, 5])
```


## References

## See also:

```
global_parameters()
```


### 4.22.5 global_parameters

global_parameters $(b, c)$
Return global parameters for a given intersection array.
Given a distance-regular graph $G$ with integers $b \_i, c_{-} i, i=0, \ldots, d$ such that for any 2 vertices $x, y$ in $G$ at a distance $\mathrm{i}=\mathrm{d}(\mathrm{x}, \mathrm{y})$, there are exactly $\mathrm{c} \_\mathrm{i}$ neighbors of y at a distance of $\mathrm{i}-1$ from x and $\mathrm{b}_{-} \mathrm{i}$ neighbors of y at a distance of $\mathrm{i}+1$ from x .

Thus, a distance regular graph has the global parameters, [[c_0,a_0,b_0],[c_1,a_1,b_1],....., [c_d,a_d,b_d]] for the intersection array $\left[b \_0, b \_1, \ldots . . . b \_\{d-1\} ; c \_1, c \_2, \ldots . . . c \_d\right]$ where $a \_i+b \_i+c \_i=k, k=$ degree of every vertex.

## Parameters

- b (list)
- $\mathbf{c}$ (list)

Returns An iterable over three tuples.
Return type iterable

## Examples

```
>>> G = nx.dodecahedral_graph()
>>> b, c = nx.intersection_array(G)
>>> list(nx.global_parameters(b, c))
[(0, 0, 3), (1, 0, 2), (1, 1, 1), (1, 1, 1), (2, 0, 1), (3, 0, 0)]
```


## References

## See also:

```
intersection_array()
```


### 4.23 Dominance

Dominance algorithms.

| immediate_dominators $(G$, start $)$ | Returns the immediate dominators of all nodes of a directed <br> graph. |
| :--- | :--- |
| dominance_frontiers(G, start) | Returns the dominance frontiers of all nodes of a directed <br> graph. |

### 4.23.1 immediate_dominators

immediate_dominators ( $G$, start)
Returns the immediate dominators of all nodes of a directed graph.

## Parameters

- G (a DiGraph or MultiDiGraph ) - The graph where dominance is to be computed.
- start (node) - The start node of dominance computation.

Returns idom - A dict containing the immediate dominators of each node reachable from start.
Return type dict keyed by nodes

## Raises

- NetworkXNot Implemented - If G is undirected.
- NetworkXError - If start is not in G.


## Notes

Except for start, the immediate dominators are the parents of their corresponding nodes in the dominator tree.

## Examples

```
>>> G = nx.DiGraph([(1, 2), (1, 3), (2, 5), (3, 4), (4, 5)])
>>> sorted(nx.immediate_dominators(G, 1).items())
[(1, 1), (2, 1), (3, 1), (4, 3), (5, 1)]
```


## References

### 4.23.2 dominance_frontiers

dominance_frontiers ( $G$, start)
Returns the dominance frontiers of all nodes of a directed graph.

## Parameters

- G (a DiGraph or MultiDiGraph) - The graph where dominance is to be computed.
- start (node) - The start node of dominance computation.

Returns df - A dict containing the dominance frontiers of each node reachable from start as lists.
Return type dict keyed by nodes

## Raises

- NetworkXNot Implemented - If G is undirected.
- NetworkXError - If start is not in G.


## Examples

```
>>> G = nx.DiGraph([(1, 2), (1, 3), (2, 5), (3, 4), (4, 5)])
>>> sorted((u, sorted(df)) for u, df in nx.dominance_frontiers(G, 1).items())
[(1, []), (2, [5]), (3, [5]), (4, [5]), (5, [])]
```


## References

### 4.24 Dominating Sets

Functions for computing dominating sets in a graph.

| dominating_set $(\mathrm{G}[$, start_with $])$ | Finds a dominating set for the graph G. |
| :--- | :--- |
| is_dominating_set $(\mathrm{G}$, nbunch $)$ | Checks if nbunch is a dominating set for G. |

### 4.24.1 dominating_set

dominating_set ( $G$, start_with=None)
Finds a dominating set for the graph G.
A dominating set for a graph with node set $V$ is a subset $D$ of $V$ such that every node not in $D$ is adjacent to at least one member of $D^{1}$.

## Parameters

- G (NetworkX graph)
- start_with (node (default=None)) - Node to use as a starting point for the algorithm.

Returns D-A dominating set for $G$.
Return type set

## Notes

This function is an implementation of algorithm 7 in ${ }^{2}$ which finds some dominating set, not necessarily the smallest one.

## See also:

is_dominating_set()

[^73]
## References

### 4.24.2 is_dominating_set

is_dominating_set ( $G$, nbunch)
Checks if nbunch is a dominating set for $G$.
A dominating set for a graph with node set $V$ is a subset $D$ of $V$ such that every node not in $D$ is adjacent to at least one member of $D^{1}$.

## Parameters

- G (NetworkX graph)
- nbunch (iterable) - An iterable of nodes in the graph G.

See also:

```
dominating_set()
```


## References

### 4.25 Efficiency

Provides functions for computing the efficiency of nodes and graphs.

| efficiency $(\mathrm{G}, \mathrm{u}, \mathrm{v})$ | Returns the efficiency of a pair of nodes in a graph. |
| :--- | :--- |
| local_efficiency $(\mathrm{G})$ | Returns the average local efficiency of the graph. |
| global_efficiency $(\mathrm{G})$ | Returns the average global efficiency of the graph. |

### 4.25.1 efficiency

efficiency ( $G, u, v$ )
Returns the efficiency of a pair of nodes in a graph.
The efficiency of a pair of nodes is the multiplicative inverse of the shortest path distance between the nodes ${ }^{1}$.

## Parameters

- G (networkx. Graph) - An undirected graph for which to compute the average local efficiency.
- u, $\mathbf{v}$ (node) - Nodes in the graph G.

Returns Multiplicative inverse of the shortest path distance between the nodes.
Return type float

## Notes

Edge weights are ignored when computing the shortest path distances.

[^74]
## See also:

```
local_efficiency(),global_efficiency()
```


## References

### 4.25.2 local_efficiency

## local_efficiency ( $G$ )

Returns the average local efficiency of the graph.
The efficiency of a pair of nodes in a graph is the multiplicative inverse of the shortest path distance between the nodes. The local efficiency of a node in the graph is the average global efficiency of the subgraph induced by the neighbors of the node. The average local efficiency is the average of the local efficiencies of each node ${ }^{1}$.

Parameters G (networkx. Graph) - An undirected graph for which to compute the average local efficiency.

Returns The average local efficiency of the graph.
Return type float

## Notes

Edge weights are ignored when computing the shortest path distances.

## See also:

```
global_efficiency()
```


## References

### 4.25.3 global_efficiency

## global_efficiency $(G)$

Returns the average global efficiency of the graph.
The efficiency of a pair of nodes in a graph is the multiplicative inverse of the shortest path distance between the nodes. The average global efficiency of a graph is the average efficiency of all pairs of nodes ${ }^{1}$.

Parameters G (networkx. Graph) - An undirected graph for which to compute the average global efficiency.

Returns The average global efficiency of the graph.
Return type float

[^75]
## Notes

Edge weights are ignored when computing the shortest path distances.
See also:

```
local_efficiency()
```


## References

### 4.26 Eulerian

Eulerian circuits and graphs.

| is_eulerian $(\mathrm{G})$ | Returns True if and only if G is Eulerian. |
| :--- | :--- |
| eulerian_circuit(G[, source]) | Returns an iterator over the edges of an Eulerian circuit in |
|  | G. |

### 4.26.1 is_eulerian

## is_eulerian ( $G$ )

Returns True if and only if G is Eulerian.
An graph is Eulerian if it has an Eulerian circuit. An Eulerian circuit is a closed walk that includes each edge of a graph exactly once.

Parameters G (NetworkX graph) - A graph, either directed or undirected.

## Examples

```
>>> nx.is_eulerian(nx.DiGraph({0: [3], 1: [2], 2: [3], 3: [0, 1]}))
True
>>> nx.is_eulerian(nx.complete_graph(5))
True
>>> nx.is_eulerian(nx.petersen_graph())
False
```


## Notes

If the graph is not connected (or not strongly connected, for directed graphs), this function returns False.

### 4.26.2 eulerian_circuit

eulerian_circuit ( $G$, source=None)
Returns an iterator over the edges of an Eulerian circuit in G.
An Eulerian circuit is a closed walk that includes each edge of a graph exactly once.

## Parameters

- G (NetworkX graph) - A graph, either directed or undirected.
- source (node, optional) - Starting node for circuit.

Returns edges - An iterator over edges in the Eulerian circuit.
Return type iterator
Raises NetworkXError - If the graph is not Eulerian.

## See also:

```
is_eulerian()
```


## Notes

This is a linear time implementation of an algorithm adapted from ${ }^{1}$.
For general information about Euler tours, see ${ }^{2}$.

## References

## Examples

To get an Eulerian circuit in an undirected graph:

```
>>> G = nx.complete_graph(3)
>>> list(nx.eulerian_circuit(G))
[(0, 2), (2, 1), (1, 0)]
>>> list(nx.eulerian_circuit(G, source=1))
[(1, 2), (2, 0), (0, 1)]
```

To get the sequence of vertices in an Eulerian circuit:

```
>>> [u for u, v in nx.eulerian_circuit(G)]
[0, 2, 1]
```


### 4.27 Flows

### 4.27.1 Maximum Flow

| maximum_flow $(\mathrm{G}, \mathrm{s}, \mathrm{t}[$, capacity, flow_func $])$ | Find a maximum single-commodity flow. |
| :--- | :--- |
| maximum_flow_value(G, s, $[$, capacity, ... $])$ | Find the value of maximum single-commodity flow. |
| minimum_cut $(\mathrm{G}, \mathrm{s}, \mathrm{t}[$, capacity, flow_func $])$ | Compute the value and the node partition of a minimum (s, <br> t)-cut. |
| minimum_cut_value(G, s, $[$ [, capacity, flow_func $])$ | Compute the value of a minimum (s, t)-cut. |

maximum_flow
maximum_flow ( $G, s, t$, capacity='capacity', flow_func=None, **kwargs)
Find a maximum single-commodity flow.

[^76]
## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- s (node) - Source node for the flow.
- $\mathbf{t}$ (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- kwargs (Any other keyword parameter is passed to the function that) - computes the maximum flow.


## Returns

- flow_value (integer, float) - Value of the maximum flow, i.e., net outflow from the source.
- flow_dict (dict) - A dictionary containing the value of the flow that went through each edge.


## Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

```
maximum_flow_value(), minimum_cut(), minimum_cut_value(), edmonds_karp(),
preflow_push(),shortest_augmenting_path()
```


## Notes

The function used in the flow_func paramter has to return a residual network that follows NetworkX conventions:
The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$ or zero otherwise. If the capacity is infinite, $R[u][v][' c a p a c i t y ']$ will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge $(u, v)$ in $R, R[u][v][$ 'flow'] represents the flow function of ( $u, v$ ) and satisfies R[u][v]['flow'] == -R[v][u]['flow'].
The flow value, defined as the total flow into $t$, the sink, is stored in R.graph ['flow_value']. Reachability to $t$ using only edges ( $u, v$ ) such that $R[u][v][' f l o w ']<R[u][v][' c a p a c i t y ']$ induces a minimum s-t cut.

Specific algorithms may store extra data in $R$.

The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

## Examples

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
```

maximum_flow returns both the value of the maximum flow and a dictionary with all flows.

```
>>> flow_value, flow_dict = nx.maximum_flow(G, 'x', 'y')
>>> flow_value
3.0
>>> print(flow_dict['x']['b'])
1.0
```

You can also use alternative algorithms for computing the maximum flow by using the flow_func parameter.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> flow_value == nx.maximum_flow(G, 'x', 'y',
... flow_func=shortest_augmenting_path)[0]
True
```


## maximum_flow_value

maximum_flow_value ( $G, s, t$, capacity='capacity', flow_func=None, **kwargs)
Find the value of maximum single-commodity flow.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push ()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- kwargs (Any other keyword parameter is passed to the function that) - computes the maximum flow.

Returns flow_value - Value of the maximum flow, i.e., net outflow from the source.
Return type integer, float

## Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

maximum_flow(), minimum_cut(), minimum_cut_value(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

## Notes

The function used in the flow_func paramter has to return a residual network that follows NetworkX conventions:
The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][' c a p a c i t y ']$ is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge $(u, v)$ in $R, R[u][v][' f l o w '] ~ r e p r e s e n t s ~ t h e ~ f l o w ~ f u n c t i o n ~ o f ~(u, v) ~ a n d ~ s a t i s f i e s ~ R[u][v][' f l o w '] ~$ $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph ['flow_value']. Reachability to $t$ using only edges ( $u, v$ ) such that $R[u][v][' f l o w ']<R[u][v][' c a p a c i t y '] ~ i n d u c e s ~ a ~$ minimum s-t cut.

Specific algorithms may store extra data in R.
The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

## Examples

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
```

maximum_flow_value computes only the value of the maximum flow:

```
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
```

You can also use alternative algorithms for computing the maximum flow by using the flow_func parameter.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> flow_value == nx.maximum_flow_value(G, 'x', 'y',
... flow_func=shortest_augmenting_path)
True
```


## minimum_cut

minimum_cut ( $G, s, t$, capacity='capacity', flow_func=None, **kwargs)
Compute the value and the node partition of a minimum ( $\mathrm{s}, \mathrm{t}$ )-cut.
Use the max-flow min-cut theorem, i.e., the capacity of a minimum capacity cut is equal to the flow value of a maximum flow.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push ()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- kwargs (Any other keyword parameter is passed to the function that) - computes the maximum flow.


## Returns

- cut_value (integer, float) - Value of the minimum cut.
- partition (pair of node sets) - A partitioning of the nodes that defines a minimum cut.

Raises NetworkXUnbounded - If the graph has a path of infinite capacity, all cuts have infinite capacity and the function raises a NetworkXError.

## See also:

```
maximum_flow(), maximum_flow_value(), minimum_cut_value(), edmonds_karp(),
preflow_push(), shortest_augmenting_path()
```


## Notes

The function used in the flow_func paramter has to return a residual network that follows NetworkX conventions:
The residual network $R$ from an input graph $G$ has the same nodes as $G . R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff ( $u, v$ ) is not a self-loop, and at least one of ( $u, v$ ) and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge ( $u, v$ ) in $R, R[u][v][' f l o w ']$ represents the flow function of ( $u, v$ ) and satisfies $R[u][v][$ flow'] == -R[v][u]['flow'].

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph ['flow_value']. Reacha-
 minimum s-t cut.

Specific algorithms may store extra data in R .
The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

## Examples

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity = 3.0)
>>> G.add_edge('x','b', capacity = 1.0)
>>> G.add_edge('a','c', capacity = 3.0)
>>> G.add_edge('b','c', capacity = 5.0)
>>> G.add_edge('b','d', capacity = 4.0)
>>> G.add_edge('d','e', capacity = 2.0)
>>> G.add_edge('c','y', capacity = 2.0)
>>> G.add_edge('e','y', capacity = 3.0)
```

minimum_cut computes both the value of the minimum cut and the node partition:

```
>>> cut_value, partition = nx.minimum_cut(G, 'x', 'y')
>>> reachable, non_reachable = partition
```

'partition' here is a tuple with the two sets of nodes that define the minimum cut. You can compute the cut set of edges that induce the minimum cut as follows:

```
>>> cutset = set()
>>> for u, nbrs in ((n, G[n]) for n in reachable):
... cutset.update((u, v) for v in nbrs if v in non_reachable)
>>> print(sorted(cutset))
[('c', 'y'), ('x', 'b')]
>>> cut_value == sum(G.edge[u][v]['capacity'] for (u, v) in cutset)
True
```

You can also use alternative algorithms for computing the minimum cut by using the flow_func parameter.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> cut_value == nx.minimum_cut(G, 'x', 'y',
    flow_func=shortest_augmenting_path) [0]
True
```

minimum_cut_value
minimum_cut_value ( $G, s, t$, capacity='capacity', flow_func=None, **kwargs)
Compute the value of a minimum ( $\mathrm{s}, \mathrm{t}$ )-cut.
Use the max-flow min-cut theorem, i.e., the capacity of a minimum capacity cut is equal to the flow value of a maximum flow.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- flow_func (function) - A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- kwargs (Any other keyword parameter is passed to the function that) - computes the maximum flow.

Returns cut_value - Value of the minimum cut.
Return type integer, float
Raises NetworkXUnbounded - If the graph has a path of infinite capacity, all cuts have infinite capacity and the function raises a NetworkXError.

## See also:

```
maximum_flow(), maximum_flow_value(), minimum_cut(), edmonds_karp(),
preflow_push(),shortest_augmenting_path()
```


## Notes

The function used in the flow_func paramter has to return a residual network that follows NetworkX conventions:
The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][' c a p a c i t y ']$ is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph ['inf']. For each edge $(u, v)$ in $R, R[u][v]\left[' f l o w^{\prime}\right]$ represents the flow function of $(u, v)$ and satisfies $R[u][v][' f l o w ']$ $==-R[v][u][' f l o w ']$.
The flow value, defined as the total flow into $t$, the sink, is stored in R.graph ['flow_value']. Reachability to $t$ using only edges (u,v) such that $R[u][v][' f l o w ']<R[u][v][' c a p a c i t y '] i n d u c e s a ~$ minimum s-t cut.

Specific algorithms may store extra data in $R$.
The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

## Examples

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity = 3.0)
>>> G.add_edge('x','b', capacity = 1.0)
>>> G.add_edge('a','c', capacity = 3.0)
>>> G.add_edge('b','c', capacity = 5.0)
>>> G.add_edge('b','d', capacity = 4.0)
>>> G.add_edge('d','e', capacity = 2.0)
>>> G.add_edge('c','y', capacity = 2.0)
>>> G.add_edge('e','y', capacity = 3.0)
```

minimum_cut_value computes only the value of the minimum cut:

```
>>> cut_value = nx.minimum_cut_value(G, 'x', 'y')
>>> cut_value
3.0
```

You can also use alternative algorithms for computing the minimum cut by using the flow_func parameter.

```
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> cut_value == nx.minimum_cut_value(G, 'x', 'y',
Mrue
```


### 4.27.2 Edmonds-Karp

| edmonds_karp(G, s, $\mathrm{t}[$, capacity, residual, ...]) | Find a maximum single-commodity flow using the <br> Edmonds-Karp algorithm. |
| :--- | :--- |

## edmonds_karp

edmonds_karp ( $G, s, t$, capacity='capacity', residual=None, value_only=False, cutoff=None)
Find a maximum single-commodity flow using the Edmonds-Karp algorithm.
This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of $O\left(n m^{\wedge} 2\right)$ for $n$ nodes and $m$ edges.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- s (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- residual (NetworkX graph) - Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- value_only (bool) - If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- cutoff (integer, float) - If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

Returns $\mathbf{R}$ - Residual network after computing the maximum flow.
Return type NetworkX DiGraph

## Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

```
maximum_flow(),minimum_cut(), preflow_push(), shortest_augmenting_path()
```


## Notes

The residual network $R$ from an input graph $G$ has the same nodes as $G . R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph ['inf']. For each edge $(u, v)$ in $R, R[u][v]\left[' f l o w^{\prime}\right]$ represents the flow function of ( $u, v$ ) and satisfies $R[u][v][' f l o w ']$ $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to $t$ using only edges (u,v) such that R[u][v]['flow'] < R[u][v]['capacity'] induces a minimum s-t cut.

## Examples

```
>>> import networkx as nx
>>> from networkx.algorithms.flow import edmonds_karp
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
```

```
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = edmonds_karp(G, 'x', 'Y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'Y')
>>> flow_value
3.0
>>> flow_value == R.graph['flow_value']
True
```


### 4.27.3 Shortest Augmenting Path

| shortest_augmenting_path $(\mathrm{G}, \mathrm{s}, \mathrm{t}[, \ldots])$ | Find a maximum single-commodity flow using the shortest <br> augmenting path algorithm. |
| :--- | :--- |

## shortest_augmenting_path

shortest_augmenting_path $(G, \quad s, \quad t, \quad$ capacity='capacity', residual=None, value_only=False, two_phase $=$ False, cutoff $=$ None )
Find a maximum single-commodity flow using the shortest augmenting path algorithm.
This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.
This algorithm has a running time of $O\left(n^{\wedge} 2 m\right)$ for $n$ nodes and $m$ edges.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- residual (NetworkX graph) - Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- value_only (bool) - If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- two_phase (bool) - If True, a two-phase variant is used. The two-phase variant improves the running time on unit-capacity networks from $O(n m)$ to $O\left(\min \left(n^{\wedge}\{2 / 3\}, m^{\wedge}\{1 / 2\}\right)\right.$ m). Default value: False.
- cutoff (integer, float) - If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.
Returns $\mathbf{R}$ - Residual network after computing the maximum flow.


## Return type NetworkX DiGraph

## Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

maximum_flow(), minimum_cut (), edmonds_karp(), preflow_push()

## Notes

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$ or zero otherwise. If the capacity is infinite, $R[u][v][' c a p a c i t y '] ~ w i l l ~ h a v e ~ a ~ h i g h ~ a r b i t r a r y ~ f i n i t e ~ v a l u e ~$ that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge $(u, v)$ in $R, R[u][v]\left[' f l o w^{\prime}\right]$ represents the flow function of $(u, v)$ and satisfies $R[u][v]\left[' f l o w^{\prime}\right]$ $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to $t$ using only edges (u,v) such that R[u][v]['flow'] < R[u][v]['capacity'] induces a minimum s-t cut.

## Examples

```
>>> import networkx as nx
>>> from networkx.algorithms.flow import shortest_augmenting_path
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = shortest_augmenting_path(G, 'x', 'Y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'Y')
>>> flow_value
3.0
>>> flow_value == R.graph['flow_value']
True
```


### 4.27.4 Preflow-Push

| preflow_push(G, s, t[, capacity, residual, ...]) | Find a maximum single-commodity flow using the highest- <br> label preflow-push algorithm. |
| :--- | :--- |

## preflow_push

preflow_push ( $G, s, t$, capacity='capacity', residual=None, global_relabel_freq=1, value_only=False)
Find a maximum single-commodity flow using the highest-label preflow-push algorithm.
This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of $O\left(n^{\wedge} 2 \operatorname{sqrt}\{m\}\right)$ for $n$ nodes and $m$ edges.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- residual (NetworkX graph) - Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- global_relabel_freq (integer, float) - Relative frequency of applying the global relabeling heuristic to speed up the algorithm. If it is None, the heuristic is disabled. Default value: 1.
- value_only (bool) - If False, compute a maximum flow; otherwise, compute a maximum preflow which is enough for computing the maximum flow value. Default value: False.

Returns $\mathbf{R}$ - Residual network after computing the maximum flow.
Return type NetworkX DiGraph

## Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

maximum_flow(), minimum_cut(), edmonds_karp(), shortest_augmenting_path()

## Notes

The residual network $R$ from an input graph $G$ has the same nodes as $G . R$ is a DiGraph that contains a pair of edges $(u, v)$ and ( $v, u)$ iff ( $u, v$ ) is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$. For each node $u$ in R, R. node [u] ['excess'] represents the difference between flow into $u$ and flow out of $u$.

For each edge ( $u, v$ ) in $R, R[u][v][' c a p a c i t y '] ~ i s ~ e q u a l ~ t o ~ t h e ~ c a p a c i t y ~ o f ~(~ u, v) ~ i n ~ G ~ i f ~ i t ~ e x i s t s ~ i n ~ G ~$
 that does not affect the solution of the problem. This value is stored in R.graph ['inf']. For each edge ( $u, v$ ) in $R, R[u][v][' f l o w ']$ represents the flow function of ( $u, v$ ) and satisfies $R[u][v][$ flow'] $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph ['flow_value']. Reachability to $t$ using only edges (u,v) such that $R[u][v][' f l o w ']<R[u][v][' c a p a c i t y '] i n d u c e s ~ a ~$ minimum s-t cut.

## Examples

```
>>> import networkx as nx
>>> from networkx.algorithms.flow import preflow_push
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = preflow_push(G, 'x', 'Y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value == R.graph['flow_value']
True
>>> # preflow_push also stores the maximum flow value
>>> # in the excess attribute of the sink node t
>>> flow_value == R.node['y']['excess']
True
>>> # For some problems, you might only want to compute a
>>> # maximum preflow.
>>> R = preflow_push(G, 'x', 'y', value_only=True)
>>> flow_value == R.graph['flow_value']
True
>>> flow_value == R.node['y']['excess']
True
```


### 4.27.5 Dinitz

Find a maximum single-commodity flow using Dinitz' algorithm.

## dinitz

dinitz ( $G, s, t$, capacity='capacity', residual=None, value_only=False, cutoff=None)
Find a maximum single-commodity flow using Dinitz' algorithm.
This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of $\mathrm{O}\left(\mathrm{n}^{\wedge} 2 \mathrm{~m}\right)$ for n nodes and m edges _[1].

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- $\mathbf{t}$ (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- residual (NetworkX graph) - Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- value_only (bool) - If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- cutoff (integer, float) - If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

Returns $\mathbf{R}$ - Residual network after computing the maximum flow.
Return type NetworkX DiGraph
Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

maximum_flow(), minimum_cut(), preflow_push(), shortest_augmenting_path()

## Notes

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$ or zero otherwise. If the capacity is infinite, $R[u][v][$ 'capacity'] will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in R.graph ['inf']. For each edge $(u, v)$ in $R, R[u][v][' f l o w ']$ represents the flow function of ( $u, v$ ) and satisfies R[u][v]['flow'] == -R[v][u]['flow'].

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to $t$ using only edges ( $u, v$ ) such that $R[u][v]\left[f^{\prime} f l o w '\right]$ < $R[u][v][$ 'capacity'] induces a minimum s-t cut.

## Examples

```
>>> import networkx as nx
>>> from networkx.algorithms.flow import dinitz
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = dinitz(G, 'x', 'Y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'Y')
>>> flow_value
3.0
>>> flow_value == R.graph['flow_value']
True
```


## References

### 4.27.6 Boykov-Kolmogorov

| boykov_kolmogorov(G, s, t[, capacity, ...]) | Find a maximum single-commodity flow using Boykov- <br> Kolmogorov algorithm. |
| :--- | :--- |

## boykov_kolmogorov

boykov_kolmogorov ( $G, s, t$, capacity='capacity', residual=None, value_only=False, cutoff=None)
Find a maximum single-commodity flow using Boykov-Kolmogorov algorithm.
This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has worse case complexity $O\left(n^{\wedge} 2 \mathrm{~m}|\mathrm{C}|\right)$ for n nodes, m edges, and $|\mathrm{C}|$ the cost of the minimum cut ${ }^{1}$. This implementation uses the marking heuristic defined in ${ }^{2}$ which improves its running time in many practical problems.

## Parameters

- G (NetworkX graph) - Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- $\mathbf{s}$ (node) - Source node for the flow.
- t (node) - Sink node for the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- residual (NetworkX graph) - Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.

[^77]- value_only (bool) - If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- cutoff (integer, float) - If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

Returns $\mathbf{R}$ - Residual network after computing the maximum flow.
Return type NetworkX DiGraph
Raises

- NetworkXError - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- NetworkXUnbounded - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.


## See also:

maximum_flow(), minimum_cut(), preflow_push(), shortest_augmenting_path()

## Notes

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge $(u, v)$ in $R, R[u][v][' f l o w ']$ represents the flow function of ( $u, v$ ) and satisfies $R[u][v]\left[' f l o w^{\prime}\right]$ $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to $t$ using only edges (u,v) such that R[u][v]['flow'] < $R[u][v][' c a p a c i t y '] ~ i n d u c e s ~ a ~ m i n i m u m ~ s-t ~ c u t . ~$

## Examples

```
>>> import networkx as nx
>>> from networkx.algorithms.flow import boykov_kolmogorov
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = boykov_kolmogorov(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'Y')
>>> flow_value
```

```
3.0
>>> flow_value == R.graph['flow_value']
True
```

A nice feature of the Boykov-Kolmogorov algorithm is that a partition of the nodes that defines a minimum cut can be easily computed based on the search trees used during the algorithm. These trees are stored in the graph attribute trees of the residual network.

```
>>> source_tree, target_tree = R.graph['trees']
>>> partition = (set(source_tree), set(G) - set(source_tree))
```

Or equivalently:

```
>>> partition = (set(G) - set(target_tree), set(target_tree))
```


## References

### 4.27.7 Utils

build_residual_network(G, capacity) Build a residual network and initialize a zero flow.
build_residual_network
build_residual_network (G, capacity)
Build a residual network and initialize a zero flow.
The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge ( $u, v$ ) in $R, R[u][v][$ 'capacity'] is equal to the capacity of ( $u, v$ ) in $G$ if it exists in $G$
 that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge $(u, v)$ in $R, R[u][v][' f l o w ']$ represents the flow function of ( $u, v$ ) and satisfies $R[u][v][' f l o w ']$ $==-R[v][u][' f l o w ']$.

The flow value, defined as the total flow into $t$, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to $t$ using only edges (u,v) such that R[u][v]['flow'] < R[u][v]['capacity'] induces a minimum s-t cut.

### 4.27.8 Network Simplex

| network_simplex(G[, demand, capacity, weight $])$ | Find a minimum cost flow satisfying all demands in di- <br> graph G. |
| :--- | :--- |
| min_cost_flow_cost(G[, demand, capacity, weight $])$ | Find the cost of a minimum cost flow satisfying all de- <br> mands in digraph G. |
| min_cost_flow(G[, demand, capacity, weight $])$ | Return a minimum cost flow satisfying all demands in di- <br> graph G. |
| cost_of_flow(G, flowDict[, weight $])$ | Compute the cost of the flow given by flowDict on graph <br>  <br> G. |
| max_flow_min_cost(G, s, $[$ [, capacity, weight $])$ | Return a maximum (s, t)-flow of minimum cost. |

## network_simplex

network_simplex ( $G$, demand='demand', capacity='capacity', weight='weight')
Find a minimum cost flow satisfying all demands in digraph G.
This is a primal network simplex algorithm that uses the leaving arc rule to prevent cycling.
$G$ is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.

## Parameters

- G (NetworkX graph) - DiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) - Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: 'demand'.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- weight (string) - Edges of the graph $G$ are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.


## Returns

- flowCost (integer, float) - Cost of a minimum cost flow satisfying all demands.
- flowDict (dictionary) - Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge ( $u, v$ ).


## Raises

- NetworkXError - This exception is raised if the input graph is not directed, not connected or is a multigraph.
- NetworkXUnfeasible - This exception is raised in the following situations:
- The sum of the demands is not zero. Then, there is no flow satisfying all demands.
- There is no flow satisfying all demand.
- NetworkXUnbounded - This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.


## Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

## See also:

cost_of_flow(), max_flow_min_cost(), min_cost_flow(),min_cost_flow_cost()

## Examples

A simple example of a min cost flow problem.

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand=-5)
>>> G.add_node('d', demand=5)
>>> G.add_edge('a', 'b', weight=3, capacity=4)
>>> G.add_edge('a', 'c', weight=6, capacity=10)
>>> G.add_edge('b', 'd', weight=1, capacity=9)
>>> G.add_edge('c', 'd', weight=2, capacity=5)
>>> flowCost, flowDict = nx.network_simplex(G)
>>> flowCost
24
>>> flowDict
{'a': {'c': 1, 'b': 4}, 'c': {'d': 1}, 'b': {'d': 4}, 'd': {}}
```

The mincost flow algorithm can also be used to solve shortest path problems. To find the shortest path between two nodes $u$ and $v$, give all edges an infinite capacity, give node $u$ a demand of -1 and node $v$ a demand a 1 . Then run the network simplex. The value of a min cost flow will be the distance between $u$ and $v$ and edges carrying positive flow will indicate the path.

```
>>> G=nx.DiGraph()
>>> G.add_weighted_edges_from([('s', 'u' ,10), ('s' ,'x' ,5),
... ('u', 'v' ,1), ('u' ,'x' ,2),
... ('v', 'y' ,1), ('x' ,'u' ,3),
... ('x', 'v' ,5), ('x','y' ,2),
... ('y', 's' ,7), ('y' ,'v' , 6)])
>>> G.add_node('s', demand = -1)
>>> G.add_node('v', demand = 1)
>>> flowCost, flowDict = nx.network_simplex(G)
>>> flowCost == nx.shortest_path_length(G, 's', 'v', weight='weight')
True
>>> sorted([(u, v) for u in flowDict for v in flowDict[u] if flowDict[u][v] > 0])
[('s', 'x'), ('u', 'v'), ('x', 'u')]
>>> nx.shortest_path(G, 's', 'v', weight = 'weight')
['s', 'x', 'u', 'v']
```

It is possible to change the name of the attributes used for the algorithm.

```
>>> G = nx.DiGraph()
>>> G.add_node('p', spam=-4)
>>> G.add_node('q', spam=2)
>>> G.add_node('a', spam=-2)
>>> G.add_node('d', spam=-1)
>>> G.add_node('t', spam=2)
>>> G.add_node('w', spam=3)
>>> G.add_edge('p', 'q', cost=7, vacancies=5)
>>> G.add_edge('p', 'a', cost=1, vacancies=4)
>>> G.add_edge('q', 'd', cost=2, vacancies=3)
>>> G.add_edge('t', 'q', cost=1, vacancies=2)
>>> G.add_edge('a', 't', cost=2, vacancies=4)
>>> G.add_edge('d', 'w', cost=3, vacancies=4)
>>> G.add_edge('t', 'w', cost=4, vacancies=1)
>>> flowCost, flowDict = nx.network_simplex(G, demand='spam',
... capacity='vacancies',
... weight='cost')
```

```
>>> flowCost
37
>>> flowDict
{'a': {'t': 4}, 'd': {'w': 2}, 'q': {'d': 1}, 'p': {'q': 2, 'a': 2}, 't': {'q': 1,
-> 'w': 1}, 'w': {}}
```


## References

```
min_cost_flow_cost
```

min_cost_flow_cost ( $G$, demand='demand', capacity='capacity', weight='weight')

Find the cost of a minimum cost flow satisfying all demands in digraph G.
$G$ is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.

## Parameters

- $\mathbf{G}$ (NetworkX graph) - DiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) - Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: 'demand'.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- weight (string) - Edges of the graph G are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.
Returns flowCost - Cost of a minimum cost flow satisfying all demands.
Return type integer, float


## Raises

- NetworkXError - This exception is raised if the input graph is not directed or not connected.
- NetworkXUnfeasible - This exception is raised in the following situations:
- The sum of the demands is not zero. Then, there is no flow satisfying all demands.
- There is no flow satisfying all demand.
- NetworkXUnbounded - This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.


## See also:

```
cost_of_flow(),max_flow_min_cost(),min_cost_flow(),network_simplex()
```


## Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

## Examples

A simple example of a min cost flow problem.

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add_edge('a', 'b', weight = 3, capacity = 4)
>>> G.add_edge('a', 'c', weight = 6, capacity = 10)
>>> G.add_edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd', weight = 2, capacity = 5)
>>> flowCost = nx.min_cost_flow_cost(G)
>>> flowCost
24
```

min_cost_flow
min_cost_flow (G, demand='demand', capacity='capacity', weight='weight')
Return a minimum cost flow satisfying all demands in digraph $G$.
G is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.

## Parameters

- G (NetworkX graph) - DiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) - Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: 'demand'.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- weight (string) - Edges of the graph $G$ are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.

Returns flowDict - Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge ( $u, v$ ).

Return type dictionary

## Raises

- NetworkXError - This exception is raised if the input graph is not directed or not connected.
- NetworkXUnfeasible - This exception is raised in the following situations:
- The sum of the demands is not zero. Then, there is no flow satisfying all demands.
- There is no flow satisfying all demand.
- NetworkXUnbounded - This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.


## See also:

```
cost_of_flow(),max_flow_min_cost(),min_cost_flow_cost(), network_simplex()
```


## Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

## Examples

A simple example of a min cost flow problem.

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add_edge('a', 'b', weight = 3, capacity = 4)
>>> G.add_edge('a', 'c', weight = 6, capacity = 10)
>>> G.add_edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd', weight = 2, capacity = 5)
>>> flowDict = nx.min_cost_flow(G)
```

```
cost_of_flow
```

cost_of_flow (G, flowDict, weight='weight')

Compute the cost of the flow given by flowDict on graph G.
Note that this function does not check for the validity of the flow flowDict. This function will fail if the graph G and the flow don't have the same edge set.

## Parameters

- G (NetworkX graph) - DiGraph on which a minimum cost flow satisfying all demands is to be found.
- weight (string) - Edges of the graph $G$ are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.
- flowDict (dictionary) - Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge $(u, v)$.

Returns cost - The total cost of the flow. This is given by the sum over all edges of the product of the edge's flow and the edge's weight.
Return type Integer, float

## See also:

max_flow_min_cost(), min_cost_flow(), min_cost_flow_cost(), network_simplex()

## Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

```
max_flow_min_cost
```

max_flow_min_cost $(G, s, t$, capacity='capacity', weight='weight')
Return a maximum ( $\mathrm{s}, \mathrm{t}$ )-flow of minimum cost.
$G$ is a digraph with edge costs and capacities. There is a source node $s$ and a sink node $t$. This function finds a maximum flow from s to $t$ whose total cost is minimized.

## Parameters

- G (NetworkX graph) - DiGraph on which a minimum cost flow satisfying all demands is to be found.
- s (node label) - Source of the flow.
- $\mathbf{t}$ (node label) - Destination of the flow.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- weight (string) - Edges of the graph $G$ are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.

Returns flowDict - Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge ( $u$, v).

Return type dictionary

## Raises

- NetworkXError - This exception is raised if the input graph is not directed or not connected.
- NetworkXUnbounded - This exception is raised if there is an infinite capacity path from $s$ to $t$ in G. In this case there is no maximum flow. This exception is also raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow is unbounded below.


## See also:

```
cost_of_flow(),min_cost_flow(),min_cost_flow_cost(), network_simplex()
```


## Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

## Examples

```
>>> G = nx.DiGraph()
>>> G.add_edges_from([(1, 2, {'capacity': 12, 'weight': 4}),
... (1, 3, {'capacity': 20, 'weight': 6}),
... (2, 3, {'capacity': 6, 'weight': -3}),
... (2, 6, {'capacity': 14, 'weight': 1}),
... (3, 4, {'weight': 9}),
... (3, 5, {'capacity': 10, 'weight': 5}),
... (4, 2, {'capacity': 19, 'weight': 13}),
... (4, 5, {'capacity': 4, 'weight': 0}),
... (5, 7, {'capacity': 28, 'weight': 2}),
... (6, 5, {'capacity': 11, 'weight': 1}),
... (6, 7, {'weight': 8}),
... (7, 4, {'capacity': 6, 'weight': 6})])
>>> mincostFlow = nx.max_flow_min_cost(G, 1, 7)
>>> mincost = nx.cost_of_flow(G, mincostFlow)
>>> mincost
373
>>> from networkx.algorithms.flow import maximum_flow
>>> maxFlow = maximum_flow(G, 1, 7) [1]
>>> nx.cost_of_flow(G, maxFlow) >= mincost
True
>>> mincostFlowValue = (sum((mincostFlow[u][7] for u in G.predecessors(7)))
... - sum((mincostFlow[7][v] for v in G.successors(7))))
>>> mincostFlowValue == nx.maximum_flow_value(G, 1, 7)
True
```


### 4.27.9 Capacity Scaling Minimum Cost Flow

```
    capacity_scaling(G[, demand, capacity, ...])
```

Find a minimum cost flow satisfying all demands in digraph G.

## capacity_scaling

capacity_scaling (G, demand='demand', capacity='capacity', weight='weight', heap=<class 'networkx.utils.heaps.BinaryHeap'>)
Find a minimum cost flow satisfying all demands in digraph G.
This is a capacity scaling successive shortest augmenting path algorithm.
$G$ is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph $G$ satisfies all demand if the net flow into each node is equal to the demand of that node.

## Parameters

- G (NetworkX graph) - DiGraph or MultiDiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) - Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: 'demand'.
- capacity (string) - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: 'capacity'.
- weight (string) - Edges of the graph $G$ are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0 . Default value: 'weight'.
- heap (class) - Type of heap to be used in the algorithm. It should be a subclass of MinHeap or implement a compatible interface.

If a stock heap implementation is to be used, BinaryHeap is recommeded over PairingHeap for Python implementations without optimized attribute accesses (e.g., CPython) despite a slower asymptotic running time. For Python implementations with optimized attribute accesses (e.g., PyPy), PairingHeap provides better performance. Default value: BinaryHeap.

## Returns

- flowCost (integer) - Cost of a minimum cost flow satisfying all demands.
- flowDict (dictionary) - If G is a digraph, a dict-of-dicts keyed by nodes such that flow$\operatorname{Dict}[u][v]$ is the flow on edge ( $u$, v). If $G$ is a MultiDiGraph, a dict-of-dicts-of-dicts keyed by nodes so that flowDict[ $u][v][$ key $]$ is the flow on edge ( $u$, $v$, key).


## Raises

- NetworkXError - This exception is raised if the input graph is not directed, not connected.
- NetworkXUnfeasible - This exception is raised in the following situations:
- The sum of the demands is not zero. Then, there is no flow satisfying all demands.
- There is no flow satisfying all demand.
- NetworkXUnbounded - This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.


## Notes

This algorithm does not work if edge weights are floating-point numbers.

## See also:

network_simplex()

## Examples

A simple example of a min cost flow problem.

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>>G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add__edge('a', 'b', weight = 3, capacity = 4)
>>> G.add__edge('a', 'c', weight = 6, capacity = 10)
>>> G.add__edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd'', weight = 2, capacity = 5)
>>> flowCost, flowDict = nx.capacity_scaling(G)
>>> flowCost
24
>>> flowDict
{'a': {'c': 1, 'b': 4}, 'c': {'d': 1}, 'b': {'d': 4}, 'd': {}}
```

It is possible to change the name of the attributes used for the algorithm.

```
>>> G = nx.DiGraph()
>>> G.add_node('p', spam = -4)
>>> G.add_node('q', spam = 2)
>>> G.add_node('a', spam = -2)
>>> G.add_node('d', spam = -1)
>>> G.add_node('t', spam = 2)
>>> G.add_node('w', spam = 3)
>>> G.add_edge('p', 'q', cost = 7, vacancies = 5)
>>> G.add_edge('p', 'a', cost = 1, vacancies = 4)
>>> G.add_edge('q', 'd', cost = 2, vacancies = 3)
>>> G.add_edge('t', 'q', cost = 1, vacancies = 2)
>>> G.add_edge('a', 't', cost = 2, vacancies = 4)
>>> G.add_edge('d', 'w', cost = 3, vacancies = 4)
>>> G.add_edge('t', 'w', cost = 4, vacancies = 1)
>>> flowCost, flowDict = nx.capacity_scaling(G, demand = 'spam',
... capacity = 'vacancies',
... weight = 'cost')
>>> flowCost
37
>>> flowDict
{'a': {'t': 4}, 'd': {'w': 2}, 'q': {'d': 1}, 'p': {'q': 2, 'a': 2}, 't': {'q': 1,
\hookrightarrow 'w': 1}, 'w': {}}
```


### 4.28 Graphical degree sequence

Test sequences for graphiness.

| is_graphical(sequence[, method]) | Returns True if sequence is a valid degree sequence. |
| :--- | :--- |
| is_digraphical(in_sequence, out_sequence) | Returns True if some directed graph can realize the in- and <br> out-degree sequences. |
| is_multigraphical(sequence) | Returns True if some multigraph can realize the sequence. |
| is_pseudographical(sequence) | Returns True if some pseudograph can realize the se- <br> quence. |
| is_valid_degree_sequence_havel_hakimi(...) | Returns True if deg_sequence can be realized by a simple <br> graph. |
| is_valid_degree_sequence_erdos_gallai(...) | Returns True if deg_sequence can be realized by a simple <br> graph. |

### 4.28.1 is_graphical

is_graphical (sequence, method='eg')
Returns True if sequence is a valid degree sequence.
A degree sequence is valid if some graph can realize it.

## Parameters

- sequence (list or iterable container) - A sequence of integer node degrees
- method ("eg"| "hh") - The method used to validate the degree sequence. "eg" corresponds to the Erdős-Gallai algorithm, and "hh" to the Havel-Hakimi algorithm.
Returns valid - True if the sequence is a valid degree sequence and False if not.
Return type bool


## Examples

```
>>> G = nx.path_graph(4)
>>>}\mathrm{ sequence = (d for n, d in G.degree())
>>> nx.is__valid_degree_sequence(sequence)
True
```


## References

Erdốs-Gallai [EG1960], [choudum1986]
Havel-Hakimi [havel1955], [hakimi1962], [CL1996]

### 4.28.2 is_digraphical

is_digraphical (in_sequence, out_sequence)
Returns True if some directed graph can realize the in- and out-degree sequences.

## Parameters

- in_sequence (list or iterable container) - A sequence of integer node in-degrees
- out_sequence (list or iterable container) - A sequence of integer node out-degrees

Returns valid - True if in and out-sequences are digraphic False if not.
Return type bool

## Notes

This algorithm is from Kleitman and Wang ${ }^{1}$. The worst case runtime is $\mathrm{O}(\mathrm{s} * \log \mathrm{n})$ where s and n are the sum and length of the sequences respectively.

[^78]
## References

### 4.28.3 is_multigraphical

## is_multigraphical (sequence)

Returns True if some multigraph can realize the sequence.
Parameters deg_sequence (list) - A list of integers
Returns valid - True if deg_sequence is a multigraphic degree sequence and False if not.
Return type bool

## Notes

The worst-case run time is $\mathrm{O}(\mathrm{n})$ where n is the length of the sequence.

## References

### 4.28.4 is_pseudographical

## is_pseudographical (sequence)

Returns True if some pseudograph can realize the sequence.
Every nonnegative integer sequence with an even sum is pseudographical (see ${ }^{1}$ ).
Parameters sequence (list or iterable container) - A sequence of integer node degrees
Returns valid - True if the sequence is a pseudographic degree sequence and False if not.
Return type bool

## Notes

The worst-case run time is $\mathrm{O}(\mathrm{n})$ where n is the length of the sequence.

## References

### 4.28.5 is_valid_degree_sequence_havel_hakimi

```
is_valid_degree_sequence_havel_hakimi (deg_sequence)
```

Returns True if deg_sequence can be realized by a simple graph.
The validation proceeds using the Havel-Hakimi theorem. Worst-case run time is: $\mathrm{O}(\mathrm{s})$ where s is the sum of the sequence.

Parameters deg_sequence (list) - A list of integers where each element specifies the degree of a node in a graph.
Returns valid - True if deg_sequence is graphical and False if not.
Return type bool

[^79]
## Notes

The ZZ condition says that for the sequence d if

$$
|d|>=\frac{(\max (d)+\min (d)+1)^{2}}{4 * \min (d)}
$$

then $d$ is graphical. This was shown in Theorem 6 in ${ }^{1}$.

## References

[havel1955], [hakimi1962], [CL1996]

### 4.28.6 is_valid_degree_sequence_erdos_gallai

is_valid_degree_sequence_erdos_gallai (deg_sequence)
Returns True if deg_sequence can be realized by a simple graph.
The validation is done using the Erdós-Gallai theorem [EG1960].
Parameters deg_sequence (list) - A list of integers
Returns valid - True if deg_sequence is graphical and False if not.
Return type bool

## Notes

This implementation uses an equivalent form of the Erdős-Gallai criterion. Worst-case run time is: O(n) where n is the length of the sequence.

Specifically, a sequence d is graphical if and only if the sum of the sequence is even and for all strong indices k in the sequence,

$$
\sum_{i=1}^{k} d_{i} \leq k(k-1)+\sum_{j=k+1}^{n} \min \left(d_{i}, k\right)=k(n-1)-\left(k \sum_{j=0}^{k-1} n_{j}-\sum_{j=0}^{k-1} j n_{j}\right)
$$

A strong index $k$ is any index where $d_{-} k>=k$ and the value $n_{\_} j$ is the number of occurrences of $j$ in $d$. The maximal strong index is called the Durfee index.
This particular rearrangement comes from the proof of Theorem 3 in ${ }^{2}$.
The ZZ condition says that for the sequence d if

$$
|d|>=\frac{(\max (d)+\min (d)+1)^{2}}{4 * \min (d)}
$$

then d is graphical. This was shown in Theorem 6 in ${ }^{2}$.

[^80]
## References

[EG1960], [choudum1986]

### 4.29 Hierarchy

Flow Hierarchy.

### 4.29.1 flow_hierarchy

flow_hierarchy (G, weight=None)
Returns the flow hierarchy of a directed network.
Flow hierarchy is defined as the fraction of edges not participating in cycles in a directed graph ${ }^{1}$.

## Parameters

- G (DiGraph or MultiDiGraph) - A directed graph
- weight (key,optional (default=None)) - Attribute to use for node weights. If None the weight defaults to 1 .

Returns h-Flow heirarchy value
Return type float

## Notes

The algorithm described in ${ }^{1}$ computes the flow hierarchy through exponentiation of the adjacency matrix. This function implements an alternative approach that finds strongly connected components. An edge is in a cycle if and only if it is in a strongly connected component, which can be found in $O(m)$ time using Tarjan's algorithm.

## References

### 4.30 Hybrid

Provides functions for finding and testing for locally ( $k, l$ ) -connected graphs.

| kl_connected_subgraph(G, k, l[, low_memory, ...]) | Returns the maximum locally ( $\mathrm{k}, \mathrm{l}$ ) -connected subgraph of G . |
| :---: | :---: |
| is_kl_connected(G, k, l[, low_memory]) | Returns True if and only if G is locally ( $\mathrm{k}, \mathrm{l})$-connected. |

[^81]
### 4.30.1 kl_connected_subgraph

kl_connected_subgraph $(G, k, l$, low_memory=False, same_as_graph=False)
Returns the maximum locally $(k, 1)$-connected subgraph of $G$.
A graph is locally $(k, l)$-connected if for each edge $(u, v)$ in the graph there are at least 1 edge-disjoint paths of length at most $k$ joining $u$ to $v$.

## Parameters

- G (NetworkX graph) - The graph in which to find a maximum locally ( $k, 1$ ) -connected subgraph.
- $\mathbf{k}$ (integer) - The maximum length of paths to consider. A higher number means a looser connectivity requirement.
- l (integer) - The number of edge-disjoint paths. A higher number means a stricter connectivity requirement.
- low_memory (bool) - If this is True, this function uses an algorithm that uses slightly more time but less memory.
- same_as_graph (bool) - If True then return a tuple of the form (H, is_same), where H is the maximum locally $(k, l)$-connected subgraph and is_same is a Boolean representing whether $G$ is locally $(k, l)$-connected (and hence, whether $H$ is simply a copy of the input graph G).
Returns If same_as_graph is True, then this function returns a two-tuple as described above. Otherwise, it returns only the maximum locally ( $k, l$ ) -connected subgraph.
Return type NetworkX graph or two-tuple


## See also:

is_kl_connected()

## References

### 4.30.2 is_kl_connected

is_kl_connected ( $G, k, l$, low_memory=False)
Returns True if and only if $G$ is locally $(k, l)$-connected.
A graph is locally $(k, l)$-connected if for each edge $(u, v)$ in the graph there are at least 1 edge-disjoint paths of length at most $k$ joining $u$ to $v$.

## Parameters

- G (NetworkX graph) - The graph to test for local $(k, l)$-connectedness.
- $\mathbf{k}$ (integer) - The maximum length of paths to consider. A higher number means a looser connectivity requirement.
- l (integer) - The number of edge-disjoint paths. A higher number means a stricter connectivity requirement.
- low_memory (bool) - If this is True, this function uses an algorithm that uses slightly more time but less memory.

Returns Whether the graph is locally ( $k, l$ )-connected subgraph.
Return type bool

## See also:

```
kl_connected_subgraph()
```


## References

### 4.31 Isolates

Functions for identifying isolate (degree zero) nodes.

| is_isolate $(\mathbf{G}, \mathrm{n})$ | Determines whether a node is an isolate. |
| :--- | :--- |
| isolates $(\mathrm{G})$ | Iterator over isolates in the graph. |

### 4.31.1 is_isolate

is_isolate ( $G, n$ )
Determines whether a node is an isolate.
An isolate is a node with no neighbors (that is, with degree zero). For directed graphs, this means no in-neighbors and no out-neighbors.

## Parameters

- G (NetworkX graph)
- n (node) - A node in G.

Returns is_isolate - True if and only if n has no neighbors.
Return type bool

## Examples

```
>>>G=nx.Graph()
>>> G.add__edge (1,2)
>>> G.add__node (3)
>>> nx.is_isolate(G, 2)
False
>>> nx.is_isolate(G, 3)
True
```


### 4.31.2 isolates

## isolates ( $G$ )

Iterator over isolates in the graph.
An isolate is a node with no neighbors (that is, with degree zero). For directed graphs, this means no in-neighbors and no out-neighbors.

Parameters G (NetworkX graph)
Returns An iterator over the isolates of G .
Return type iterator

## Examples

To get a list of all isolates of a graph, use the list constructor:

```
>>> G = nx.Graph()
>>> G.add_edge(1, 2)
>>> G.add_node (3)
>>> list(nx.isolates(G))
[3]
```

To remove all isolates in the graph, first create a list of the isolates, then use Graph.remove_nodes_from():

```
>>> G.remove_nodes_from(list(nx.isolates(G)))
>>> list(G)
[1, 2]
```

For digraphs, isolates have zero in-degree and zero out_degre:

```
>>>G = nx.DiGraph([(0, 1), (1, 2)])
>>> G.add_node(3)
>>> list(nx.isolates(G))
[3]
```


### 4.32 Isomorphism

| is_isomorphic(G1, G2[, node_match, edge_match]) | Returns True if the graphs G1 and G2 are isomorphic and <br> False otherwise. |
| :--- | :--- |
| could_be_isomorphic(G1, G2) | Returns False if graphs are definitely not isomorphic. |
| fast_could_be_isomorphic(G1, G2) | Returns False if graphs are definitely not isomorphic. |
| faster_could_be_isomorphic(G1, G2) | Returns False if graphs are definitely not isomorphic. |

### 4.32.1 is_isomorphic

is_isomorphic (G1,G2, node_match=None, edge_match=None)
Returns True if the graphs G1 and G2 are isomorphic and False otherwise.

## Parameters

- G1, G2 (graphs) - The two graphs G1 and G2 must be the same type.
- node_match (callable) - A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during the isomorphism test. If node_match is not specified then node attributes are not considered.

The function will be called like

> node_match(G1.node[n1], G2.node[n2]).

That is, the function will receive the node attribute dictionaries for n 1 and n 2 as inputs.

- edge_match (callable) - A function that returns True if the edge attribute dictionary for the pair of nodes ( $\mathrm{u} 1, \mathrm{v} 1$ ) in G1 and ( $\mathrm{u} 2, \mathrm{v} 2$ ) in G 2 should be considered equal during the isomorphism test. If edge_match is not specified then edge attributes are not considered.

The function will be called like

```
edge_match(G1[u1][v1], G2[u2][v2]).
```

That is, the function will receive the edge attribute dictionaries of the edges under consideration.

## Notes

Uses the vf2 algorithm ${ }^{1}$.

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
```

For digraphs G1 and G2, using 'weight' edge attribute (default: 1)

```
>>> G1 = nx.DiGraph()
>>> G2 = nx.DiGraph()
>>> nx.add_path(G1, [1,2,3,4], weight=1)
>>> nx.add_path(G2, [10, 20,30,40], weight=2)
>>> em = iso.numerical_edge_match('weight', 1)
>>> nx.is_isomorphic(G1, G2) # no weights considered
True
>>> nx.is_isomorphic(G1, G2, edge_match=em) # match weights
False
```

For multidigraphs G1 and G2, using 'fill' node attribute (default: '’)

```
>>> G1 = nx.MultiDiGraph()
>>> G2 = nx.MultiDiGraph()
>>> G1.add_nodes_from([1,2,3], fill='red')
>>> G2.add_nodes_from([10,20,30,40], fill='red')
>>> nx.add_path(G1, [1,2,3,4], weight=3, linewidth=2.5)
>>> nx.add_path(G2, [10, 20,30,40], weight=3)
>>> nm = iso.categorical_node_match('fill', 'red')
>>> nx.is_isomorphic(G1, G2, node_match=nm)
True
```

For multidigraphs G1 and G2, using 'weight' edge attribute (default: 7)

```
>>> G1.add_edge(1,2, weight=7)
1
>>> G2.add_edge(10,20)
1
>>> em = iso.numerical_multiedge_match('weight', 7, rtol=1e-6)
>>> nx.is_isomorphic(G1, G2, edge_match=em)
True
```

For multigraphs G1 and G2, using 'weight' and 'linewidth’ edge attributes with default values 7 and 2.5 . Also using 'fill' node attribute with default value 'red'.

[^82]```
>>> em = iso.numerical_multiedge_match(['weight', 'linewidth'], [7, 2.5])
>>> nm = iso.categorical_node_match('fill', 'red')
>>> nx.is_isomorphic(G1, G2, edge_match=em, node_match=nm)
True
```


## See also:

```
numerical_node_match(), numerical_edge_match(), numerical_multiedge_match(),
categorical_node_match(), categorical_edge_match(), categorical_multiedge_match()
```


## References

### 4.32.2 could_be_isomorphic

could_be_isomorphic ( $G 1, G 2$ )
Returns False if graphs are definitely not isomorphic. True does NOT guarantee isomorphism.
Parameters G1, G2 (graphs) - The two graphs G1 and G2 must be the same type.

## Notes

Checks for matching degree, triangle, and number of cliques sequences.

### 4.32.3 fast_could_be_isomorphic

fast_could_be_isomorphic (G1, G2)
Returns False if graphs are definitely not isomorphic.
True does NOT guarantee isomorphism.
Parameters G1, G2 (graphs) - The two graphs G1 and G2 must be the same type.

## Notes

Checks for matching degree and triangle sequences.

### 4.32.4 faster_could_be_isomorphic

faster_could_be_isomorphic (G1, G2)
Returns False if graphs are definitely not isomorphic.
True does NOT guarantee isomorphism.
Parameters G1, G2 (graphs) - The two graphs G1 and G2 must be the same type.

## Notes

Checks for matching degree sequences.

### 4.32.5 Advanced Interface to VF2 Algorithm

## VF2 Algorithm

## VF2 Algorithm

An implementation of VF2 algorithm for graph ismorphism testing.
The simplest interface to use this module is to call networkx.is_isomorphic().

## Introduction

The GraphMatcher and DiGraphMatcher are responsible for matching graphs or directed graphs in a predetermined manner. This usually means a check for an isomorphism, though other checks are also possible. For example, a subgraph of one graph can be checked for isomorphism to a second graph.

Matching is done via syntactic feasibility. It is also possible to check for semantic feasibility. Feasibility, then, is defined as the logical AND of the two functions.

To include a semantic check, the ( Di )GraphMatcher class should be subclassed, and the semantic_feasibility() function should be redefined. By default, the semantic feasibility function always returns True. The effect of this is that semantics are not considered in the matching of G1 and G2.

## Examples

Suppose G1 and G2 are isomorphic graphs. Verification is as follows:

```
>>> from networkx.algorithms import isomorphism
>>> G1 = nx.path_graph(4)
>>> G2 = nx.path_graph(4)
>>> GM = isomorphism.GraphMatcher(G1,G2)
>>> GM.is_isomorphic()
True
```

GM.mapping stores the isomorphism mapping from G1 to G2.

```
>>> GM.mapping
{0: 0, 1: 1, 2: 2, 3: 3}
```

Suppose G1 and G2 are isomorphic directed graphs graphs. Verification is as follows:

```
>>> G1 = nx.path_graph(4, create_using=nx.DiGraph())
>>> G2 = nx.path_graph(4, create_using=nx.DiGraph())
>>> DiGM = isomorphism.DiGraphMatcher(G1,G2)
>>> DiGM.is_isomorphic()
True
```

DiGM.mapping stores the isomorphism mapping from G1 to G 2 .

```
>>> DiGM.mapping
{0: 0, 1: 1, 2: 2, 3: 3}
```


## Subgraph Isomorphism

Graph theory literature can be ambiguious about the meaning of the above statement, and we seek to clarify it now.
In the VF2 literature, a mapping $M$ is said to be a graph-subgraph isomorphism iff $M$ is an isomorphism between G2 and a subgraph of G1. Thus, to say that G1 and G2 are graph-subgraph isomorphic is to say that a subgraph of G1 is isomorphic to G2.

Other literature uses the phrase 'subgraph isomorphic' as in 'G1 does not have a subgraph isomorphic to G2'. Another use is as an in adverb for isomorphic. Thus, to say that G1 and G2 are subgraph isomorphic is to say that a subgraph of G1 is isomorphic to G2.
Finally, the term 'subgraph' can have multiple meanings. In this context, 'subgraph' always means a 'node-induced subgraph'. Edge-induced subgraph isomorphisms are not directly supported, but one should be able to perform the check by making use of nx.line_graph(). For subgraphs which are not induced, the term 'monomorphism' is preferred over 'isomorphism'. Currently, it is not possible to check for monomorphisms.

Let $\mathrm{G}=(\mathrm{N}, \mathrm{E})$ be a graph with a set of nodes N and set of edges E .
If $G^{\prime}=\left(\mathbf{N}^{\prime}, \mathbf{E}^{\prime}\right)$ is a subgraph, then: $N^{\prime}$ is a subset of $N E$ ' is a subset of $E$
If $G^{\prime}=\left(\mathbf{N}^{\prime}, \mathbf{E}^{\prime}\right)$ is a node-induced subgraph, then: $N^{\prime}$ is a subset of $N E^{\prime}$ is the subset of edges in $E$ relating nodes in N'

If $\mathbf{G}^{\prime}=\left(\mathbf{N}^{\prime}, \mathbf{E}^{\prime}\right)$ is an edge-induced subgraph, then: $N^{\prime}$ is the subset of nodes in $N$ related by edges in $E^{\prime} E^{\prime}$ is a subset of $E$

## References

[1] Luigi P. Cordella, Pasquale Foggia, Carlo Sansone, Mario Vento, "A (Sub)Graph Isomorphism Algorithm for Matching Large Graphs", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 26, no. 10, pp. 1367-1372, Oct., 2004. http://ieeexplore.ieee.org/iel5/34/29305/01323804.pdf
[2] L. P. Cordella, P. Foggia, C. Sansone, M. Vento, "An Improved Algorithm for Matching Large Graphs", 3rd IAPR-TC15 Workshop on Graph-based Representations in Pattern Recognition, Cuen, pp. 149-159, 2001. http://amalfi.dis.unina.it/graph/db/papers/vf-algorithm.pdf

## See also:

syntactic_feasibliity, semantic_feasibility

## Notes

Modified to handle undirected graphs. Modified to handle multiple edges.
In general, this problem is NP-Complete.

## Graph Matcher

| GraphMatcher.__init__(G1, G2[, node_match,...]) | Initialize graph matcher. |
| :--- | :--- |
| GraphMatcher.initialize() | Reinitializes the state of the algorithm. |
| GraphMatcher.is_isomorphic() | Returns True if G1 and G2 are isomorphic graphs. |
| GraphMatcher.subgraph_is_isomorphic() | Returns True if a subgraph of G1 is isomorphic to G2. |
| GraphMatcher.isomorphisms_iter() | Generator over isomorphisms between G1 and G2. |
|  |  |

Table 4.87 - continued from previous page

| GraphMatcher.subgraph_isomorphisms_iter() | Generator over isomorphisms between a subgraph of G1 <br> and G2. |
| :--- | :--- |
| GraphMatcher.candidate_pairs_iter() | Iterator over candidate pairs of nodes in G1 and G2. |
| GraphMatcher.match() | Extends the isomorphism mapping. |
| GraphMatcher.semantic_feasibility(G1_node, Returns True if mapping G1_node to G2_node is semanti- <br> cally feasible.  |  |
| GraphMatcher.syntactic_feasibility(G1_node, Returns True if adding (G1_node, G2_node) is syntacti-  <br> (..) cally feasible. |  |

$\qquad$
nit__

GraphMatcher.__init__(G1, G2, node_match=None, edge_match=None)
Initialize graph matcher.

## Parameters

- G1, G2 (graph) - The graphs to be tested.
- node_match (callable) - A function that returns True iff node n 1 in G1 and n2 in G2 should be considered equal during the isomorphism test. The function will be called like:
node_match(G1.node[n1], G2.node[n2])
That is, the function will receive the node attribute dictionaries of the nodes under consideration. If None, then no attributes are considered when testing for an isomorphism.
- edge_match (callable) - A function that returns True iff the edge attribute dictionary for the pair of nodes ( $\mathrm{u} 1, \mathrm{v} 1$ ) in G1 and ( $\mathrm{u} 2, \mathrm{v} 2$ ) in G2 should be considered equal during the isomorphism test. The function will be called like:

```
edge_match(G1[u1][v1], G2[u2][v2])
```

That is, the function will receive the edge attribute dictionaries of the edges under consideration. If None, then no attributes are considered when testing for an isomorphism.
initialize

## GraphMatcher.initialize()

Reinitializes the state of the algorithm.
This method should be redefined if using something other than GMState. If only subclassing GraphMatcher, a redefinition is not necessary.

## is_isomorphic

```
GraphMatcher.is_isomorphic()
```

Returns True if G1 and G2 are isomorphic graphs.
subgraph_is_isomorphic
GraphMatcher.subgraph_is_isomorphic()
Returns True if a subgraph of G1 is isomorphic to G2.
isomorphisms_iter

GraphMatcher.isomorphisms_iter ()
Generator over isomorphisms between G1 and G2.
subgraph_isomorphisms_iter

GraphMatcher.subgraph_isomorphisms_iter()
Generator over isomorphisms between a subgraph of G1 and G2.
candidate_pairs_iter

GraphMatcher. candidate_pairs_iter()
Iterator over candidate pairs of nodes in G1 and G2.
match

GraphMatcher.match()
Extends the isomorphism mapping.
This function is called recursively to determine if a complete isomorphism can be found between G1 and G2. It cleans up the class variables after each recursive call. If an isomorphism is found, we yield the mapping.

```
semantic_feasibility
```


## GraphMatcher.semantic_feasibility (G1_node, G2_node)

Returns True if mapping G1_node to G2_node is semantically feasible.

## syntactic_feasibility

GraphMatcher.syntactic_feasibility (G1_node, G2_node)
Returns True if adding (G1_node, G2_node) is syntactically feasible.
This function returns True if it is adding the candidate pair to the current partial isomorphism mapping is allowable. The addition is allowable if the inclusion of the candidate pair does not make it impossible for an isomorphism to be found.

## DiGraph Matcher

| DiGraphMatcher.__init__(G1, G2[, ...]) | Initialize graph matcher. |
| :--- | :--- |
| DiGraphMatcher.initialize() | Reinitializes the state of the algorithm. |
| DiGraphMatcher.is_isomorphic() | Returns True if G1 and G2 are isomorphic graphs. |
| DiGraphMatcher.subgraph_is_isomorphic() | Returns True if a subgraph of G1 is isomorphic to G2. |
| DiGraphMatcher.isomorphisms_iter() | Generator over isomorphisms between G1 and G2. |
| DiGraphMatcher.subgraph_isomorphisms_iter_( | and G2. |
|  | and over isomorphisms between a subgraph of G1 |
| DiGraphMatcher.candidate_pairs_iter() | Iterator over candidate pairs of nodes in G1 and G2. |
|  |  |

Table 4.88 - continued from previous page

| DiGraphMatcher.match() | Extends the isomorphism mapping. |
| :--- | :--- |
| DiGraphMatcher.semantic_feasibility(G1_nodReturns True if mapping G1_node to G2_node is semanti- |  |
| cally feasible. |  |

$\qquad$
DiGraphMatcher.__init__(G1, G2, node_match=None, edge_match=None)
Initialize graph matcher.

## Parameters

- G1, G2 (graph) - The graphs to be tested.
- node_match (callable) - A function that returns True iff node n1 in G1 and n2 in G2 should be considered equal during the isomorphism test. The function will be called like:
node_match(G1.node[n1], G2.node[n2])
That is, the function will receive the node attribute dictionaries of the nodes under consideration. If None, then no attributes are considered when testing for an isomorphism.
- edge_match (callable) - A function that returns True iff the edge attribute dictionary for the pair of nodes ( $\mathrm{u} 1, \mathrm{v} 1$ ) in G1 and ( $\mathrm{u} 2, \mathrm{v} 2$ ) in G2 should be considered equal during the isomorphism test. The function will be called like:

```
edge_match(G1[u1][v1], G2[u2][v2])
```

That is, the function will receive the edge attribute dictionaries of the edges under consideration. If None, then no attributes are considered when testing for an isomorphism.

## initialize

## DiGraphMatcher.initialize()

Reinitializes the state of the algorithm.
This method should be redefined if using something other than DiGMState. If only subclassing GraphMatcher, a redefinition is not necessary.

## is_isomorphic

DiGraphMatcher.is_isomorphic()
Returns True if G1 and G2 are isomorphic graphs.

## subgraph_is_isomorphic

Returns True if a subgraph of G1 is isomorphic to G2.

## isomorphisms_iter

DiGraphMatcher.isomorphisms_iter()
Generator over isomorphisms between G1 and G2.
subgraph_isomorphisms_iter

DiGraphMatcher.subgraph_isomorphisms_iter()
Generator over isomorphisms between a subgraph of G1 and G2.
candidate_pairs_iter

DiGraphMatcher.candidate_pairs_iter()
Iterator over candidate pairs of nodes in G1 and G2.
match

## DiGraphMatcher.match()

Extends the isomorphism mapping.
This function is called recursively to determine if a complete isomorphism can be found between G1 and G2. It cleans up the class variables after each recursive call. If an isomorphism is found, we yield the mapping.

## semantic_feasibility

DiGraphMatcher.semantic_feasibility (G1_node, G2_node)
Returns True if mapping G1_node to G2_node is semantically feasible.

## syntactic_feasibility

DiGraphMatcher.syntactic_feasibility (G1_node, G2_node)
Returns True if adding (G1_node, G2_node) is syntactically feasible.
This function returns True if it is adding the candidate pair to the current partial isomorphism mapping is allowable. The addition is allowable if the inclusion of the candidate pair does not make it impossible for an isomorphism to be found.

## Match helpers

| categorical_node_match(attr, default) | Returns a comparison function for a categorical node at- <br> tribute. |
| :--- | :--- |
| categorical_edge_match(attr, default) | Returns a comparison function for a categorical edge at- <br> tribute. |
| categorical_multiedge_match(attr, default) | Returns a comparison function for a categorical edge at- <br> tribute. |
| numerical_node_match(attr, default[, rtol, atol]) | Returns a comparison function for a numerical node at- <br> tribute. |

Table 4.89 - continued from previous page

| numerical_edge_match(attr, default[, rtol, atol]) | Returns a comparison function for a numerical edge at- <br> tribute. |
| :--- | :--- |
| numerical_multiedge_match(attr, default[, ...]) | Returns a comparison function for a numerical edge at- <br> tribute. |
| generic_node_match(attr, default, op) | Returns a comparison function for a generic attribute. |
| generic_edge_match(attr, default, op) | Returns a comparison function for a generic attribute. |
| generic_multiedge_match(attr, default, op) | Returns a comparison function for a generic attribute. |

## categorical_node_match

categorical_node_match (attr, default)
Returns a comparison function for a categorical node attribute.
The value(s) of the attr(s) must be hashable and comparable via the $==$ operator since they are placed into a $\operatorname{set}([])$ object. If the sets from G1 and G2 are the same, then the constructed function returns True.

## Parameters

- attr (string I list) - The categorical node attribute to compare, or a list of categorical node attributes to compare.
- default (value | list) - The default value for the categorical node attribute, or a list of default values for the categorical node attributes.

Returns match - The customized, categorical node_match function.
Return type function

## Examples

>>> import networkx.algorithms.isomorphism as iso
$\ggg \mathrm{nm}=$ iso.categorical_node_match('size', 1)
>>> nm = iso.categorical_node_match(['color', 'size'], ['red', 2])

## categorical_edge_match

## categorical_edge_match (attr, default)

Returns a comparison function for a categorical edge attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ must be hashable and comparable via the $==$ operator since they are placed into a $\operatorname{set}([])$ object. If the sets from G1 and G2 are the same, then the constructed function returns True.

## Parameters

- attr (string \| list) - The categorical edge attribute to compare, or a list of categorical edge attributes to compare.
- default (value I list) - The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

Returns match - The customized, categorical edge_match function.
Return type function

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_edge_match('size', 1)
>>> nm = iso.categorical_edge_match(['color', 'size'], ['red', 2])
```


## categorical_multiedge_match

categorical_multiedge_match (attr, default)
Returns a comparison function for a categorical edge attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ must be hashable and comparable via the $==$ operator since they are placed into a $\operatorname{set}([])$ object. If the sets from G1 and G2 are the same, then the constructed function returns True.

## Parameters

- attr (string I list) - The categorical edge attribute to compare, or a list of categorical edge attributes to compare.
- default (value I list) - The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

Returns match - The customized, categorical edge_match function.
Return type function

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_multiedge_match('size', 1)
>>> nm = iso.categorical_multiedge_match(['color', 'size'], ['red', 2])
```


## numerical_node_match

numerical_node_match (attr, default, rtol=1e-05, atol=le-08)
Returns a comparison function for a numerical node attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

## Parameters

- attr (string I list) - The numerical node attribute to compare, or a list of numerical node attributes to compare.
- default (value I list) - The default value for the numerical node attribute, or a list of default values for the numerical node attributes.
- rtol (float) - The relative error tolerance.
- atol (float) - The absolute error tolerance.

Returns match - The customized, numerical node_match function.
Return type function

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_node_match('weight', 1.0)
>>> nm = iso.numerical_node_match(['weight', 'linewidth'], [.25, .5])
```


## numerical_edge_match

numerical_edge_match (attr, default, rtol=1e-05, atol=1e-08)
Returns a comparison function for a numerical edge attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

## Parameters

- attr (string | list) - The numerical edge attribute to compare, or a list of numerical edge attributes to compare.
- default (value I list) - The default value for the numerical edge attribute, or a list of default values for the numerical edge attributes.
- rtol (float) - The relative error tolerance.
- atol (float) - The absolute error tolerance.

Returns match - The customized, numerical edge_match function.
Return type function

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_edge_match('weight', 1.0)
>>> nm = iso.numerical_edge_match(['weight', 'linewidth'], [.25, .5])
```


## numerical_multiedge_match

numerical_multiedge_match (attr, default, rtol=1e-05, atol=le-08)
Returns a comparison function for a numerical edge attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

## Parameters

- attr (string | list) - The numerical edge attribute to compare, or a list of numerical edge attributes to compare.
- default (value | list) - The default value for the numerical edge attribute, or a list of default values for the numerical edge attributes.
- rtol (float) - The relative error tolerance.
- atol (float) - The absolute error tolerance.

Returns match - The customized, numerical edge_match function.

Return type function

## Examples

```
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_multiedge_match('weight', 1.0)
>>> nm = iso.numerical_multiedge_match(['weight', 'linewidth'], [.25, .5])
```

generic_node_match
generic_node_match (attr, default, op )
Returns a comparison function for a generic attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ are compared using the specified operators. If all the attributes are equal, then the constructed function returns True.

## Parameters

- attr (string | list) - The node attribute to compare, or a list of node attributes to compare.
- default (value I list) - The default value for the node attribute, or a list of default values for the node attributes.
- op (callable | list) - The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.

Returns match - The customized, generic node_match function.
Return type function

## Examples

```
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_node_match
>>> nm = generic_node_match('weight', 1.0, close)
>>> nm = generic_node_match('color', 'red', eq)
>>> nm = generic_node_match(['weight', 'color'], [1.0, 'red'], [close, eq])
```

generic_edge_match
generic_edge_match (attr, default, op)
Returns a comparison function for a generic attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ are compared using the specified operators. If all the attributes are equal, then the constructed function returns True.

## Parameters

- attr (string | list) - The edge attribute to compare, or a list of edge attributes to compare.
- default (value I list) - The default value for the edge attribute, or a list of default values for the edge attributes.
- op (callable | list) - The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.
Returns match - The customized, generic edge_match function.
Return type function


## Examples

```
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_edge_match
>>> nm = generic_edge_match('weight', 1.0, close)
>>> nm = generic_edge_match('color', 'red', eq)
>>> nm = generic_edge_match(['weight', 'color'], [1.0, 'red'], [close, eq])
```


## generic_multiedge_match

generic_multiedge_match (attr, default, op)
Returns a comparison function for a generic attribute.
The value(s) of the $\operatorname{attr}(\mathrm{s})$ are compared using the specified operators. If all the attributes are equal, then the constructed function returns True. Potentially, the constructed edge_match function can be slow since it must verify that no isomorphism exists between the multiedges before it returns False.

## Parameters

- attr (string | list) - The edge attribute to compare, or a list of node attributes to compare.
- default (value I list) - The default value for the edge attribute, or a list of default values for the dgeattributes.
- op (callable | list) - The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.

Returns match - The customized, generic edge_match function.
Return type function

## Examples

```
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_node_match
>>> nm = generic_node_match('weight', 1.0, close)
>>> nm = generic_node_match('color', 'red', eq)
>>> nm = generic_node_match(['weight', 'color'],
... [1.0, 'red'],
... [close, eq])
```

. .

### 4.33 Link Analysis

### 4.33.1 PageRank

PageRank analysis of graph structure.

| pagerank $(\mathrm{G}[$, alpha, personalization, ...]) | Return the PageRank of the nodes in the graph. |
| :--- | :--- |
| pagerank__numpy $(\mathrm{G}[$, alpha, personalization, ...]) | Return the PageRank of the nodes in the graph. |
| pagerank_scipy $(\mathrm{G}[$, alpha, personalization, ... $])$ | Return the PageRank of the nodes in the graph. |
| google_matrix $(\mathrm{G}[$, alpha, personalization, ...]) | Return the Google matrix of the graph. |

## pagerank

pagerank ( $G$, alpha $=0.85$, personalization=None, max_iter $=100$, tol $=1 e-06$, nstart $=$ None, weight $=$ 'weight', dangling $=$ None)
Return the PageRank of the nodes in the graph.
PageRank computes a ranking of the nodes in the graph G based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

## Parameters

- G (graph) - A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- alpha (float, optional) - Damping parameter for PageRank, default=0.85.
- personalization (dict, optional) - The "personalization vector" consisting of a dictionary with a key for every graph node and personalization value for each node. At least one personalization value must be non-zero. By default, a uniform distribution is used.
- max_iter (integer, optional) - Maximum number of iterations in power method eigenvalue solver.
- tol (float, optional) - Error tolerance used to check convergence in power method solver.
- nstart (dictionary, optional) - Starting value of PageRank iteration for each node.
- weight (key, optional) - Edge data key to use as weight. If None weights are set to 1 .
- dangling (dict, optional) - The outedges to be assigned to any "dangling" nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified). This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.

Returns pagerank - Dictionary of nodes with PageRank as value
Return type dictionary

## Examples

```
>>> G = nx.DiGraph(nx.path_graph(4))
>>> pr = nx.pagerank(G, alpha=0.9)
```


## Notes

The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after an error tolerance of $\operatorname{len}(G) *$ tol has been reached. If the number of iterations exceed max_iter, a networkx.exception.PowerIterationFailedConvergence exception is raised.

The PageRank algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs by converting each edge in the directed graph to two edges.

## See also:

pagerank_numpy(), pagerank_scipy(), google_matrix()
Raises PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

## References

## pagerank_numpy

pagerank_numpy ( $G$, alpha $=0.85$, personalization=None, weight='weight', dangling=None)
Return the PageRank of the nodes in the graph.
PageRank computes a ranking of the nodes in the graph $G$ based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

## Parameters

- G (graph) - A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- alpha (float, optional) - Damping parameter for PageRank, default=0.85.
- personalization (dict, optional) - The "personalization vector" consisting of a dictionary with a key for every graph node and nonzero personalization value for each node. By default, a uniform distribution is used.
- weight (key, optional) - Edge data key to use as weight. If None weights are set to 1 .
- dangling (dict, optional) - The outedges to be assigned to any "dangling" nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.

Returns pagerank - Dictionary of nodes with PageRank as value.
Return type dictionary

## Examples

```
>>> G = nx.DiGraph(nx.path_graph(4))
>>> pr = nx.pagerank_numpy(G, alpha=0.9)
```


## Notes

The eigenvector calculation uses NumPy's interface to the LAPACK eigenvalue solvers. This will be the fastest and most accurate for small graphs.

This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

## See also:

pagerank(), pagerank_scipy(), google_matrix()

## References

## pagerank_scipy

pagerank_scipy ( $G$, alpha $=0.85$, personalization $=$ None, max_iter $=100$, tol $=1 e-06$, weight='weight', dangling=None)
Return the PageRank of the nodes in the graph.
PageRank computes a ranking of the nodes in the graph G based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

## Parameters

- G (graph) - A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- alpha (float, optional) - Damping parameter for PageRank, default=0.85.
- personalization (dict, optional) - The "personalization vector" consisting of a dictionary with a key for every graph node and nonzero personalization value for each node. By default, a uniform distribution is used.
- max_iter (integer, optional) - Maximum number of iterations in power method eigenvalue solver.
- tol (float, optional) - Error tolerance used to check convergence in power method solver.
- weight (key, optional) - Edge data key to use as weight. If None weights are set to 1 .
- dangling (dict, optional) - The outedges to be assigned to any "dangling" nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.
Returns pagerank - Dictionary of nodes with PageRank as value
Return type dictionary


## Examples

```
>>> G = nx.DiGraph(nx.path_graph(4))
>>> pr = nx.pagerank_scipy(G, alpha=0.9)
```


## Notes

The eigenvector calculation uses power iteration with a SciPy sparse matrix representation.
This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

## See also:

```
pagerank(), pagerank_numpy(), google_matrix()
```

Raises PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

## References

```
google_matrix
```

google_matrix (G, alpha=0.85, personalization=None, nodelist=None, weight='weight', dangling=None)
Return the Google matrix of the graph.

## Parameters

- G (graph) - A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- alpha (float) - The damping factor.
- personalization (dict, optional) - The "personalization vector" consisting of a dictionary with a key for every graph node and nonzero personalization value for each node. By default, a uniform distribution is used.
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- weight (key, optional) - Edge data key to use as weight. If None weights are set to 1 .
- dangling (dict, optional) - The outedges to be assigned to any "dangling" nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes below). It may be common to have the dangling dict to be the same as the personalization dict.
Returns A-Google matrix of the graph
Return type NumPy matrix


## Notes

The matrix returned represents the transition matrix that describes the Markov chain used in PageRank. For PageRank to converge to a unique solution (i.e., a unique stationary distribution in a Markov chain), the transition matrix must be irreducible. In other words, it must be that there exists a path between every pair of nodes in the graph, or else there is the potential of "rank sinks."
This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

## See also:

pagerank(), pagerank_numpy(), pagerank_scipy()

### 4.33.2 Hits

Hubs and authorities analysis of graph structure.

| hits(G[, max_iter, tol, nstart, normalized]) | Return HITS hubs and authorities values for nodes. |
| :--- | :--- |
| hits_numpy $(\mathrm{G}[$, normalized]) | Return HITS hubs and authorities values for nodes. |
| hits_scipy $(\mathrm{G}[$, max_iter, tol, normalized] $)$ | Return HITS hubs and authorities values for nodes. |
| hub_matrix(G[, nodelist]) | Return the HITS hub matrix. |
| authority_matrix(G[, nodelist] | Return the HITS authority matrix. |

hits
hits ( $G$, max_iter $=100$, tol=1e-08, nstart=None, normalized=True)
Return HITS hubs and authorities values for nodes.
The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

## Parameters

- G (graph) - A NetworkX graph
- max_iter (interger, optional) - Maximum number of iterations in power method.
- tol (float, optional) - Error tolerance used to check convergence in power method iteration.
- nstart (dictionary, optional) - Starting value of each node for power method iteration.
- normalized (bool (default=True)) - Normalize results by the sum of all of the values.

Returns (hubs,authorities) - Two dictionaries keyed by node containing the hub and authority values.

Return type two-tuple of dictionaries
Raises PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

## Examples

```
>>> G=nx.path_graph(4)
>>> h,a=nx.hits(G)
```


## Notes

The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after max_iter iterations or an error tolerance of number_of_nodes $(\mathrm{G}) *$ tol has been reached.

The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

## References

hits_numpy
hits_numpy ( $G$, normalized=True)
Return HITS hubs and authorities values for nodes.
The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

## Parameters

- G (graph) - A NetworkX graph
- normalized (bool (default=True)) - Normalize results by the sum of all of the values.

Returns (hubs,authorities) - Two dictionaries keyed by node containing the hub and authority values.

Return type two-tuple of dictionaries

## Examples

>>> G=nx.path_graph (4)
>>> h, a=nx.hits (G)

## Notes

The eigenvector calculation uses NumPy's interface to LAPACK.
The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

## References

## hits_scipy

hits_scipy ( $G$, max_iter=100, tol=1e-06, normalized=True)
Return HITS hubs and authorities values for nodes.
The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

## Parameters

- G (graph) - A NetworkX graph
- max_iter (interger, optional) - Maximum number of iterations in power method.
- tol (float, optional) - Error tolerance used to check convergence in power method iteration.
- nstart (dictionary, optional) - Starting value of each node for power method iteration.
- normalized (bool (default=True)) - Normalize results by the sum of all of the values.

Returns (hubs,authorities) - Two dictionaries keyed by node containing the hub and authority values.

Return type two-tuple of dictionaries

## Examples

```
>>> G=nx.path__graph(4)
>>> h,a=nx.hits(G)
```


## Notes

This implementation uses SciPy sparse matrices.
The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after max_iter iterations or an error tolerance of number_of_nodes $(\mathrm{G}) *$ tol has been reached.

The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

Raises PowerIterationFailedConvergence - If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

## References

```
hub_matrix
```

hub_matrix ( $G$, nodelist=None)
Return the HITS hub matrix.

```
authority_matrix
```

authority_matrix (G, nodelist=None)

Return the HITS authority matrix.

### 4.34 Link Prediction

Link prediction algorithms.

| resource_allocation_index(G[, ebunch]) | Compute the resource allocation index of all node pairs in <br> ebunch. |
| :--- | :--- |
| jaccard_coefficient(G[, ebunch]) | Compute the Jaccard coefficient of all node pairs in ebunch. |
| adamic_adar_index(G[, ebunch]) | Compute the Adamic-Adar index of all node pairs in <br> ebunch. |
| preferential_attachment(G[, ebunch]) | Compute the preferential attachment score of all node pairs <br> in ebunch. |
| cn_soundarajan_hopcroft(G[, ebunch, commu- <br> nity]) | Count the number of common neighbors of all node pairs <br> in ebunch using community information. |
| ra_index_soundarajan_hopcroft(G[, ebunch,, | Compute the resource allocation index of all node pairs in <br> ebunch using community information. |
| within_inter_cluster(G[, ebunch, delta, ...]) | Compute the ratio of within- and inter-cluster common <br> neighbors of all node pairs in ebunch. |

### 4.34.1 resource_allocation_index

resource_allocation_index ( $G$, ebunch=None)
Compute the resource allocation index of all node pairs in ebunch.
Resource allocation index of $u$ and $v$ is defined as

$$
\sum_{w \in \Gamma(u) \cap \Gamma(v)} \frac{1}{|\Gamma(w)|}
$$

where $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - A NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None) $)$ - Resource allocation index will be computed for each pair of nodes given in the iterable. The pairs must be given as 2 -tuples $(u, v)$ where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their resource allocation index.
Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.resource_allocation_index(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
... '(%d, %d) -> %.8f' % (u, v, p)
'(0, 1) -> 0.75000000'
'(2, 3) -> 0.75000000'
```


## References

### 4.34.2 jaccard_coefficient

jaccard_coefficient ( $G$, ebunch=None)
Compute the Jaccard coefficient of all node pairs in ebunch.
Jaccard coefficient of nodes $u$ and $v$ is defined as

$$
\frac{|\Gamma(u) \cap \Gamma(v)|}{|\Gamma(u) \cup \Gamma(v)|}
$$

where $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - A NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None)) - Jaccard coefficient will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples $(u, v)$ where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their Jaccard coefficient.
Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.jaccard_coefficient(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
... '(%d, %d) -> %.8f' % (u, v, p)
'(0, 1) -> 0.60000000'
'(2, 3) -> 0.60000000'
```


## References

### 4.34.3 adamic_adar_index

adamic_adar_index ( $G$, ebunch=None)
Compute the Adamic-Adar index of all node pairs in ebunch.
Adamic-Adar index of $u$ and $v$ is defined as

$$
\sum_{w \in \Gamma(u) \cap \Gamma(v)} \frac{1}{\log |\Gamma(w)|}
$$

where $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None $)$ ) Adamic-Adar index will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples $(u, v)$ where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their Adamic-Adar index.

Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.adamic_adar_index(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
... '(%d, %d) -> %.8f' % (u, v, p)
...
'(0, 1) -> 2.16404256'
'(2, 3) -> 2.16404256'
```


## References

### 4.34.4 preferential_attachment

## preferential_attachment (G,ebunch=None)

Compute the preferential attachment score of all node pairs in ebunch.
Preferential attachment score of $u$ and $v$ is defined as

$$
|\Gamma(u)||\Gamma(v)|
$$

where $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None $)$ ) - Preferential attachment score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples ( $u, v$ ) where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their preferential attachment score.

Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.preferential_attachment(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
... '(%d, %d) -> %d' % (u, v, p)
\cdots
'(0, 1) -> 16'
'(2, 3) -> 16'
```


## References

### 4.34.5 cn_soundarajan_hopcroft

cn_soundarajan_hopcroft ( $G$, ebunch=None, community='community')
Count the number of common neighbors of all node pairs in ebunch using community information.
For two nodes $u$ and $v$, this function computes the number of common neighbors and bonus one for each common neighbor belonging to the same community as $u$ and $v$. Mathematically,

$$
|\Gamma(u) \cap \Gamma(v)|+\sum_{w \in \Gamma(u) \cap \Gamma(v)} f(w)
$$

where $f(w)$ equals 1 if $w$ belongs to the same community as $u$ and $v$ or 0 otherwise and $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - A NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None $)$ ) The score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples (u, v) where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.
- community (string, optional (default $=$ 'community')) - Nodes attribute name containing the community information. $\mathrm{G}[\mathrm{u}][$ community] identifies which community u belongs to. Each node belongs to at most one community. Default value: 'community'.

Returns piter - An iterator of 3-tuples in the form $(u, v, p)$ where $(u, v)$ is a pair of nodes and $p$ is their score.

Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.path_graph (3)
>>> G.node[0]['community'] = 0
>>> G.node[1]['community'] = 0
>>> G.node[2]['community'] = 0
>>> preds = nx.cn_soundarajan_hopcroft(G, [(0, 2)])
>> for u, v, p in preds:
... '(%d, %d) -> %d' % (u, v, p)
'(0, 2) -> 2'
```


## References

### 4.34.6 ra_index_soundarajan_hopcroft

ra_index_soundarajan_hopcroft ( $G$, ebunch=None, community='community')
Compute the resource allocation index of all node pairs in ebunch using community information.
For two nodes $u$ and $v$, this function computes the resource allocation index considering only common neighbors belonging to the same community as $u$ and $v$. Mathematically,

$$
\sum_{w \in \Gamma(u) \cap \Gamma(v)} \frac{f(w)}{|\Gamma(w)|}
$$

where $f(w)$ equals 1 if $w$ belongs to the same community as $u$ and $v$ or 0 otherwise and $\Gamma(u)$ denotes the set of neighbors of $u$.

## Parameters

- G (graph) - A NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None) $)$ - The score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples (u, v) where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.
- community (string, optional (default = 'community')) - Nodes attribute name containing the community information. $\mathrm{G}[\mathrm{u}][$ community] identifies which community u belongs to. Each node belongs to at most one community. Default value: 'community'.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their score.
Return type iterator

## Examples

```
>>> import networkx as nx
>>> G = nx.Graph()
>>> G.add_edges_from([(0, 1), (0, 2), (1, 3), (2, 3)])
>>> G.node[0]['community'] = 0
>>> G.node[1]['community'] = 0
>>> G.node[2]['community'] = 1
>>> G.node[3]['community'] = 0
>>> preds = nx.ra_index_soundarajan_hopcroft(G, [(0, 3)])
>>> for u, v, p in preds:
... '(%d, %d) -> %.8f' % (u, v, p)
...
'(0, 3) -> 0.500000000'
```


## References

### 4.34.7 within_inter_cluster

within_inter_cluster ( $G$, ebunch=None, delta=0.001, community='community')
Compute the ratio of within- and inter-cluster common neighbors of all node pairs in ebunch.
For two nodes $u$ and $v$, if a common neighbor $w$ belongs to the same community as them, $w$ is considered as within-cluster common neighbor of $u$ and $v$. Otherwise, it is considered as inter-cluster common neighbor of $u$ and $v$. The ratio between the size of the set of within- and inter-cluster common neighbors is defined as the WIC measure.

## Parameters

- G (graph) - A NetworkX undirected graph.
- ebunch (iterable of node pairs, optional (default $=$ None)) - The WIC measure will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples $(u, v)$ where $u$ and $v$ are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.
- delta (float, optional (default $=0.001)$ ) - Value to prevent division by zero in case there is no inter-cluster common neighbor between two nodes. See ${ }^{1}$ for details. Default value: 0.001 .
- community (string, optional (default $=$ 'community')) - Nodes attribute name containing the community information. $\mathrm{G}[\mathrm{u}][$ community $]$ identifies which community $u$ belongs to. Each node belongs to at most one community. Default value: 'community'.

Returns piter - An iterator of 3-tuples in the form ( $u, v, p$ ) where $(u, v)$ is a pair of nodes and $p$ is their WIC measure.
Return type iterator

[^83]
## Examples

```
>>> import networkx as nx
>>> G = nx.Graph()
>>> G.add_edges_from([(0, 1), (0, 2), (0, 3), (1, 4), (2, 4), (3, 4)])
>>> G.node[0]['community'] = 0
>>> G.node[1]['community'] = 1
>>> G.node[2]['community'] = 0
>>> G.node[3]['community'] = 0
>>> G.node[4]['community'] = 0
>>> preds = nx.within_inter_cluster(G, [(0, 4)])
>>> for u, v, p in preds:
... '(%d, %d) -> %.8f' % (u, v, p)
. . .
'(0, 4) -> 1.99800200'
>>> preds = nx.within_inter_cluster(G, [(0, 4)], delta=0.5)
>>> for u, v, p in preds:
... '(%d, %d) -> %. 8f' % (u, v, p)
•••
'(0, 4) -> 1.33333333'
```


## References

### 4.35 Matching

Functions for computing and verifying matchings in a graph.

| is_matching(G, matching) | Decides whether the given set or dictionary represents a <br> valid matching in G. |
| :--- | :--- |
| is_maximal_matching(G, matching) | Decides whether the given set or dictionary represents a <br> valid maximal matching in G. |
| maximal_matching(G) | Find a maximal matching in the graph. |
| max_weight_matching(G[, maxcardinality, weight $])$ | Compute a maximum-weighted matching of G. |

### 4.35.1 is_matching

## is_matching (G, matching)

Decides whether the given set or dictionary represents a valid matching in G.
A matching in a graph is a set of edges in which no two distinct edges share a common endpoint.

## Parameters

- G (NetworkX graph)
- matching (dict or set) - A dictionary or set representing a matching. If a dictionary, it must have matching[u] == $v$ and matching $[v]==u$ for each edge ( $u, v$ ) in the matching. If a set, it must have elements of the form $(u, v)$, where $(u, v)$ is an edge in the matching.

Returns Whether the given set or dictionary represents a valid matching in the graph.
Return type bool

### 4.35.2 is_maximal_matching

## is_maximal_matching (G, matching)

Decides whether the given set or dictionary represents a valid maximal matching in $G$.
A maximal matching in a graph is a matching in which adding any edge would cause the set to no longer be a valid matching.

## Parameters

- G (NetworkX graph)
- matching (dict or set) - A dictionary or set representing a matching. If a dictionary, it must have matching[u] == $v$ and matching $[v]==u$ for each edge ( $u, v$ ) in the matching. If a set, it must have elements of the form $(u, v)$, where $(u, v)$ is an edge in the matching.

Returns Whether the given set or dictionary represents a valid maximal matching in the graph.
Return type bool

### 4.35.3 maximal_matching

maximal_matching ( $G$ )
Find a maximal matching in the graph.
A matching is a subset of edges in which no node occurs more than once. A maximal matching cannot add more edges and still be a matching.

Parameters G (NetworkX graph) - Undirected graph
Returns matching - A maximal matching of the graph.
Return type set

## Notes

The algorithm greedily selects a maximal matching $M$ of the graph $G$ (i.e. no superset of $M$ exists). It runs in $O(|E|)$ time.

### 4.35.4 max_weight_matching

max_weight_matching ( $G$, maxcardinality=False, weight='weight')
Compute a maximum-weighted matching of $G$.
A matching is a subset of edges in which no node occurs more than once. The weight of a matching is the sum of the weights of its edges. A maximal matching cannot add more edges and still be a matching. The cardinality of a matching is the number of matched edges.

## Parameters

- G (NetworkX graph) - Undirected graph
- maxcardinality (bool, optional (default=False)) - If maxcardinality is True, compute the maximum-cardinality matching with maximum weight among all maximum-cardinality matchings.
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight. If key not found, uses 1 as weight.

Returns mate - The matching is returned as a dictionary, mate, such that mate[v] $==\mathrm{w}$ if node v is matched to node w. Unmatched nodes do not occur as a key in mate.
Return type dictionary

## Notes

If $G$ has edges with weight attributes the edge data are used as weight values else the weights are assumed to be 1.

This function takes time O (number_of_nodes ** 3).
If all edge weights are integers, the algorithm uses only integer computations. If floating point weights are used, the algorithm could return a slightly suboptimal matching due to numeric precision errors.
This method is based on the "blossom" method for finding augmenting paths and the "primal-dual" method for finding a matching of maximum weight, both methods invented by Jack Edmonds ${ }^{1}$.
Bipartite graphs can also be matched using the functions present in networkx.algorithms.bipartite.matching.

## References

### 4.36 Minors

Provides functions for computing minors of a graph.

| contracted_edge(G, edge[, self_loops]) | Returns the graph that results from contracting the specified <br> edge. |
| :--- | :--- |
| contracted_nodes(G, u, v[, self_loops]) | Returns the graph that results from contracting u and v. |
| identified_nodes(G, u, v[, self_loops]) | Returns the graph that results from contracting u and v. |
| quotient_graph(G, partition[, ...]) | Returns the quotient graph of G under the specified equiva- <br> lence relation on nodes. |
| blockmodel(G, partition[, multigraph $])$ | Returns a reduced graph constructed using the generalized <br> block modeling technique. |

### 4.36.1 contracted_edge

contracted_edge (G, edge, self_loops=True)
Returns the graph that results from contracting the specified edge.
Edge contraction identifies the two endpoints of the edge as a single node incident to any edge that was incident to the original two nodes. A graph that results from edge contraction is called a minor of the original graph.

## Parameters

- G (NetworkX graph) - The graph whose edge will be contracted.
- edge (tuple) - Must be a pair of nodes in G.
- self_loops (Boolean) - If this is True, any edges (including edge) joining the endpoints of edge in $G$ become self-loops on the new node in the returned graph.

[^84]Returns A new graph object of the same type as $G$ (leaving $G$ unmodified) with endpoints of edge identified in a single node. The right node of edge will be merged into the left one, so only the left one will appear in the returned graph.

Return type Networkx graph
Raises ValueError - If edge is not an edge in G.

## Examples

Attempting to contract two nonadjacent nodes yields an error:

```
>>> import networkx as nx
>>> G = nx.cycle_graph(4)
>>> nx.contracted_edge(G, (1, 3))
Traceback (most recent call last):
ValueError: Edge (1, 3) does not exist in graph G; cannot contract it
```

Contracting two adjacent nodes in the cycle graph on $n$ nodes yields the cycle graph on $n-1$ nodes:

```
>>> import networkx as nx
>>> C5 = nx.cycle_graph(5)
>>> C4 = nx.cycle_graph(4)
>>> M = nx.contracted_edge(C5, (0, 1), self_loops=False)
>>> nx.is_isomorphic(M, C4)
True
```


## See also:

```
contracted_nodes(), quotient_graph()
```


### 4.36.2 contracted_nodes

contracted_nodes $(G, u, v$, self_loops=True)
Returns the graph that results from contracting $u$ and $v$.
Node contraction identifies the two nodes as a single node incident to any edge that was incident to the original two nodes.

## Parameters

- G (NetworkX graph) - The graph whose nodes will be contracted.
- u, $\mathbf{v}$ (nodes) - Must be nodes in G.
- self_loops (Boolean) - If this is True, any edges joining $u$ and $v$ in $G$ become self-loops on the new node in the returned graph.

Returns A new graph object of the same type as $G$ (leaving $G$ unmodified) with $u$ and $v$ identified in a single node. The right node $v$ will be merged into the node $u$, so only $u$ will appear in the returned graph.
Return type Networkx graph

## Examples

Contracting two nonadjacent nodes of the cycle graph on four nodes C_4 yields the path graph (ignoring parallel edges):

```
>>> import networkx as nx
>>> G = nx.cycle_graph(4)
>>> M = nx.contracted_nodes(G, 1, 3)
>>> P3 = nx.path_graph(3)
>>> nx.is_isomorphic(M, P3)
True
```


## See also:

```
contracted_edge(), quotient_graph()
```


## Notes

This function is also available as identified_nodes.

### 4.36.3 identified_nodes

identified_nodes ( $G, u, v$, self_loops=True)
Returns the graph that results from contracting $u$ and $v$.
Node contraction identifies the two nodes as a single node incident to any edge that was incident to the original two nodes.

## Parameters

- G (NetworkX graph) - The graph whose nodes will be contracted.
- $\mathbf{u}, \mathbf{v}$ (nodes) - Must be nodes in G.
- self_loops (Boolean) - If this is True, any edges joining $u$ and $v$ in $G$ become self-loops on the new node in the returned graph.

Returns A new graph object of the same type as $G$ (leaving $G$ unmodified) with $u$ and $v$ identified in a single node. The right node $v$ will be merged into the node $u$, so only $u$ will appear in the returned graph.

## Return type Networkx graph

## Examples

Contracting two nonadjacent nodes of the cycle graph on four nodes C_4 yields the path graph (ignoring parallel edges):

```
>>> import networkx as nx
>>> G = nx.cycle_graph(4)
>>> M = nx.contracted_nodes(G, 1, 3)
>>> P3 = nx.path_graph(3)
>>> nx.is_isomorphic(M, P3)
True
```


## See also:

contracted_edge(), quotient_graph()

## Notes

This function is also available as identified_nodes.

### 4.36.4 quotient_graph

quotient_graph (G, partition, edge_relation=None, node_data=None, edge_data=None, relabel=False, create_using=None)
Returns the quotient graph of $G$ under the specified equivalence relation on nodes.

## Parameters

- G (NetworkX graph) - The graph for which to return the quotient graph with the specified node relation.
- partition (function or list of sets) - If a function, this function must represent an equivalence relation on the nodes of G. It must take two arguments $u$ and $v$ and return True exactly when $u$ and $v$ are in the same equivalence class. The equivalence classes form the nodes in the returned graph.

If a list of sets, the list must form a valid partition of the nodes of the graph. That is, each node must be in exactly one block of the partition.

- edge_relation (Boolean function with two arguments) - This function must represent an edge relation on the blocks of G in the partition induced by node_relation. It must take two arguments, $B$ and $C$, each one a set of nodes, and return True exactly when there should be an edge joining block $B$ to block $C$ in the returned graph.
If edge_relation is not specified, it is assumed to be the following relation. Block $B$ is related to block $C$ if and only if some node in $B$ is adjacent to some node in $C$, according to the edge set of $G$.
- edge_data (function) - This function takes two arguments, $B$ and $C$, each one a set of nodes, and must return a dictionary representing the edge data attributes to set on the edge joining $B$ and $C$, should there be an edge joining $B$ and $C$ in the quotient graph (if no such edge occurs in the quotient graph as determined by edge_relation, then the output of this function is ignored).

If the quotient graph would be a multigraph, this function is not applied, since the edge data from each edge in the graph $G$ appears in the edges of the quotient graph.

- node_data (function) - This function takes one argument, $B$, a set of nodes in G , and must return a dictionary representing the node data attributes to set on the node representing $B$ in the quotient graph. If None, the following node attributes will be set:
- 'graph', the subgraph of the graph G that this block represents,
- 'nnodes', the number of nodes in this block,
- 'nedges', the number of edges within this block,
- 'density', the density of the subgraph of G that this block represents.
- relabel (bool) - If True, relabel the nodes of the quotient graph to be nonnegative integers. Otherwise, the nodes are identified with frozenset instances representing the blocks given in partition.
- create_using (NetworkX graph) - If specified, this must be an instance of a NetworkX graph class. The nodes and edges of the quotient graph will be added to this graph and returned. If not specified, the returned graph will have the same type as the input graph.
Returns The quotient graph of $G$ under the equivalence relation specified by partition. If the partition were given as a list of set instances and relabel is False, each node will be a frozenset corresponding to the same set.


## Return type NetworkX graph

Raises NetworkXException - If the given partition is not a valid partition of the nodes of G .

## Examples

The quotient graph of the complete bipartite graph under the "same neighbors" equivalence relation is K_2. Under this relation, two nodes are equivalent if they are not adjacent but have the same neighbor set:

```
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph (2, 3)
>>> same_neighbors = lambda u, v: (u not in G[v] and v not in G[u]
    and G[u] == G[v])
>>> Q = nx.quotient_graph(G, same_neighbors)
>>> K2 = nx.complete_graph(2)
>>> nx.is_isomorphic(Q, K2)
True
```

The quotient graph of a directed graph under the "same strongly connected component" equivalence relation is the condensation of the graph (see condensation()). This example comes from the Wikipedia article 'Strongly connected component'_:

```
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> edges = ['ab', 'be', 'bf', 'bc', 'cg', 'cd', 'dc', 'dh', 'ea',
... 'ef', 'fg', 'gf', 'hd', 'hf']
>>> G.add_edges_from(tuple(x) for x in edges)
>>> components = list(nx.strongly_connected_components(G))
>>> sorted(sorted(component) for component in components)
[['a', 'b', 'e'], ['c', 'd', 'h'], ['f', 'g']]
>>>
>>> C = nx.condensation(G, components)
>>> component_of = C.graph['mapping']
>>> same_component = lambda u, v: component_of[u] == component_of[v]
>>> Q = nx.quotient_graph(G, same_component)
>>> nx.is_isomorphic(C, Q)
True
```

Node identification can be represented as the quotient of a graph under the equivalence relation that places the two nodes in one block and each other node in its own singleton block:

```
>>> import networkx as nx
>>> K24 = nx.complete_bipartite_graph(2, 4)
>>> K34 = nx.complete_bipartite_graph(3, 4)
>>> C = nx.contracted_nodes(K34, 1, 2)
>>> nodes = {1, 2}
>>> is_contracted = lambda u, v: u in nodes and v in nodes
>>> Q = nx.quotient_graph(K34, is_contracted)
>>> nx.is_isomorphic(Q, C)
```

```
True
>>> nx.is_isomorphic(Q, K24)
True
```

The blockmodeling technique described in ${ }^{1}$ can be implemented as a quotient graph:

```
>>> G = nx.path_graph(6)
>>> partition = [{0, 1}, {2, 3}, {4, 5}]
>>> M = nx.quotient_graph(G, partition, relabel=True)
>>> list(M.edges())
[(0, 1), (1, 2)]
```


## References

### 4.36.5 blockmodel

## blockmodel ( $G$, partition, multigraph=False)

Returns a reduced graph constructed using the generalized block modeling technique.
The blockmodel technique collapses nodes into blocks based on a given partitioning of the node set. Each partition of nodes (block) is represented as a single node in the reduced graph. Edges between nodes in the block graph are added according to the edges in the original graph. If the parameter multigraph is False (the default) a single edge is added with a weight equal to the sum of the edge weights between nodes in the original graph The default is a weight of 1 if weights are not specified. If the parameter multigraph is True then multiple edges are added each with the edge data from the original graph.

## Parameters

- G (graph) - A networkx Graph or DiGraph
- partition (list of lists, or list of sets) - The partition of the nodes. Must be non-overlapping.
- multigraph (bool, optional) - If True return a MultiGraph with the edge data of the original graph applied to each corresponding edge in the new graph. If False return a Graph with the sum of the edge weights, or a count of the edges if the original graph is unweighted.


## Returns blockmodel

Return type a Networkx graph object

## Examples

```
>>> G = nx.path_graph(6)
>>> partition = [[0,1],[2,3],[4,5]]
>>> M = nx.blockmodel(G,partition)
```


## References

Note: Deprecated in NetworkX v1.11

[^85]blockmodel will be removed in NetworkX 2.0. Instead use quotient_graph with keyword argument relabel=True, and create_using=nx.MultiGraph() for multigraphs.

### 4.37 Maximal independent set

Algorithm to find a maximal (not maximum) independent set.
maximal_independent_set(G[, nodes])
Return a random maximal independent set guaranteed to contain a given set of nodes.

### 4.37.1 maximal_independent_set

maximal_independent_set ( $G$, nodes=None)
Return a random maximal independent set guaranteed to contain a given set of nodes.
An independent set is a set of nodes such that the subgraph of $G$ induced by these nodes contains no edges. A maximal independent set is an independent set such that it is not possible to add a new node and still get an independent set.

## Parameters

- G (NetworkX graph)
- nodes (list or iterable) - Nodes that must be part of the independent set. This set of nodes must be independent.

Returns indep_nodes - List of nodes that are part of a maximal independent set.
Return type list
Raises NetworkXUnfeasible - If the nodes in the provided list are not part of the graph or do not form an independent set, an exception is raised.

## Examples

```
>>> G = nx.path_graph(5)
>>> nx.maximal_independent_set(G)
[4, 0, 2]
>>> nx.maximal_independent_set(G, [1])
[1, 3]
```


## Notes

This algorithm does not solve the maximum independent set problem.

### 4.38 Operators

Unary operations on graphs

| complement $(\mathrm{G}[$, name $])$ | Return the graph complement of G. |
| :--- | :--- |
| reverse $(\mathrm{G}[$, copy $])$ | Return the reverse directed graph of G. |

### 4.38.1 complement

complement ( $G$, name=None)
Return the graph complement of G.

## Parameters

- G (graph) - A NetworkX graph
- name (string) - Specify name for new graph


## Returns GC

Return type A new graph.

## Notes

Note that complement() does not create self-loops and also does not produce parallel edges for MultiGraphs.
Graph, node, and edge data are not propagated to the new graph.

### 4.38.2 reverse

reverse ( $G$, copy=True)
Return the reverse directed graph of G.

## Parameters

- G (directed graph ) - A NetworkX directed graph
- copy (bool) - If True, then a new graph is returned. If False, then the graph is reversed in place.
Returns H - The reversed G.
Return type directed graph
Operations on graphs including union, intersection, difference.

| compose $(\mathrm{G}, \mathrm{H}[$, name $])$ | Return a new graph of G composed with H. |
| :--- | :--- |
| union $(\mathrm{G}, \mathrm{H}[$, rename, name $])$ | Return the union of graphs G and H. |
| disjoint_union $(\mathrm{G}, \mathrm{H})$ | Return the disjoint union of graphs G and H. |
| intersection $(\mathrm{G}, \mathrm{H})$ | Return a new graph that contains only the edges that exist <br> in both G and H. |
| difference $(\mathrm{G}, \mathrm{H})$ | Return a new graph that contains the edges that exist in G <br> but not in H. |
| symmetric_difference(G, H) | Return new graph with edges that exist in either G or H but <br> not both. |

### 4.38.3 compose

```
compose ( \(G, H\), name \(=\) None )
```

Return a new graph of G composed with H.
Composition is the simple union of the node sets and edge sets. The node sets of G and H do not need to be disjoint.

## Parameters

- G,H (graph) - A NetworkX graph
- name (string) - Specify name for new graph


## Returns C

Return type A new graph with the same type as G

## Notes

It is recommended that G and H be either both directed or both undirected. Attributes from H take precedent over attributes from G.

For MultiGraphs, the edges are identified by incident nodes AND edge-key. This can cause surprises (i.e., edge $(1,2)$ may or may not be the same in two graphs) if you use MultiGraph without keeping track of edge keys.

### 4.38.4 union

union $(G, H$, rename $=($ None, None $)$, name $=$ None $)$
Return the union of graphs G and H .
Graphs G and H must be disjoint, otherwise an exception is raised.

## Parameters

- G,H (graph) - A NetworkX graph
- create_using (NetworkX graph) - Use specified graph for result. Otherwise
- rename (bool, default=(None, None)) - Node names of G and H can be changed by specifying the tuple rename $=\left({ }^{\prime} G-‘,{ }^{\prime}{ }^{-}\right.$') (for example). Node "u" in G is then renamed "G-u" and " $v$ " in $H$ is renamed " $H-v$ ".
- name (string) - Specify the name for the union graph


## Returns U

Return type A union graph with the same type as G.

## Notes

To force a disjoint union with node relabeling, use disjoint_union( $\mathrm{G}, \mathrm{H}$ ) or convert_node_labels_to integers().
Graph, edge, and node attributes are propagated from $G$ and $H$ to the union graph. If a graph attribute is present in both G and H the value from H is used.

## See also:

disjoint_union()

### 4.38.5 disjoint_union

## disjoint_union ( $G, H$ )

Return the disjoint union of graphs G and H .
This algorithm forces distinct integer node labels.
Parameters G,H (graph) - A NetworkX graph

## Returns U

Return type A union graph with the same type as G.

## Notes

A new graph is created, of the same class as G . It is recommended that G and H be either both directed or both undirected.

The nodes of G are relabeled 0 to len(G)-1, and the nodes of H are relabeled len(G) to len(G)+len(H)-1.
Graph, edge, and node attributes are propagated from G and H to the union graph. If a graph attribute is present in both G and H the value from H is used.

### 4.38.6 intersection

## intersection ( $G, H$ )

Return a new graph that contains only the edges that exist in both G and H.
The node sets of H and G must be the same.
Parameters G,H (graph) - A NetworkX graph. G and H must have the same node sets.

## Returns GH

Return type A new graph with the same type as G.

## Notes

Attributes from the graph, nodes, and edges are not copied to the new graph. If you want a new graph of the intersection of G and H with the attributes (including edge data) from G use remove_nodes_from() as follows

```
>>> G=nx.path_graph(3)
>>> H=nx.path_graph (5)
>>> R=G.copy ()
>>> R.remove_nodes_from(n for }n\mathrm{ in }G\mathrm{ if n not in H)
```


### 4.38.7 difference

## difference ( $G, H$ )

Return a new graph that contains the edges that exist in G but not in H .
The node sets of H and G must be the same.
Parameters G,H (graph) - A NetworkX graph. G and H must have the same node sets.

## Returns D

Return type A new graph with the same type as G.

## Notes

Attributes from the graph, nodes, and edges are not copied to the new graph. If you want a new graph of the difference of G and H with with the attributes (including edge data) from G use remove_nodes_from() as follows:

```
>>> G = nx.path_graph(3)
>>> H = nx.path_graph(5)
>>> R = G.copy()
>>> R.remove_nodes_from(n for n in G if n in H)
```


### 4.38.8 symmetric_difference

## symmetric_difference $(G, H)$

Return new graph with edges that exist in either G or H but not both.
The node sets of H and G must be the same.
Parameters G,H (graph) - A NetworkX graph. G and H must have the same node sets.

## Returns D

Return type A new graph with the same type as G.

## Notes

Attributes from the graph, nodes, and edges are not copied to the new graph.
Operations on many graphs.

| compose_all(graphs[, name]) | Return the composition of all graphs. |
| :--- | :--- |
| union_all(graphs[, rename, name]) | Return the union of all graphs. |
| disjoint_union_all(graphs) | Return the disjoint union of all graphs. |
| intersection_all(graphs) | Return a new graph that contains only the edges that exist <br> in all graphs. |

### 4.38.9 compose_all

## compose_all (graphs, name=None)

Return the composition of all graphs.
Composition is the simple union of the node sets and edge sets. The node sets of the supplied graphs need not be disjoint.

## Parameters

- graphs (list) - List of NetworkX graphs
- name (string) - Specify name for new graph


## Returns C

Return type A graph with the same type as the first graph in list

## Notes

It is recommended that the supplied graphs be either all directed or all undirected.
Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple graphs, then the value from the last graph in the list with that attribute is used.

### 4.38.10 union_all

union_all (graphs, rename $=($ None, $)$, name $=$ None $)$
Return the union of all graphs.
The graphs must be disjoint, otherwise an exception is raised.

## Parameters

- graphs (list of graphs) - List of NetworkX graphs
- rename (bool, default=(None, None)) - Node names of G and H can be changed by specifying the tuple rename=('G-','H-‘) (for example). Node "u" in G is then renamed "G-u" and " v " in H is renamed " $\mathrm{H}-\mathrm{v}$ ".
- name (string) - Specify the name for the union graph@ not_implemnted_for('direct


## Returns U

Return type a graph with the same type as the first graph in list

## Notes

To force a disjoint union with node relabeling, use disjoint_union_all(G,H) or convert_node_labels_to integers().
Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple graphs, then the value from the last graph in the list with that attribute is used.

## See also:

union(), disjoint_union_all()

### 4.38.11 disjoint_union_all

## disjoint_union_all (graphs)

Return the disjoint union of all graphs.
This operation forces distinct integer node labels starting with 0 for the first graph in the list and numbering consecutively.

Parameters graphs (list) - List of NetworkX graphs

## Returns U

Return type A graph with the same type as the first graph in list

## Notes

It is recommended that the graphs be either all directed or all undirected.
Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple graphs, then the value from the last graph in the list with that attribute is used.

### 4.38.12 intersection_all

## intersection_all (graphs)

Return a new graph that contains only the edges that exist in all graphs.
All supplied graphs must have the same node set.
Parameters graphs_list (list) - List of NetworkX graphs

## Returns R

Return type A new graph with the same type as the first graph in list

## Notes

Attributes from the graph, nodes, and edges are not copied to the new graph.

## Graph products.

| cartesian_product $(\mathrm{G}, \mathrm{H})$ | Return the Cartesian product of G and H. |
| :--- | :--- |
| lexicographic_product $(\mathrm{G}, \mathrm{H})$ | Return the lexicographic product of G and H. |
| strong_product $(\mathrm{G}, \mathrm{H})$ | Return the strong product of G and H. |
| tensor_product $(\mathrm{G}, \mathrm{H})$ | Return the tensor product of G and H. |
| power $(\mathbf{G}, \mathrm{k})$ | Returns the specified power of a graph. |

### 4.38.13 cartesian_product

```
cartesian_product (G,H)
```

Return the Cartesian product of G and H .
The Cartesian product P of the graphs G and H has a node set that is the Cartesian product of the node sets, $V(P)=V(G)$ imes $V(H)$. P has an edge $((\mathrm{u}, \mathrm{v}),(\mathrm{x}, \mathrm{y}))$ if and only if either u is equal to x and $\mathrm{v} \& \mathrm{y}$ are adjacent in H or if v is equal to y and $\mathrm{u} \& \mathrm{x}$ are adjacent in G .

Parameters G, H (graphs) - Networkx graphs.
Returns $\mathbf{P}$ - The Cartesian product of G and H . P will be a multi-graph if either G or H is a multigraph. Will be a directed if G and H are directed, and undirected if G and H are undirected.

Return type NetworkX graph
Raises NetworkXError - If G and H are not both directed or both undirected.

## Notes

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.

## Examples

```
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a',a2='Spam')
>>> P = nx.cartesian_product (G,H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

### 4.38.14 lexicographic_product

## lexicographic_product ( $G, H$ )

Return the lexicographic product of G and H .
The lexicographical product P of the graphs G and H has a node set that is the Cartesian product of the node sets, $\$ \mathrm{~V}(\mathrm{P})=\mathrm{V}(\mathrm{G})$ imes $\mathrm{V}(\mathrm{H}) \$$. P has an edge $((\mathrm{u}, \mathrm{v}),(\mathrm{x}, \mathrm{y}))$ if and only if $(\mathrm{u}, \mathrm{v})$ is an edge in G or $\mathrm{u}==\mathrm{v}$ and $(\mathrm{x}, \mathrm{y})$ is an edge in H .

Parameters G, H (graphs) - Networkx graphs.
Returns $\mathbf{P}$ - The Cartesian product of G and H . P will be a multi-graph if either G or H is a multigraph. Will be a directed if G and H are directed, and undirected if G and H are undirected.
Return type NetworkX graph
Raises NetworkXError - If G and H are not both directed or both undirected.

## Notes

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.

## Examples

```
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a',a2='Spam')
>>> P = nx.lexicographic_product (G,H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

### 4.38.15 strong_product

strong_product ( $G, H$ )
Return the strong product of G and H .
The strong product P of the graphs G and H has a node set that is the Cartesian product of the node sets, $\$ \mathrm{~V}(\mathrm{P})=\mathrm{V}(\mathrm{G})$ imes $\mathrm{V}(\mathrm{H}) \$$. P has an edge $((\mathrm{u}, \mathrm{v}),(\mathrm{x}, \mathrm{y}))$ if and only if $\mathrm{u}==\mathrm{v}$ and $(\mathrm{x}, \mathrm{y})$ is an edge in H , or $\mathrm{x}==\mathrm{y}$ and $(u, v)$ is an edge in $G$, or $(u, v)$ is an edge in $G$ and $(x, y)$ is an edge in $H$.

Parameters G, H (graphs) - Networkx graphs.
Returns $\mathbf{P}$ - The Cartesian product of G and H. P will be a multi-graph if either G or H is a multigraph. Will be a directed if G and H are directed, and undirected if G and H are undirected.
Return type NetworkX graph
Raises NetworkXError - If G and H are not both directed or both undirected.

## Notes

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.

## Examples

```
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0,a1=True)
>>> H.add_node('a',a2='Spam')
>>> P = nx.strong_product (G,H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

### 4.38.16 tensor_product

## tensor_product ( $G, H$ )

Return the tensor product of G and H .
The tensor product P of the graphs G and H has a node set that is the tensor product of the node sets, $V(P)=$ $V(G) \times V(H)$. P has an edge $((\mathrm{u}, \mathrm{v}),(\mathrm{x}, \mathrm{y}))$ if and only if $(\mathrm{u}, \mathrm{x})$ is an edge in G and $(\mathrm{v}, \mathrm{y})$ is an edge in H .

Tensor product is sometimes also referred to as the categorical product, direct product, cardinal product or conjunction.

Parameters G, H (graphs) - Networkx graphs.
Returns $\mathbf{P}$ - The tensor product of G and H . P will be a multi-graph if either G or H is a multi-graph, will be a directed if G and H are directed, and undirected if G and H are undirected.
Return type NetworkX graph
Raises NetworkXError - If G and H are not both directed or both undirected.

## Notes

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.

## Examples

```
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a',a2='Spam')
>>> P = nx.tensor_product(G,H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

### 4.38.17 power

power ( $G, k$ )
Returns the specified power of a graph.
The k`th power of a simple graph ` $G$, denoted $G^{k}$, is a graph on the same set of nodes in which two distinct nodes $u$ and $v$ are adjacent in $G^{k}$ if and only if the shortest path distance between $u$ and $v$ in G is at most k.

## Parameters

- G (graph) - A NetworkX simple graph object.
- $\mathbf{k}$ (positive integer) - The power to which to raise the graph G.

Returns G to the power k .
Return type NetworkX simple graph

## Raises

- ValueError - If the exponent $k$ is not positive.
- NetworkXNot Implemented - If G is not a simple graph.


## Examples

The number of edges will never decrease when taking successive powers:

```
>>> G = nx.path_graph(4)
>>> list(nx.power(G, 2).edges())
[(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)]
>>> list(nx.power(G, 3).edges())
[(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)]
```

The $k$ th power of a cycle graph on $* n *$ nodes is the complete graph on *n* nodes, if `k is at least $\mathrm{n} / / \mathrm{2}$ :

```
>>> G = nx.cycle_graph(5)
>>> H = nx.complete_graph(5)
>>> nx.is_isomorphic(nx.power(G, 2), H)
True
>>> G = nx.cycle_graph(8)
>>> H = nx.complete_graph(8)
>>> nx.is_isomorphic(nx.power(G, 4), H)
True
```


## References

## Notes

This definition of "power graph" comes from Exercise 3.1.6 of Graph Theory by Bondy and Murty ${ }^{1}$.

### 4.39 Reciprocity

Algorithms to calculate reciprocity in a directed graph.

| reciprocity $(\mathrm{G}[$, nodes $])$ | Compute the reciprocity in a directed graph. |
| :--- | :--- |
| overall_reciprocity $(\mathrm{G})$ | Compute the reciprocity for the whole graph. |

### 4.39.1 reciprocity

reciprocity ( $G$, nodes=None)
Compute the reciprocity in a directed graph.
The reciprocity of a directed graph is defined as the ratio of the number of edges pointing in both directions to the total number of edges in the graph. Formally, $r=|(u, v) \in G|(v, u) \in G|/|(u, v) \in G|$.

The reciprocity of a single node $u$ is defined similarly, it is the ratio of the number of edges in both directions to the total number of edges attached to node $u$.

## Parameters

- G (graph) - A networkx directed graph
- nodes (container of nodes, optional (default=whole graph)) - Compute reciprocity for nodes in this container.

Returns out - Reciprocity keyed by node label.
Return type dictionary

## Notes

The reciprocity is not defined for isolated nodes. In such cases this function will return None.

### 4.39.2 overall_reciprocity

```
overall_reciprocity (G)
```

Compute the reciprocity for the whole graph.
See the doc of reciprocity for the definition.
Parameters G (graph) - A networkx graph

[^86]
### 4.40 Rich Club

Functions for computing rich-club coefficients.
rich_club_coefficient(G[, normalized, Q]) Returns the rich-club coefficient of the graph G.

### 4.40.1 rich_club_coefficient

rich_club_coefficient ( $G$, normalized $=$ True, $Q=100$ )
Returns the rich-club coefficient of the graph $G$.
For each degree $k$, the rich-club coefficient is the ratio of the number of actual to the number of potential edges for nodes with degree greater than $k$ :

$$
\phi(k)=\frac{2 E_{k}}{N_{k}\left(N_{k}-1\right)}
$$

where $N \_k$ is the number of nodes with degree larger than $k$, and $E \_k$ is the number of edges among those nodes.

## Parameters

- G (NetworkX graph) - Undirected graph with neither parallel edges nor self-loops.
- normalized (bool (optional)) - Normalize using randomized network as in ${ }^{1}$
- Q (float (optional, default=100)) - If normalized is True, perform Q * m double-edge swaps, where $m$ is the number of edges in $G$, to use as a null-model for normalization.

Returns rc - A dictionary, keyed by degree, with rich-club coefficient values.
Return type dictionary

## Examples

```
>>> G = nx.Graph([ (0, 1), (0, 2), (1, 2), (1, 3), (1, 4), (4, 5)])
>>> rc = nx.rich_club_coefficient(G, normalized=False)
>>> rc[0]
0.4
```


## Notes

The rich club definition and algorithm are found in ${ }^{1}$. This algorithm ignores any edge weights and is not defined for directed graphs or graphs with parallel edges or self loops.
Estimates for appropriate values of $Q$ are found in ${ }^{2}$.

[^87]
## References

### 4.41 Shortest Paths

Compute the shortest paths and path lengths between nodes in the graph.
These algorithms work with undirected and directed graphs.

| shortest_path(G[, source, target, weight $])$ | Compute shortest paths in the graph. |
| :--- | :--- |
| all_shortest_paths(G, source, target[, weight $])$ | Compute all shortest paths in the graph. |
| shortest_path_length(G[, source, target, weight $])$ | Compute shortest path lengths in the graph. |
| average_shortest_path_length $(\mathbf{G}[$, weight $])$ | Return the average shortest path length. |
| has_path(G, source, target $)$ | Return True if $G$ has a path from source to target. |

### 4.41.1 shortest_path

shortest_path $(G$, source $=$ None, target=None, weight=None)
Compute shortest paths in the graph.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Starting node for path. If not specified, compute shortest paths for each possible starting node.
- target (node, optional) - Ending node for path. If not specified, compute shortest paths to all possible nodes.
- weight (None or string, optional (default = None)) - If None, every edge has weight/distance/cost 1 . If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1 .


## Returns

path - All returned paths include both the source and target in the path.
If the source and target are both specified, return a single list of nodes in a shortest path from the source to the target.
If only the source is specified, return a dictionary keyed by targets with a list of nodes in a shortest path from the source to one of the targets.
If only the target is specified, return a dictionary keyed by sources with a list of nodes in a shortest path from one of the sources to the target.

If neither the source nor target are specified return a dictionary of dictionaries with path[source][target]=[list of nodes in path].

Return type list or dictionary

## Examples

```
>>> G = nx.path_graph(5)
>>> print(nx.shortest_path(G, source=0, target=4))
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(G, source=0) # target not specified
```

```
>>> p[4]
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(G, target=4) # source not specified
>>> p[0]
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(G) # source, target not specified
>>> p[0][4]
[0, 1, 2, 3, 4]
```


## Notes

There may be more than one shortest path between a source and target. This returns only one of them.

## See also:

```
all_pairs_shortest_path(), all_pairs__dijkstra_path(),
single_source_shortest_path(), single_source_dijkstra_path()
```


### 4.41.2 all_shortest_paths

all_shortest_paths ( $G$, source, target, weight=None)
Compute all shortest paths in the graph.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path.
- target (node) - Ending node for path.
- weight (None or string, optional (default $=$ None)) - If None, every edge has weight/distance/cost 1. If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1 .

Returns paths - A generator of all paths between source and target.
Return type generator of lists

## Examples

```
>>> G = nx.Graph()
>>> nx.add_path(G, [0, 1, 2])
>>> nx.add_path(G, [0, 10, 2])
>>> print([p for p in nx.all_shortest_paths(G, source=0, target=2)])
[[0, 1, 2], [0, 10, 2]]
```


## Notes

There may be many shortest paths between the source and target.

## See also:

```
shortest_path(), single_source_shortest_path(), all_pairs_shortest_path()
```


### 4.41.3 shortest_path_length

shortest_path_length (G, source=None, target=None, weight=None)
Compute shortest path lengths in the graph.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Starting node for path. If not specified, compute shortest path lengths using all nodes as source nodes.
- target (node, optional) - Ending node for path. If not specified, compute shortest path lengths using all nodes as target nodes.
- weight (None or string, optional (default $=$ None)) - If None, every edge has weight/distance/cost 1 . If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1 .


## Returns

length - If the source and target are both specified, return the length of the shortest path from the source to the target.

If only the source is specified, return a tuple (target, shortest path length) iterator, where shortest path lengths are the lengths of the shortest path from the source to one of the targets.

If only the target is specified, return a tuple (source, shortest path length) iterator, where shortest path lengths are the lengths of the shortest path from one of the sources to the target.
If neither the source nor target are specified, return a (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.
Return type int or iterator
Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G = nx.path_graph(5)
>>> nx.shortest_path_length(G, source=0, target=4)
4
>>> p = nx.shortest_path_length(G, source=0) # target not specified
>>> dict(p)[4]
4
>>> p = nx.shortest_path_length(G, target=4) # source not specified
>>> dict(p)[0]
4
>>> p = nx.shortest_path_length(G) # source,target not specified
>>> dict(p)[0][4]
4
```


## Notes

The length of the path is always 1 less than the number of nodes involved in the path since the length measures the number of edges followed.

For digraphs this returns the shortest directed path length. To find path lengths in the reverse direction use G.reverse(copy=False) first to flip the edge orientation.

## See also:

```
all_pairs_shortest_path_length(), all_pairs_dijkstra_path_length(),
single_source_shortest_path_length(), single_source_dijkstra_path_length()
```


### 4.41.4 average_shortest_path_length

average_shortest_path_length (G, weight=None)
Return the average shortest path length.
The average shortest path length is

$$
a=\sum_{s, t \in V} \frac{d(s, t)}{n(n-1)}
$$

where $V$ is the set of nodes in $G, d(s, t)$ is the shortest path from $s$ to $t$, and $n$ is the number of nodes in $G$.

## Parameters

- G (NetworkX graph)
- weight (None or string, optional (default $=$ None)) - If None, every edge has weight/distance/cost 1 . If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1 .


## Raises

- NetworkXPointlessConcept - If G is the null graph (that is, the graph on zero nodes).
- NetworkXError - If $G$ is not connected (or not weakly connected, in the case of a directed graph).


## Examples

```
>>> G = nx.path_graph(5)
>>> nx.average_shortest_path_length(G)
2.0
```

For disconnected graphs, you can compute the average shortest path length for each component

```
>>> G = nx.Graph([(1, 2), (3, 4)])
>>> for C in nx.connected_component_subgraphs(G):
... print(nx.average_shortest_path_length(C))
1.0
1.0
```


### 4.41.5 has_path

has_path ( $G$, source, target)
Return True if $G$ has a path from source to target.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- target (node) - Ending node for path


### 4.41.6 Advanced Interface

Shortest path algorithms for unweighted graphs.

| single_source_shortest_path(G, source[, cut- <br> off] $)$ | Compute shortest path between source and all other nodes <br> reachable from source. |
| :--- | :--- |
| single_source_shortest_path_length(G, <br> source $)$ | Compute the shortest path lengths from source to all reach- <br> able nodes. |
| all_pairs_shortest_path $(\mathbf{G}[$, cutoff $])$ | Compute shortest paths between all nodes. |
| all_pairs_shortest_path_length(G[, cutoff]) | Computes the shortest path lengths between all nodes in G. |
| predecessor(G, source[, target, cutoff, ...]) | Returns dictionary of predecessors for the path from source <br> to all nodes in G. |

## single_source_shortest_path

single_source_shortest_path ( $G$, source, cutoff=None)
Compute shortest path between source and all other nodes reachable from source.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- cutoff (integer, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns lengths - Dictionary, keyed by target, of shortest paths.
Return type dictionary

## Examples

```
>>> G=nx.path_graph(5)
>>> path=nx.single_source_shortest_path(G,0)
>>> path[4]
[0, 1, 2, 3, 4]
```


## Notes

The shortest path is not necessarily unique. So there can be multiple paths between the source and each target node, all of which have the same 'shortest' length. For each target node, this function returns only one of those paths.

## See also:

```
shortest_path()
```

single_source_shortest_path_length
single_source_shortest_path_length (G, source, cutoff=None)
Compute the shortest path lengths from source to all reachable nodes.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- cutoff (integer, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns lengths - (target, shortest path length) iterator
Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = nx.single_source_shortest_path_length(G, 0)
>>> dict(length)
{0: 0, 1: 1, 2: 2, 3: 3, 4: 4}
```


## See also:

```
shortest_path_length()
```

all_pairs_shortest_path
all_pairs_shortest_path (G, cutoff=None)
Compute shortest paths between all nodes.

## Parameters

- G (NetworkX graph)
- cutoff (integer, optional) - Depth at which to stop the search. Only paths of length at most cutoff are returned.

Returns lengths - Dictionary, keyed by source and target, of shortest paths.
Return type dictionary

## Examples

```
>>> G = nx.path_graph(5)
```

>>> path = nx.all_pairs_shortest_path (G)
>>> print(path[0][4])
$[0,1,2,3,4]$

## See also:

floyd_warshall()
all_pairs_shortest_path_length
all_pairs_shortest_path_length ( $G$, cutoff=None)
Computes the shortest path lengths between all nodes in G.

## Parameters

- G (NetworkX graph)
- cutoff (integer, optional) - Depth at which to stop the search. Only paths of length at most cutoff are returned.

Returns lengths - (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

Return type iterator

## Notes

The iterator returned only has reachable node pairs.

## Examples

```
>>> G = nx.path_graph(5)
>>> length = nx.all_pairs_shortest_path_length(G)
>>> dict(length) [1]
{0: 1, 1: 0, 2: 1, 3: 2, 4: 3}
```


## predecessor

predecessor ( $G$, source, target=None, cutoff=None, return_seen=None)
Returns dictionary of predecessors for the path from source to all nodes in G.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- target (node label, optional) - Ending node for path. If provided only predecessors between source and target are returned
- cutoff (integer, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns pred - Dictionary, keyed by node, of predecessors in the shortest path.
Return type dictionary

## Examples

```
>>> G = nx.path_graph(4)
>>> list(G)
[0, 1, 2, 3]
>>> nx.predecessor(G, 0)
{0: [], 1: [0], 2: [1], 3: [2]}
```

Shortest path algorithms for weighed graphs.
dijkstra_predecessor_and_distance(G, Compute weighted shortest path length and predecessors.
source)

Table 4.104 - continued from previous page

| dijkstra_path(G, source, target[, weight]) | Returns the shortest weighted path from source to target in G. |
| :---: | :---: |
| dijkstra_path_length(G, source, target[, weight]) | Returns the shortest weighted path length in G from source to target. |
| single_source_dijkstra(G, source[, target, ...]) | Find shortest weighted paths and lengths from a source node. |
| single_source_dijkstra_path(G, source[, | Find shortest weighted paths in G from a source node. |
| single_source_dijkstra_path_length(G, source) | Find shortest weighted path lengths in G from a source node. |
| multi_source_dijkstra_path(G, sources[, ...]) | Find shortest weighted paths in G from a given set of source nodes. |
| multi_source_dijkstra_path_length(G, sources) | Find shortest weighted path lengths in G from a given set of source nodes. |
| all_pairs_dijkstra_path(G[, cutoff, weight]) | Compute shortest paths between all nodes in a weighted graph. |
| all_pairs_dijkstra_path_length(G[, cutoff, ...]) | Compute shortest path lengths between all nodes in a weighted graph. |
| bidirectional_dijkstra(G, source, target[, ..]) | Dijkstra's algorithm for shortest paths using bidirectional search. |
| bellman_ford_path(G, | Returns the shortest path from source to target in a weighted graph G. |
| be | Returns the shortest path length from source to target in a weighted graph. |
| $\qquad$ | Compute shortest path between source and all other reachable nodes for a weighted graph. |
| single_source_bellman_ford_path_length(G, source) | Compute the shortest path length between source and all other reachable nodes for a weighted graph. |
| all_pairs_bellman_ford_path(G[, cutoff, weight]) | Compute shortest paths between all nodes in a weighted graph. |
| ```all_pairs_bellman_ford_path_length(G[, ...])``` | Compute shortest path lengths between all nodes in a weighted graph. |
| single_source_bellman_ford(G, source[, ...]) | Compute shortest paths and lengths in a weighted graph G. |
| bellman_ford_predecessor_and_distance(G, source) | Compute shortest path lengths and predecessors on shortest paths in weighted graphs. |
| negative_edge_cycle(G[, weight]) | Return True if there exists a negative edge cycle anywhere in G. |
| johnson(G[, weight]) | Uses Johnson's Algorithm to compute shortest paths. |

## dijkstra_predecessor_and_distance

dijkstra_predecessor_and_distance (G, source, cutoff=None, weight='weight')
Compute weighted shortest path length and predecessors.
Uses Dijkstra's Method to obtain the shortest weighted paths and return dictionaries of predecessors for each node and distance for each node from the source.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns pred, distance - Returns two dictionaries representing a list of predecessors of a node and the distance to each node.

Return type dictionaries

## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The list of predecessors contains more than one element only when there are more than one shortest paths to the key node.

## dijkstra_path

dijkstra_path ( $G$, source, target, weight='weight')
Returns the shortest weighted path from source to target in G.
Uses Dijkstra's Method to compute the shortest weighted path between two nodes in a graph.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node
- target (node) - Ending node
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns path - List of nodes in a shortest path.
Return type list
Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G=nx.path_graph(5)
>>> print(nx.dijkstra_path(G,0,4))
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if d['color']=="red" else None will find the shortest red path.

The weight function can be used to include node weights. " def func(u, v, d):
return G.node[u].get('node_weight', 1 ) $2+$ G.node[v].get('node_weight', 1 ) $/ 2+\operatorname{d.get}($ 'weight', 1 )
"' In this example we take the average of start and end node weights of an edge and add it to the weight of the edge.
See also:
bidirectional_dijkstra(), bellman_ford_path()

## dijkstra_path_length

dijkstra_path_length ( $G$, source, target, weight='weight')
Returns the shortest weighted path length in G from source to target.
Uses Dijkstra's Method to compute the shortest weighted path length between two nodes in a graph.

## Parameters

- G (NetworkX graph)
- source (node label) - starting node for path
- target (node label) - ending node for path
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns length - Shortest path length.
Return type number
Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G=nx.path_graph(5)
>>> print(nx.dijkstra_path_length(G,0,4))
4
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if d['color']=="red" else None will find the shortest red path.

## See also:

bidirectional_dijkstra(), bellman_ford_path_length()

## single_source_dijkstra

single_source_dijkstra ( $G$, source, target=None, cutoff=None, weight='weight')
Find shortest weighted paths and lengths from a source node.
Compute the shortest path length between source and all other reachable nodes for a weighted graph.
Uses Dijkstra's algorithm to compute shortest paths and lengths between a source and all other reachable nodes in a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- target (node label, optional) - Ending node for path
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge [u][v] [weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns distance,path - Returns a tuple of two dictionaries keyed by node. The first dictionary stores distance from the source. The second stores the path from the source to that node.

Return type dictionaries

Examples

```
>>> G=nx.path_graph(5)
>>> length,path=nx.single_source_dijkstra(G,0)
>>> print(length[4])
4
>>> print(length)
{0: 0, 1: 1, 2: 2, 3: 3, 4: 4}
>>> path[4]
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if d['color']=="red" else None will find the shortest red path.
Based on the Python cookbook recipe (119466) at http://aspn.activestate.com/ASPN/Cookbook/Python/Recipe/ 119466

This algorithm is not guaranteed to work if edge weights are negative or are floating point numbers (overflows and roundoff errors can cause problems).

## See also:

```
single_source_dijkstra_path(), single_source_dijkstra_path_length(),
single_source_bellman_ford()
```

```
single_source_dijkstra_path
```

single_source_dijkstra_path (G, source, cutoff=None, weight='weight')

Find shortest weighted paths in G from a source node.
Compute shortest path between source and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path.
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length $<=$ cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge [u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns paths - Dictionary of shortest path lengths keyed by target.
Return type dictionary

## Examples

```
>>> G=nx.path_graph(5)
>>> path=nx.single_source_dijkstra_path(G,0)
>>> path[4]
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if d['color']=="red" else None will find the shortest red path.

## See also:

single_source_dijkstra(), single_source_bellman_ford()

## single_source_dijkstra_path_length

single_source_dijkstra_path_length (G, source, cutoff=None, weight='weight')
Find shortest weighted path lengths in G from a source node.
Compute the shortest path length between source and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns length - (target, shortest path length) iterator
Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = dict(nx.single_source_dijkstra_path_length(G, 0))
>>> length[4]
4
>>> print(length)
{0: 0, 1: 1, 2: 2, 3: 3, 4: 4}
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if d['color']=="red" else None will find the shortest red path.

## See also:

```
single_source_dijkstra(), single_source_bellman_ford_path_length()
```

```
multi_source_dijkstra_path
```

multi_source_dijkstra_path (G, sources, cutoff=None, weight='weight')

Find shortest weighted paths in G from a given set of source nodes.
Compute shortest path between any of the source nodes and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- sources (non-empty set of nodes) - Starting nodes for paths. If this is just a set containing a single node, then all paths computed by this function will start from that node. If there are two or more nodes in the set, the computed paths may begin from any one of the start nodes.
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns paths - Dictionary of shortest paths keyed by target.
Return type dictionary

## Examples

```
>>> G = nx.path_graph(5)
>>> path = nx.multi__source_dijkstra_path(G, {0, 4})
>>> path[1]
[0, 1]
>>> path[3]
[4, 3]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if $\mathrm{d}[$ 'color']=="red" else None will find the shortest red path.

Raises ValueError-If sources is empty.

## See also:

```
multi__source_dijkstra(), multi__source_bellman_ford()
```

```
multi_source_dijkstra_path_length
```

multi_source_dijkstra_path_length (G, sources, cutoff=None, weight='weight')

Find shortest weighted path lengths in $G$ from a given set of source nodes.
Compute the shortest path length between any of the source nodes and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- sources (non-empty set of nodes) - Starting nodes for paths. If this is just a set containing a single node, then all paths computed by this function will start from that node. If there are two or more nodes in the set, the computed paths may begin from any one of the start nodes.
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns length - (target, shortest path length) iterator
Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = dict(nx.multi_source_dijkstra_path_length(G, {0, 4}))
>>> length
{0: 0, 1: 1, 2: 2, 3: 1, 4: 0}
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight $=$ lambda $u, v, d$ : 1 if $\mathrm{d}[$ 'color']=="red" else None will find the shortest red path.

Raises ValueError-If sources is empty.

## See also:

multi_source_dijkstra()
all_pairs_dijkstra_path
all_pairs_dijkstra_path (G, cutoff=None, weight='weight')
Compute shortest paths between all nodes in a weighted graph.

## Parameters

- G (NetworkX graph)
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge [u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Returns distance - Dictionary, keyed by source and target, of shortest paths.
Return type dictionary

## Examples

```
>>> G=nx.path_graph(5)
>>> path=nx.all_pairs_dijkstra_path(G)
>>> print(path[0][4])
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

floyd_warshall(), all_pairs_bellman_ford_path()
all_pairs_dijkstra_path_length
all_pairs_dijkstra_path_length (G, cutoff=None, weight='weight')
Compute shortest path lengths between all nodes in a weighted graph.

## Parameters

- G (NetworkX graph)
- cutoff (integer or float, optional) - Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge [u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns distance - (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = dict(nx.all_pairs_dijkstra_path_length(G))
>>> length[1][4]
3
```

```
>>> length[1]
{0: 1, 1: 0, 2: 1, 3: 2, 4: 3}
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The dictionary returned only has keys for reachable node pairs.

## bidirectional_dijkstra

bidirectional_dijkstra (G, source, target, weight='weight')
Dijkstra's algorithm for shortest paths using bidirectional search.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node.
- target (node) - Ending node.
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

## Returns

- length (number) - Shortest path length.
- Returns a tuple of two dictionaries keyed by node.
- The first dictionary stores distance from the source.
- The second stores the path from the source to that node.

Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G=nx.path_graph(5)
>>> length,path=nx.bidirectional_dijkstra(G,0,4)
>>> print(length)
4
>>> print(path)
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
In practice bidirectional Dijkstra is much more than twice as fast as ordinary Dijkstra.
Ordinary Dijkstra expands nodes in a sphere-like manner from the source. The radius of this sphere will eventually be the length of the shortest path. Bidirectional Dijkstra will expand nodes from both the source and the target, making two spheres of half this radius. Volume of the first sphere is $p i \star r * r$ while the others are $2 \star p i * r / 2 * r / 2$, making up half the volume.
This algorithm is not guaranteed to work if edge weights are negative or are floating point numbers (overflows and roundoff errors can cause problems).

## See also:

```
shortest_path(),shortest_path_length()
```


## bellman_ford_path

bellman_ford_path (G, source, target, weight='weight')
Returns the shortest path from source to target in a weighted graph G.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node
- target (node) - Ending node
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight

Returns path - List of nodes in a shortest path.
Return type list
Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G=nx.path_graph(5)
>>> print(nx.bellman_ford_path(G,0,4))
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

```
dijkstra_path(),bellman_ford_path_length()
```


## bellman_ford_path_length

bellman_ford_path_length (G, source, target, weight='weight')
Returns the shortest path length from source to target in a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node label) - starting node for path
- target (node label) - ending node for path
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight

Returns length - Shortest path length.
Return type number
Raises NetworkXNoPath - If no path exists between source and target.

Examples

```
>>> G=nx.path_graph(5)
>>> print(nx.bellman_ford_path_length(G,0,4))
4
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

dijkstra_path_length(), bellman_ford_path()
single_source_bellman_ford_path
single_source_bellman_ford_path (G, source, cutoff=None, weight='weight')
Compute shortest path between source and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path.
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight
- cutoff (integer or float, optional) - Depth to stop the search. Only paths of length <= cutoff are returned.
Returns paths - Dictionary of shortest path lengths keyed by target.
Return type dictionary


## Examples

```
>>> G=nx.path_graph(5)
>>> path=nx.single_source_bellman_ford_path(G,0)
>>> path[4]
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

```
single_source_dijkstra(), single_source_bellman_ford()
```

single_source_bellman_ford_path_length
single_source_bellman_ford_path_length ( $G$, source, cutoff=None, weight='weight')
Compute the shortest path length between source and all other reachable nodes for a weighted graph.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight.
- cutoff (integer or float, optional) - Depth to stop the search. Only paths of length <= cutoff are returned.

Returns length - (target, shortest path length) iterator
Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = dict(nx.single_source_bellman_ford_path_length(G, 0))
>>> length[4]
4
>>> print(length)
{0: 0, 1: 1, 2: 2, 3: 3, 4: 4}
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

```
single_source_dijkstra(), single_source_bellman_ford()
```

all_pairs_bellman_ford_path
all_pairs_bellman_ford_path $(G$, cutoff=None, weight='weight')
Compute shortest paths between all nodes in a weighted graph.

## Parameters

- G (NetworkX graph)
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight
- cutoff (integer or float, optional) - Depth to stop the search. Only paths of length <= cutoff are returned.

Returns distance - Dictionary, keyed by source and target, of shortest paths.
Return type dictionary

## Examples

```
>>> G=nx.path_graph(5)
>>> path=nx.all_pairs_bellman_ford_path(G)
>>> print(path[0][4])
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

floyd_warshall(), all_pairs_dijkstra_path()
all_pairs_bellman_ford_path_length
all_pairs_bellman_ford_path_length (G, cutoff=None, weight='weight')
Compute shortest path lengths between all nodes in a weighted graph.

## Parameters

- G (NetworkX graph)
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight
- cutoff (integer or float, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns distance - (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

Return type iterator

## Examples

```
>>> G = nx.path_graph(5)
>>> length = dict(nx.all_pairs_bellman_ford_path_length(G))
>>> length[1][4]
3
>>> length[1]
{0: 1, 1: 0, 2: 1, 3: 2, 4: 3}
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The dictionary returned only has keys for reachable node pairs.

```
single_source_bellman_ford
```

single_source_bellman_ford (G, source, target=None, cutoff=None, weight='weight')

Compute shortest paths and lengths in a weighted graph G.
Uses Bellman-Ford algorithm for shortest paths.

## Parameters

- G (NetworkX graph)
- source (node label) - Starting node for path
- target (node label, optional) - Ending node for path
- cutoff (integer or float, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns distance,path - Returns a tuple of two dictionaries keyed by node. The first dictionary stores distance from the source. The second stores the path from the source to that node.

Return type dictionaries

## Examples

```
>>> G=nx.path_graph(5)
>>> length,path=nx.single_source_bellman_ford(G,0)
>>> print(length[4])
4
>>> print(length)
{0: 0, 1: 1, 2: 2, 3: 3, 4: 4}
>>> path[4]
[0, 1, 2, 3, 4]
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

## See also:

```
single_source_dijkstra(), single_source_bellman_ford_path(),
single_source_bellman_ford_path_length()
```


## bellman_ford_predecessor_and_distance

bellman_ford_predecessor_and_distance ( $G$, source, target=None, cutoff=None, weight='weight')
Compute shortest path lengths and predecessors on shortest paths in weighted graphs.
The algorithm has a running time of $\mathrm{O}(\mathrm{mn})$ where n is the number of nodes and m is the number of edges. It is slower than Dijkstra but can handle negative edge weights.

## Parameters

- G (NetworkX graph) - The algorithm works for all types of graphs, including directed graphs and multigraphs.
- source (node label) - Starting node for path
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge [u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns pred, dist - Returns two dictionaries keyed by node to predecessor in the path and to the distance from the source respectively.

## Return type dictionaries

Raises NetworkXUnbounded - If the (di)graph contains a negative cost (di)cycle, the algorithm raises an exception to indicate the presence of the negative cost (di)cycle. Note: any negative weight edge in an undirected graph is a negative cost cycle.

## Examples

```
>>> import networkx as nx
>>> G = nx.path_graph(5, create_using = nx.DiGraph())
>>> pred, dist = nx.bellman_ford_predecessor_and_distance(G, 0)
>>> sorted(pred.items())
[(0, [None]), (1, [0]), (2, [1]), (3, [2]), (4, [3])]
>>> sorted(dist.items())
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)]
```

```
>>> from nose.tools import assert_raises
>>> G = nx.cycle_graph(5, create_using = nx.DiGraph())
>>> G[1][2]['weight'] = -7
>>> assert_raises(nx.NetworkXUnbounded, nx.bellman_ford_
@predecessor_and_distance, G, 0)
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The dictionaries returned only have keys for nodes reachable from the source.
In the case where the (di)graph is not connected, if a component not containing the source contains a negative cost (di)cycle, it will not be detected.

```
negative_edge_cycle
```

negative_edge_cycle ( $G$, weight='weight')

Return True if there exists a negative edge cycle anywhere in G.

## Parameters

- G (NetworkX graph)
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns negative_cycle - True if a negative edge cycle exists, otherwise False.
Return type bool

Examples

```
>>> import networkx as nx
>>> G = nx.cycle_graph(5, create_using = nx.DiGraph())
>>> print(nx.negative_edge_cycle(G))
False
>>> G[1][2]['weight'] = -7
>>> print(nx.negative_edge_cycle(G))
True
```


## Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
This algorithm uses bellman_ford_predecessor_and_distance() but finds negative cycles on any component by first adding a new node connected to every node, and starting bellman_ford_predecessor_and_distance on that node. It then removes that extra node.

## johnson

johnson ( $G$, weight='weight')
Uses Johnson's Algorithm to compute shortest paths.
Johnson's Algorithm finds a shortest path between each pair of nodes in a weighted graph even if negative weights are present.

## Parameters

- G (NetworkX graph)
- weight (string or function) - If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining $u$ to $v$ will be G.edge[u][v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns distance - Dictionary, keyed by source and target, of shortest paths.
Return type dictionary
Raises NetworkXError - If given graph is not weighted.

## Examples

```
>>> import networkx as nx
>>> graph = nx.DiGraph()
>>> graph.add_weighted_edges_from([('0', '3', 3), ('0', '1', -5),
... ('0', '2', 2), ('1', '2', 4), ('2', '3', 1)])
>>> paths = nx.johnson(graph, weight='weight')
>>> paths['0']['2']
['0', '1', '2']
```


## Notes

Johnson's algorithm is suitable even for graphs with negative weights. It works by using the Bellman-Ford algorithm to compute a transformation of the input graph that removes all negative weights, allowing Dijkstra's algorithm to be used on the transformed graph.

The time complexity of this algorithm is $O\left(n^{\wedge} 2 \log n+n m\right)$, where $n$ is the number of nodes and $m$ the number of edges in the graph. For dense graphs, this may be faster than the Floyd-Warshall algorithm.

## See also:

```
floyd_warshall_predecessor_and_distance(), floyd_warshall_numpy(),
all_pairs_shortest_path(), all_pairs_shortest_path_length(),
all_pairs_dijkstra_path(), bellman_ford_predecessor_and_distance(),
all_pairs_bellman_ford_path(),all_pairs_bellman_ford_path_length()
```


### 4.41.7 Dense Graphs

Floyd-Warshall algorithm for shortest paths.

| floyd_warshall(G[, weight]) | Find all-pairs shortest path lengths using Floyd's algorithm. |
| :--- | :---: |
| floyd_warshall_predecessor_and_distance(G[Find all-pairs shortest path lengths using Floyd's algorithm.  <br> $\ldots .])$.  <br> floyd_warshall_numpy(G[, nodelist, weight]) Find all-pairs shortest path lengths using Floyd's algorithm. |  |

floyd_warshall
floyd_warshall ( $G$, weight='weight')
Find all-pairs shortest path lengths using Floyd's algorithm.

## Parameters

- G (NetworkX graph)
- weight (string, optional (default= 'weight')) - Edge data key corresponding to the edge weight.

Returns distance - A dictionary, keyed by source and target, of shortest paths distances between nodes.

## Return type dict

## Notes

Floyd's algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra’s algorithm fails. This algorithm can still fail if there are negative cycles. It has running time $\mathrm{O}\left(\mathrm{n}^{\wedge} 3\right)$ with running space of $\mathrm{O}\left(\mathrm{n}^{\wedge} 2\right)$.

## See also:

```
floyd_warshall_predecessor_and_distance(), floyd_warshall_numpy(),
all_pairs_shortest_path(), all_pairs_shortest_path_length()
```

floyd_warshall_predecessor_and_distance
floyd_warshall_predecessor_and_distance ( $G$, weight='weight')
Find all-pairs shortest path lengths using Floyd's algorithm.

## Parameters

- G (NetworkX graph)
- weight (string, optional (default $=$ 'weight') $)$ - Edge data key corresponding to the edge weight.

Returns predecessor,distance - Dictionaries, keyed by source and target, of predecessors and distances in the shortest path.

Return type dictionaries

## Notes

Floyd's algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra's algorithm fails. This algorithm can still fail if there are negative cycles. It has running time $\mathrm{O}\left(\mathrm{n}^{\wedge} 3\right)$ with running space of $O\left(n^{\wedge} 2\right)$.

## See also:

```
floyd_warshall(), floyd_warshall_numpy(), all_pairs_shortest_path(),
all_pairs_shortest_path_length()
```


## floyd_warshall_numpy

floyd_warshall_numpy ( $G$, nodelist=None, weight='weight')
Find all-pairs shortest path lengths using Floyd's algorithm.

## Parameters

- G (NetworkX graph)
- nodelist (list, optional) - The rows and columns are ordered by the nodes in nodelist. If nodelist is None then the ordering is produced by G.nodes().
- weight (string, optional (default= 'weight')) - Edge data key corresponding to the edge weight.

Returns distance - A matrix of shortest path distances between nodes. If there is no path between to nodes the corresponding matrix entry will be Inf.
Return type NumPy matrix

## Notes

Floyd's algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra's algorithm fails. This algorithm can still fail if there are negative cycles. It has running time $\mathrm{O}\left(\mathrm{n}^{\wedge} 3\right)$ with running space of $\mathrm{O}\left(\mathrm{n}^{\wedge} 2\right)$.

### 4.41.8 A* Algorithm

Shortest paths and path lengths using the A* ("A star") algorithm.

| astar_path $(\mathbf{G}$, source, target[, heuristic,...$])$ | Return a list of nodes in a shortest path between source and <br> target using the A* ("A-star") algorithm. |
| :--- | :--- |
| astar_path_length $(\mathbf{G}$, source, target[, ...]) | Return the length of the shortest path between source and <br> target using the A* ("A-star") algorithm. |

astar_path
astar_path (G, source, target, heuristic=None, weight='weight')
Return a list of nodes in a shortest path between source and target using the A* ("A-star") algorithm.
There may be more than one shortest path. This returns only one.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- target (node) - Ending node for path
- heuristic (function) - A function to evaluate the estimate of the distance from the a node to the target. The function takes two nodes arguments and must return a number.
- weight (string, optional (default='weight')) - Edge data key corresponding to the edge weight.

Raises NetworkXNoPath - If no path exists between source and target.

## Examples

```
>>> G=nx.path_graph(5)
>>> print(nx.astar_path(G,0,4))
[0, 1, 2, 3, 4]
>>> G=nx.grid_graph(dim=[3,3]) # nodes are two-tuples (x,y)
>>> def dist(a, b):
... (x1, y1) = a
... (x2, y2) = b
... return ((x1 - x2) ** 2 + (y1 - y2) ** 2) ** 0.5
>>> print(nx.astar_path(G, (0,0), (2,2),dist))
[(0, 0), (0, 1), (1, 1), (1, 2), (2, 2)]
```


## See also:

```
shortest_path(), dijkstra_path()
```

```
astar_path_length
```

astar_path_length ( $G$, source, target, heuristic=None, weight='weight')
Return the length of the shortest path between source and target using the A* ("A-star") algorithm.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- target (node) - Ending node for path
- heuristic (function) - A function to evaluate the estimate of the distance from the a node to the target. The function takes two nodes arguments and must return a number.

Raises NetworkXNoPath - If no path exists between source and target.

## See also:

```
astar_path()
```


### 4.42 Simple Paths

| all_simple_paths(G, source, target[, cutoff]) | Generate all simple paths in the graph G from source to <br> target. |
| :--- | :--- |
| is_simple_path(G, nodes $)$ | Returns True if and only if the given nodes form a simple <br> path in G. |
| shortest_simple_paths(G, source, $\operatorname{target[,\ldots ])}$ | Generate all simple paths in the graph G from source to <br> target, starting from shortest ones. |

### 4.42.1 all_simple_paths

all_simple_paths ( $G$, source, target, cutoff=None)
Generate all simple paths in the graph G from source to target.
A simple path is a path with no repeated nodes.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- target (node) - Ending node for path
- cutoff (integer, optional) - Depth to stop the search. Only paths of length $<=$ cutoff are returned.

Returns path_generator - A generator that produces lists of simple paths. If there are no paths between the source and target within the given cutoff the generator produces no output.

## Return type generator

## Examples

This iterator generates lists of nodes:

```
>>>G G nx.complete_graph(4)
>>> for path in nx.all_simple_paths(G, source=0, target=3):
... print(path)
...
[0, 1, 2, 3]
[0, 1, 3]
[0, 2, 1, 3]
[0, 2, 3]
[0, 3]
```

You can generate only those paths that are shorter than a certain length by using the cutof $f$ keyword argument:

```
>>> paths = nx.all_simple_paths(G, source=0, target=3, cutoff=2)
>>> print(list(paths))
[[0, 1, 3], [0, 2, 3], [0, 3]]
```

To get each path as the corresponding list of edges, you can use the networkx.utils.pairwise () helper function:

```
>>> paths = nx.all_simple_paths(G, source=0, target=3)
>>> for path in map(nx.utils.pairwise, paths):
... print(list(path))
[(0, 1), (1, 2), (2, 3)]
[(0, 1), (1, 3)]
[(0, 2), (2, 1), (1, 3)]
[(0, 2), (2, 3)]
[(0, 3)]
```


## Notes

This algorithm uses a modified depth-first search to generate the paths ${ }^{1}$. A single path can be found in $\mathrm{O}(\mathrm{V}+\mathrm{E})$ time but the number of simple paths in a graph can be very large, e.g. $O(n!)$ in the complete graph of order $n$.

## References

## See also:

[^88]```
all_shortest_paths(), shortest_path()
```


### 4.42.2 is_simple_path

## is_simple_path (G, nodes)

Returns True if and only if the given nodes form a simple path in G.
A simple path in a graph is a nonempty sequence of nodes in which no node appears more than once in the sequence, and each adjacent pair of nodes in the sequence is adjacent in the graph.

Parameters nodes (list) - A list of one or more nodes in the graph G.
Returns Whether the given list of nodes represents a simple path in $G$.
Return type bool

## Notes

A list of zero nodes is not a path and a list of one node is a path. Here's an explanation why.
This function operates on node paths. One could also consider edge paths. There is a bijection between node paths and edge paths.

The length of a path is the number of edges in the path, so a list of nodes of length $n$ corresponds to a path of length $n-1$. Thus the smallest edge path would be a list of zero edges, the empty path. This corresponds to a list of one node.

To convert between a node path and an edge path, you can use code like the following:

```
>>> from networkx.utils import pairwise
>>> nodes = [0, 1, 2, 3]
>>> edges = list(pairwise(nodes))
>>> edges
[(0, 1), (1, 2), (2, 3)]
>>> nodes = [edges[0][0]] + [v for u, v in edges]
>>> nodes
[0, 1, 2, 3]
```


## Examples

```
>>> G = nx.cycle_graph(4)
>>> nx.is_simple_path(G, [2, 3, 0])
True
>>> nx.is_simple_path(G, [0, 2])
False
```


### 4.42.3 shortest_simple_paths

shortest_simple_paths ( $G$, source, target, weight=None)
Generate all simple paths in the graph G from source to target, starting from shortest ones.
A simple path is a path with no repeated nodes.
If a weighted shortest path search is to be used, no negative weights are allawed.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for path
- target (node) - Ending node for path
- weight (string) - Name of the edge attribute to be used as a weight. If None all edges are considered to have unit weight. Default value None.

Returns path_generator - A generator that produces lists of simple paths, in order from shortest to longest.

## Return type generator

## Raises

- NetworkXNoPath - If no path exists between source and target.
- NetworkXError - If source or target nodes are not in the input graph.
- NetworkXNot Implemented - If the input graph is a Multi[Di]Graph.


## Examples

```
>>> G = nx.cycle_graph(7)
>>> paths = list(nx.shortest_simple_paths(G, 0, 3))
>>> print(paths)
[[0, 1, 2, 3], [0, 6, 5, 4, 3]]
```

You can use this function to efficiently compute the k shortest/best paths between two nodes.

```
>>> from itertools import islice
>>> def k_shortest_paths(G, source, target, k, weight=None):
... return list(islice(nx.shortest_simple_paths(G, source, target,r
\hookrightarrowweight=weight), k))
>>> for path in k_shortest_paths(G, 0, 3, 2):
... print (path)
[0, 1, 2, 3]
[0, 6, 5, 4, 3]
```


## Notes

This procedure is based on algorithm by Jin Y. Yen ${ }^{1}$. Finding the first $K$ paths requires $\mathrm{O}\left(\mathrm{KN}^{\wedge} 3\right)$ operations. See also:

```
all_shortest_paths(),shortest_path(),all_simple_paths()
```


## References

### 4.43 Swap

Swap edges in a graph.

[^89]double_edge_swap(G[, nswap, max_tries])
connected_double_edge_swap(G[, nswap, ...])

Swap two edges in the graph while keeping the node degrees fixed.
Attempts the specified number of double-edge swaps in the graph G.

### 4.43.1 double_edge_swap

double_edge_swap ( $G$, nswap=1, max_tries=100)
Swap two edges in the graph while keeping the node degrees fixed.
A double-edge swap removes two randomly chosen edges $u-v$ and $x-y$ and creates the new edges $u-x$ and $v-y$ :

| $\mathrm{u}--\mathrm{v}$ | u | v |
| :--- | :--- | :--- |
| $\mathrm{x}--\mathrm{y}$ | becomes | $\mid$ |
| $\mid$ |  |  |
| x | y |  |

If either the edge $u-x$ or $v-y$ already exist no swap is performed and another attempt is made to find a suitable edge pair.

## Parameters

- G (graph) - An undirected graph
- nswap (integer (optional, default=1)) - Number of double-edge swaps to perform
- max_tries (integer (optional)) - Maximum number of attempts to swap edges

Returns G - The graph after double edge swaps.
Return type graph

## Notes

Does not enforce any connectivity constraints.
The graph $G$ is modified in place.

### 4.43.2 connected_double_edge_swap

connected_double_edge_swap ( $G$, nswap $=1$, _window_threshold $=3$ )
Attempts the specified number of double-edge swaps in the graph G.
A double-edge swap removes two randomly chosen edges $(u, v)$ and $(x, y)$ and creates the new edges $(u, x)$ and $(v, y)$ :

| $u--v$ | becomes | $\mid$ |
| :---: | :---: | :---: |
|  | l | v |
| $x--y$ | $x$ | $y$ |

If either $(u, x)$ or $(v, y)$ already exist, then no swap is performed so the actual number of swapped edges is always at most nswap.

## Parameters

- G (graph) - An undirected graph
- nswap (integer (optional, default=1)) - Number of double-edge swaps to perform
- _window_threshold (integer) - The window size below which connectedness of the graph will be checked after each swap.
The "window" in this function is a dynamically updated integer that represents the number of swap attempts to make before checking if the graph remains connected. It is an optimization used to decrease the running time of the algorithm in exchange for increased complexity of implementation.

If the window size is below this threshold, then the algorithm checks after each swap if the graph remains connected by checking if there is a path joining the two nodes whose edge was just removed. If the window size is above this threshold, then the algorithm performs do all the swaps in the window and only then check if the graph is still connected.
Returns The number of successful swaps
Return type int
Raises NetworkXError - If the input graph is not connected, or if the graph has fewer than four nodes.

## Notes

The initial graph G must be connected, and the resulting graph is connected. The graph G is modified in place.

## References

### 4.44 Tournament

Functions concerning tournament graphs.
A tournament graph is a complete oriented graph. In other words, it is a directed graph in which there is exactly one directed edge joining each pair of distinct nodes. For each function in this module that accepts a graph as input, you must provide a tournament graph. The responsibility is on the caller to ensure that the graph is a tournament graph.

To access the functions in this module, you must access them through the networkx. algorithms.tournament module:

```
>>> import networkx as nx
>>> from networkx.algorithms import tournament
>>> G = nx.DiGraph([(0, 1), (1, 2), (2, 0)])
>>> tournament.is_tournament (G)
True
```

| hamiltonian_path $(\mathrm{G})$ | Returns a Hamiltonian path in the given tournament graph. |
| :--- | :--- |
| is_reachable $(\mathbf{G}, \mathrm{s}, \mathrm{t})$ | Decides whether there is a path from s to t in the tourna- <br> ment. |
| is_strongly_connected $(\mathbf{G})$ | Decides whether the given tournament is strongly con- <br> nected. |
| is_tournament $(\mathrm{G})$ | Returns True if and only if G is a tournament. |
| random_tournament $(\mathrm{n})$ | Returns a random tournament graph on n nodes. |
| score_sequence $(\mathrm{G})$ | Returns the score sequence for the given tournament graph. |

### 4.44.1 hamiltonian_path

## hamiltonian_path ( $G$ )

Returns a Hamiltonian path in the given tournament graph.
Each tournament has a Hamiltonian path. If furthermore, the tournament is strongly connected, then the returned Hamiltonian path is a Hamiltonian cycle (by joining the endpoints of the path).

Parameters G (NetworkX graph) - A directed graph representing a tournament.
Returns Whether the given graph is a tournament graph.
Return type bool

## Notes

This is a recursive implementation with an asymptotic running time of $O\left(n^{\wedge} 2\right)$, ignoring multiplicative polylogarithmic factors, where $n$ is the number of nodes in the graph.

### 4.44.2 is_reachable

is_reachable ( $G, s, t$ )
Decides whether there is a path from $s$ to $t$ in the tournament.
This function is more theoretically efficient than the reachability checks than the shortest path algorithms in networkx.algorithms.shortest_paths.
The given graph must be a tournament, otherwise this function's behavior is undefined.

## Parameters

- G (NetworkX graph) - A directed graph representing a tournament.
- $\mathbf{s}($ node $)$ - A node in the graph.
- $\mathbf{t}$ (node) - A node in the graph.

Returns Whether there is a path from $s$ to $t$ in $G$.
Return type bool

## Notes

Although this function is more theoretically efficient than the generic shortest path functions, a speedup requires the use of parallelism. Though it may in the future, the current implementation does not use parallelism, thus you may not see much of a speedup.
This algorithm comes from [1].

## References

### 4.44.3 is_strongly_connected

```
is_strongly_connected ( \(G\) )
```

Decides whether the given tournament is strongly connected.
This function is more theoretically efficient than the is_strongly_connected() function.

The given graph must be a tournament, otherwise this function's behavior is undefined.
Parameters G (NetworkX graph) - A directed graph representing a tournament.
Returns Whether the tournament is strongly connected.
Return type bool

## Notes

Although this function is more theoretically efficient than the generic strong connectivity function, a speedup requires the use of parallelism. Though it may in the future, the current implementation does not use parallelism, thus you may not see much of a speedup.

This algorithm comes from [1].

## References

### 4.44.4 is_tournament

## is_tournament ( $G$ )

Returns True if and only if G is a tournament.
A tournament is a directed graph, with neither self-loops nor multi-edges, in which there is exactly one directed edge joining each pair of distinct nodes.

Parameters G (NetworkX graph) - A directed graph representing a tournament.
Returns Whether the given graph is a tournament graph.
Return type bool

## Notes

Some definitions require a self-loop on each node, but that is not the convention used here.

### 4.44.5 random_tournament

```
random_tournament ( }n\mathrm{ )
```

Returns a random tournament graph on $n$ nodes.
Parameters $\mathbf{n}$ (int) - The number of nodes in the returned graph.
Returns Whether the given graph is a tournament graph.
Return type bool

## Notes

This algorithm adds, for each pair of distinct nodes, an edge with uniformly random orientation. In other words, binom $\{n\}\{2\}$ flips of an unbiased coin decide the orientations of the edges in the graph.

### 4.44.6 score_sequence

## score_sequence ( $G$ )

Returns the score sequence for the given tournament graph.
The score sequence is the sorted list of the out-degrees of the nodes of the graph.
Parameters G (NetworkX graph) - A directed graph representing a tournament.
Returns A sorted list of the out-degrees of the nodes of G .
Return type list

### 4.45 Traversal

### 4.45.1 Depth First Search

Basic algorithms for depth-first searching the nodes of a graph.
Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.

| dfs_edges(G[, source]) | Produce edges in a depth-first-search (DFS). |
| :---: | :---: |
| dfs_tree(G[, source]) | Return oriented tree constructed from a depth-first-search from source. |
| dfs_predecessors(G[, source]) | Return dictionary of predecessors in depth-first-search from source. |
| dfs_successors(G[, source]) | Return dictionary of successors in depth-first-search from source. |
| dfs_preorder_nodes(G[, source]) | Produce nodes in a depth-first-search pre-ordering starting from source. |
| dfs_postorder_nodes(G[, source]) | Produce nodes in a depth-first-search post-ordering starting from source. |
| dfs_labeled_edges(G[, source]) | Produce edges in a depth-first-search (DFS) labeled by type. |

dfs_edges
dfs_edges $(G$, source $=$ None $)$
Produce edges in a depth-first-search (DFS).

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.

Returns edges - A generator of edges in the depth-first-search.
Return type generator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(list(nx.dfs_edges(G,0)))
[(0, 1), (1, 2)]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.
dfs_tree
dfs tree $(G$, source $=$ None $)$
Return oriented tree constructed from a depth-first-search from source.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search.

Returns $\mathbf{T}$ - An oriented tree
Return type NetworkX DiGraph

## Examples

```
>>> G = nx.path_graph(3)
>>> T = nx.dfs_tree (G,0)
>>> print(list(T.edges()))
[(0, 1), (1, 2)]
```

dfs_predecessors
dfs_predecessors ( $G$, source=None)
Return dictionary of predecessors in depth-first-search from source.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.

Returns pred - A dictionary with nodes as keys and predecessor nodes as values.
Return type dict

## Examples

```
>>> G = nx.path_graph(3)
>>> print(nx.dfs_predecessors(G,0))
{1: 0, 2: 1}
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

## successors

dfs_successors $(G$, source $=$ None $)$
Return dictionary of successors in depth-first-search from source.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.

Returns succ - A dictionary with nodes as keys and list of successor nodes as values.
Return type dict

## Examples

```
>>> G = nx.path_graph(3)
>>> print(nx.dfs_successors(G,0))
{0: [1], 1: [2]}
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.
dfs_preorder_nodes
dfs_preorder_nodes ( $G$, source $=$ None)
Produce nodes in a depth-first-search pre-ordering starting from source.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.

Returns nodes - A generator of nodes in a depth-first-search pre-ordering.
Return type generator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(list(nx.dfs_preorder_nodes(G,0)))
[0, 1, 2]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.
dfs_postorder_nodes
dfs_postorder_nodes ( $G$, source $=$ None )
Produce nodes in a depth-first-search post-ordering starting from source.

## Parameters

- G (NetworkX graph)
- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.

Returns nodes - A generator of nodes in a depth-first-search post-ordering.
Return type generator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(list(nx.dfs_postorder_nodes(G,0)))
[2, 1, 0]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

```
dfs_labeled_edges
```

dfs_labeled_edges ( $G$, source $=$ None )

Produce edges in a depth-first-search (DFS) labeled by type.

## Parameters

## - G (NetworkX graph)

- source (node, optional) - Specify starting node for depth-first search and return edges in the component reachable from source.
Returns edges - A generator of triples of the form $(u, v, d)$, where $(u, v)$ is the edge being explored in the depth-first search and $d$ is one of the strings 'forward', 'nontree', or 'reverse'. A 'forward' edge is one in which $u$ has been visited but $v$ has not. A 'nontree' edge is one in which both $u$ and $v$ have been visited but the edge is not in the DFS tree. A 'reverse' edge is on in which both $u$ and $v$ have been visited and the edge is in the DFS tree.


## Return type generator

## Examples

The labels reveal the complete transcript of the depth-first search algorithm in more detail than, for example,

```
dfs_edges():
```

```
>>> from pprint import pprint
>>>
>>> G = nx.DiGraph([(0, 1), (1, 2), (2, 1)])
>>> pprint(list(nx.dfs_labeled_edges(G, source=0)))
[(0, 0, 'forward'),
    (0, 1, 'forward'),
    (1, 2, 'forward'),
    (2, 1, 'nontree'),
    (1, 2, 'reverse'),
    (0, 1, 'reverse'),
    (0, 0, 'reverse')]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/DFS.py by D. Eppstein, July 2004.
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

### 4.45.2 Breadth First Search

Basic algorithms for breadth-first searching the nodes of a graph.

| $b f s_{-} e d g e s(G$, source[, reverse]) | Produce edges in a breadth-first-search starting at source. |
| :--- | :--- |
| $b £ s_{-} t r e e(G$, source[, reverse]) | Return an oriented tree constructed from of a breadth-first- <br> search starting at source. |
| $b £ s_{-}$predecessors(G, source) | Returns an iterator of predecessors in breadth-first-search <br> from source. |
| bfs_successors(G, source) | Returns an iterator of successors in breadth-first-search <br> from source. |

bfs_edges
bfs_edges $(G$, source, reverse $=$ False $)$
Produce edges in a breadth-first-search starting at source.

## Parameters

- G (NetworkX graph)
- source (node) - Specify starting node for breadth-first search and return edges in the component reachable from source.
- reverse (bool, optional) - If True traverse a directed graph in the reverse direction

Returns edges - A generator of edges in the breadth-first-search.
Return type generator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(list(nx.bfs_edges(G,0)))
[(0, 1), (1, 2)]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/BFS.py by D. Eppstein, July 2004.
bfs_tree
bfs_tree ( $G$, source, reverse $=$ False )
Return an oriented tree constructed from of a breadth-first-search starting at source.

## Parameters

- G (NetworkX graph)
- source (node) - Specify starting node for breadth-first search and return edges in the component reachable from source.
- reverse (bool, optional) - If True traverse a directed graph in the reverse direction

Returns $\mathbf{T}$ - An oriented tree
Return type NetworkX DiGraph

Examples

```
>>> G = nx.path_graph(3)
>>> print(list(nx.bfs_tree(G,1).edges()))
[(1, 0), (1, 2)]
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/BFS.py by D. Eppstein, July 2004.

## bfs_predecessors

bfs_predecessors ( $G$, source)
Returns an iterator of predecessors in breadth-first-search from source.

## Parameters

- G (NetworkX graph)
- source (node) - Specify starting node for breadth-first search and return edges in the component reachable from source.
Returns pred - (node, predecessors) iterator where predecessors is the list of predecessors of the node.

Return type iterator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(dict(nx.bfs_predecessors(G, 0)))
{1: 0, 2: 1}
>>> H = nx.Graph()
>>> H.add_edges_from([(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)])
>>> dict(nx.bfs_predecessors(H, 0))
{1: 0, 2: 0, 3: 1, 4: 1, 5: 2, 6: 2}
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/BFS.py by D. Eppstein, July 2004.
bfs_successors
bfs_successors ( $G$, source)
Returns an iterator of successors in breadth-first-search from source.

## Parameters

- G (NetworkX graph)
- source (node) - Specify starting node for breadth-first search and return edges in the component reachable from source.

Returns succ - (node, successors) iterator where successors is the list of successors of the node.
Return type iterator

## Examples

```
>>> G = nx.path_graph(3)
>>> print(dict(nx.bfs_successors(G,0)))
{0: [1], 1: [2]}
>>> H = nx.Graph()
>>> H.add_edges_from([(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)])
>>> dict(nx.bfs_successors(H, 0))
{0: [1, 2], 1: [3, 4], 2: [5, 6]}
```


## Notes

Based on http://www.ics.uci.edu/~eppstein/PADS/BFS.py by D. Eppstein, July 2004.

### 4.45.3 Beam search

Basic algorithms for breadth-first searching the nodes of a graph.
bfs_beam_edges(G, source, value[, width]) Iterates over edges in a beam search.
bfs_beam_edges
bfs_beam_edges $(G$, source, value, width=None)
Iterates over edges in a beam search.
The beam search is a generalized breadth-first search in which only the "best" $w$ neighbors of the current node are enqueued, where $w$ is the beam width and "best" is an application-specific heuristic. In general, a beam search with a small beam width might not visit each node in the graph.

## Parameters

- G (NetworkX graph)
- source (node) - Starting node for the breadth-first search; this function iterates over only those edges in the component reachable from this node.
- value (function) - A function that takes a node of the graph as input and returns a real number indicating how "good" it is. A higher value means it is more likely to be visited sooner during the search. When visiting a new node, only the width neighbors with the highest value are enqueued (in decreasing order of value).
- width $($ int $($ default $=$ None $))$ - The beam width for the search. This is the number of neighbors (ordered by value) to enqueue when visiting each new node.

Yields edge - Edges in the beam search starting from source, given as a pair of nodes.

## Examples

To give nodes with, for example, a higher centrality precedence during the search, set the value function to return the centrality value of the node:

```
>>> G = nx.karate_club_graph()
>>> centrality = nx.eigenvector_centrality(G)
```

```
>>> source = 0
>>> width = 5
>>> for u, v in nx.bfs_beam_edges(G, source, centrality.get, width):
    print((u, v))
```


### 4.45.4 Depth First Search on Edges

## Depth First Search on Edges

Algorithms for a depth-first traversal of edges in a graph.

| edge_dfs(G[, source, orientation]) | A directed, depth-first traversal of edges in G, beginning at <br> source. |
| :--- | :--- |

```
edge_dfs
```

edge_dfs ( $G$, source $=$ None, orientation='original')

A directed, depth-first traversal of edges in $G$, beginning at source.

## Parameters

- G (graph) - A directed/undirected graph/multigraph.
- source (node, list of nodes) - The node from which the traversal begins. If None, then a source is chosen arbitrarily and repeatedly until all edges from each node in the graph are searched.
- orientation ('original'। 'reverse' | 'ignore') - For directed graphs and directed multigraphs, edge traversals need not respect the original orientation of the edges. When set to 'reverse', then every edge will be traversed in the reverse direction. When set to 'ignore', then each directed edge is treated as a single undirected edge that can be traversed in either direction. For undirected graphs and undirected multigraphs, this parameter is meaningless and is not consulted by the algorithm.

Yields edge (directed edge) - A directed edge indicating the path taken by the depth-first traversal. For graphs, edge is of the form $(u, v)$ where $u$ and $v$ are the tail and head of the edge as determined by the traversal. For multigraphs, edge is of the form ( $u, v, k e y$ ), where key is the key of the edge. When the graph is directed, then $u$ and $v$ are always in the order of the actual directed edge. If orientation is 'reverse' or 'ignore', then edge takes the form ( $u, v, k e y$, direction) where direction is a string, 'forward' or 'reverse', that indicates if the edge was traversed in the forward (tail to head) or reverse (head to tail) direction, respectively.

## Examples

```
>>> import networkx as nx
>>> nodes = [0, 1, 2, 3]
>>> edges = [(0, 1), (1, 0), (1, 0), (2, 1), (3, 1)]
```

```
>>> list(nx.edge_dfs(nx.Graph(edges), nodes))
[(0, 1), (1, 2), (1, 3)]
```

```
>>> list(nx.edge_dfs(nx.DiGraph(edges), nodes))
[(0, 1), (1, 0), (2, 1), (3, 1)]
```

```
>>> list(nx.edge_dfs(nx.MultiGraph(edges), nodes))
[(0, 1, 0), (1, 0, 1), (0, 1, 2), (1, 2, 0), (1, 3, 0)]
```

```
>>> list(nx.edge_dfs(nx.MultiDiGraph(edges), nodes))
[(0, 1, 0), (1, 0, 0), (1, 0, 1), (2, 1, 0), (3, 1, 0)]
```

```
>>> list(nx.edge_dfs(nx.DiGraph(edges), nodes, orientation='ignore'))
[(0, 1, 'forward'), (1, 0, 'forward'), (2, 1, 'reverse'), (3, 1, 'reverse')]
```

```
>>> list(nx.edge_dfs(nx.MultiDiGraph(edges), nodes, orientation='ignore'))
[(0, 1, 0, 'forward'), (1, 0, 0, 'forward'), (1, 0, 1, 'reverse'), (2, 1, 0,
\hookrightarrow'reverse'), (3, 1, 0, 'reverse')]
```


## Notes

The goal of this function is to visit edges. It differs from the more familiar depth-first traversal of nodes, as provided by networkx.algorithms.traversal.depth_first_search.dfs_edges(), in that it does not stop once every node has been visited. In a directed graph with edges $[(0,1),(1,2),(2,1)]$, the edge $(2,1)$ would not be visited if not for the functionality provided by this function.

## See also:

```
dfs_edges()
```


### 4.46 Tree

### 4.46.1 Recognition

## Recognition Tests

A forest is an acyclic, undirected graph, and a tree is a connected forest. Depending on the subfield, there are various conventions for generalizing these definitions to directed graphs.

In one convention, directed variants of forest and tree are defined in an identical manner, except that the direction of the edges is ignored. In effect, each directed edge is treated as a single undirected edge. Then, additional restrictions are imposed to define branchings and arborescences.

In another convention, directed variants of forest and tree correspond to the previous convention's branchings and arborescences, respectively. Then two new terms, polyforest and polytree, are defined to correspond to the other convention's forest and tree.

Summarizing:

```
+-------------------------------
| Convention A | Convention B |
+==============================+
forest | polyforest
tree | polytree |
branching | forest |
```

```
arborescence | tree |
+--------------------------------
```

Each convention has its reasons. The first convention emphasizes definitional similarity in that directed forests and trees are only concerned with acyclicity and do not have an in-degree constraint, just as their undirected counterparts do not. The second convention emphasizes functional similarity in the sense that the directed analog of a spanning tree is a spanning arborescence. That is, take any spanning tree and choose one node as the root. Then every edge is assigned a direction such there is a directed path from the root to every other node. The result is a spanning arborescence.

NetworkX follows convention "A". Explicitly, these are:
undirected forest An undirected graph with no undirected cycles.
undirected tree A connected, undirected forest.
directed forest A directed graph with no undirected cycles. Equivalently, the underlying graph structure (which ignores edge orientations) is an undirected forest. In convention $B$, this is known as a polyforest.
directed tree A weakly connected, directed forest. Equivalently, the underlying graph structure (which ignores edge orientations) is an undirected tree. In convention B, this is known as a polytree.
branching A directed forest with each node having, at most, one parent. So the maximum in-degree is equal to 1 . In convention B , this is known as a forest.
arborescence A directed tree with each node having, at most, one parent. So the maximum in-degree is equal to 1 . In convention B, this is known as a tree.

For trees and arborescences, the adjective "spanning" may be added to designate that the graph, when considered as a forest/branching, consists of a single tree/arborescence that includes all nodes in the graph. It is true, by definition, that every tree/arborescence is spanning with respect to the nodes that define the tree/arborescence and so, it might seem redundant to introduce the notion of "spanning". However, the nodes may represent a subset of nodes from a larger graph, and it is in this context that the term "spanning" becomes a useful notion.

| is_tree $(\mathbf{G})$ | Returns True if G is a tree. |
| :--- | :--- |
| is_forest $(\mathrm{G})$ | Returns True if G is a forest. |
| is_arborescence $(\mathrm{G})$ | Returns True if G is an arborescence. |
| is_branching $(\mathrm{G})$ | Returns True if G is a branching. |

## is_tree

is_tree ( $G$ )
Returns True if G is a tree.
A tree is a connected graph with no undirected cycles.
For directed graphs, G is a tree if the underlying graph is a tree. The underlying graph is obtained by treating each directed edge as a single undirected edge in a multigraph.

Parameters G (graph) - The graph to test.
Returns $\mathbf{b}$ - A boolean that is True if G is a tree.
Return type bool

## Notes

In another convention, a directed tree is known as a polytree and then tree corresponds to an arborescence.

## See also:

```
is_arborescence()
```

is_forest
is_forest ( $G$ )
Returns True if G is a forest.
A forest is a graph with no undirected cycles.
For directed graphs, G is a forest if the underlying graph is a forest. The underlying graph is obtained by treating each directed edge as a single undirected edge in a multigraph.

Parameters G (graph) - The graph to test.
Returns b-A boolean that is True if G is a forest.
Return type bool

## Notes

In another convention, a directed forest is known as a polyforest and then forest corresponds to a branching.

## See also:

is_branching()

```
is_arborescence
```

```
is_arborescence (G)
```

Returns True if G is an arborescence.
An arborescence is a directed tree with maximum in-degree equal to 1 .
Parameters G (graph) - The graph to test.
Returns $\mathbf{b}$ - A boolean that is True if G is an arborescence.
Return type bool

## Notes

In another convention, an arborescence is known as a tree.

## See also:

```
is_tree()
```

is_branching
is_branching ( $G$ )
Returns True if G is a branching.
A branching is a directed forest with maximum in-degree equal to 1.
Parameters G (directed graph) - The directed graph to test.

Returns $\mathbf{b}$ - A boolean that is True if G is a branching.
Return type bool

## Notes

In another convention, a branching is also known as a forest.
See also:

```
is_forest()
```


### 4.46.2 Branchings and Spanning Arborescences

Algorithms for finding optimum branchings and spanning arborescences.
This implementation is based on:
J. Edmonds, Optimum branchings, J. Res. Natl. Bur. Standards 71B (1967), 233-240. URL: http:
//archive.org/details/jresv71Bn4p233

| branching_weight(G], attr, default]) | Returns the total weight of a branching. |
| :---: | :---: |
| greedy_branching(G], attr, default, kind]) | Returns a branching obtained through a greedy algorithm. |
| maximum_branching(G[, attr, default]) | Returns a maximum branching from G . |
| minimum_branching(G], attr, default]) | Returns a minimum branching from G . |
| maximum_spanning_arborescence(G[, attr, default]) | Returns a maximum spanning arborescence from G . |
| minimum_spanning_arborescence(G[, attr, default]) | Returns a minimum spanning arborescence from G . |
| Edmonds(G[, seed]) | Edmonds algorithm for finding optimal branchings and spanning arborescences. |

## branching_weight

branching_weight ( $G$, attr='weight', default=1)
Returns the total weight of a branching.

## greedy_branching

greedy_branching ( $G$, attr='weight', default=1, kind='max')
Returns a branching obtained through a greedy algorithm.
This algorithm is wrong, and cannot give a proper optimal branching. However, we include it for pedagogical reasons, as it can be helpful to see what its outputs are.

The output is a branching, and possibly, a spanning arborescence. However, it is not guaranteed to be optimal in either case.

## Parameters

- G (DiGraph) - The directed graph to scan.
- attr (str) - The attribute to use as weights. If None, then each edge will be treated equally with a weight of 1 .
- default (float) - When attr is not None, then if an edge does not have that attribute, default specifies what value it should take.
- kind (str) - The type of optimum to search for: 'min' or 'max' greedy branching.

Returns B - The greedily obtained branching.
Return type directed graph
maximum_branching
maximum_branching ( $G$, attr='weight', default=1)
Returns a maximum branching from $G$.
Parameters

- $\mathbf{G}(($ multi)digraph-like) - The graph to be searched.
- $\operatorname{attr}(s t r)$ - The edge attribute used to in determining optimality.
- default (float) - The value of the edge attribute used if an edge does not have the attribute attr.

Returns B - A maximum branching.
Return type (multi)digraph-like
minimum_branching
minimum_branching ( $G$, attr='weight', default=1)
Returns a minimum branching from $G$.
Parameters

- $\mathbf{G}(($ multi)digraph-like) - The graph to be searched.
- $\operatorname{attr}(s t r)$ - The edge attribute used to in determining optimality.
- default (float) - The value of the edge attribute used if an edge does not have the attribute attr.

Returns B - A minimum branching.
Return type (multi)digraph-like
maximum_spanning_arborescence
maximum_spanning_arborescence ( $G$, attr='weight', default=1)
Returns a maximum spanning arborescence from G.

## Parameters

- $\mathbf{G}$ ((multi)digraph-like) - The graph to be searched.
- $\operatorname{attr}(s t r)$ - The edge attribute used to in determining optimality.
- default (float) - The value of the edge attribute used if an edge does not have the attribute attr.

Returns B - A maximum spanning arborescence.
Return type (multi)digraph-like

Raises NetworkXException - If the graph does not contain a maximum spanning arborescence.

## minimum_spanning_arborescence

```
minimum_spanning_arborescence ( }G\mathrm{ ,attr='weight',default=l)
```

Returns a minimum spanning arborescence from $G$.

## Parameters

- $\mathbf{G}$ ((multi)digraph-like) - The graph to be searched.
- $\boldsymbol{\operatorname { a t t r }}(\mathrm{str})$ - The edge attribute used to in determining optimality.
- default (float) - The value of the edge attribute used if an edge does not have the attribute attr.

Returns B - A minimum spanning arborescence.
Return type (multi)digraph-like
Raises NetworkXException - If the graph does not contain a minimum spanning arborescence.

## Edmonds

class Edmonds ( $G$, seed=None)
Edmonds algorithm for finding optimal branchings and spanning arborescences.
$\qquad$ ( $G$, seed $=$ None )

## Methods

| init__(G[, seed] $)$ |  |
| :--- | :--- |
| find_optimum([attr, default, kind, style] $)$ | Returns a branching from G. |

### 4.46.3 Spanning Trees

Algorithms for calculating $\mathrm{min} / \mathrm{max}$ spanning trees/forests.

| minimum_spanning_tree $(\mathrm{G}[$, weight, algorithm $])$ | Returns a minimum spanning tree or forest on an undi- <br> rected graph $G$. |
| :--- | :--- |
| maximum_spanning_tree $(\mathrm{G}[$, weight, algorithm $])$ | Returns a maximum spanning tree or forest on an undi- <br> rected graph $G$. |
| minimum_spanning_edges $(\mathrm{G}[$, algorithm, ...]) | Generate edges in a minimum spanning forest of an undi- <br>  <br> rected weighted graph. |
| maximum_spanning_edges $(\mathrm{G}[$, algorithm, ...]) | Generate edges in a maximum spanning forest of an undi- <br> rected weighted graph. |

minimum_spanning_tree
minimum_spanning_tree ( $G$, weight='weight', algorithm='kruskal')
Returns a minimum spanning tree or forest on an undirected graph G.
Parameters

- G (undirected graph) - An undirected graph. If $G$ is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- weight (str) - Data key to use for edge weights.
- algorithm (string) - The algorithm to use when finding a minimum spanning tree. Valid choices are 'kruskal', 'prim', or 'boruvka'. The default is 'kruskal'.

Returns G-A minimum spanning tree or forest.
Return type NetworkX Graph

## Examples

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> T = nx.minimum_spanning_tree(G)
>>> sorted(T.edges(data=True))
[(0, 1, {}), (1, 2, {}), (2, 3, {})]
```


## Notes

For Borůvka's algorithm, each edge must have a weight attribute, and each edge weight must be distinct.
For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.
There may be more than one tree with the same minimum or maximum weight. See networkx.tree.recognition for more detailed definitions.
maximum_spanning_tree
maximum_spanning_tree ( $G$, weight='weight', algorithm='kruskal')
Returns a maximum spanning tree or forest on an undirected graph G.

## Parameters

- G (undirected graph) - An undirected graph. If $G$ is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- weight (str) - Data key to use for edge weights.
- algorithm (string) - The algorithm to use when finding a minimum spanning tree. Valid choices are 'kruskal', 'prim', or 'boruvka'. The default is 'kruskal'.

Returns G-A minimum spanning tree or forest.
Return type NetworkX Graph

## Examples

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> T = nx.maximum_spanning_tree(G)
>>> sorted(T.edges(data=True))
[(0, 1, {}), (0, 3, {'weight': 2}), (1, 2, {})]
```


## Notes

For Borůvka's algorithm, each edge must have a weight attribute, and each edge weight must be distinct.
For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.
There may be more than one tree with the same minimum or maximum weight. See networkx.tree.recognition for more detailed definitions.

## minimum_spanning_edges

minimum_spanning_edges (G, algorithm='kruskal', weight='weight', keys=True, data=True)
Generate edges in a minimum spanning forest of an undirected weighted graph.
A minimum spanning tree is a subgraph of the graph (a tree) with the minimum sum of edge weights. A spanning forest is a union of the spanning trees for each connected component of the graph.

## Parameters

- G (undirected Graph) - An undirected graph. If G is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- algorithm (string) - The algorithm to use when finding a minimum spanning tree. Valid choices are 'kruskal', 'prim', or 'boruvka'. The default is 'kruskal'.
- weight (string) - Edge data key to use for weight (default 'weight').
- keys (bool) - Whether to yield edge key in multigraphs in addition to the edge. If G is not a multigraph, this is ignored.
- data (bool, optional) - If True yield the edge data along with the edge.


## Returns

edges - An iterator over tuples representing edges in a minimum spanning tree of $G$.
If $G$ is a multigraph and both keys and data are True, then the tuples are four-tuples of the form ( $u, v, k, w$ ), where ( $u, v$ ) is an edge, $k$ is the edge key identifying the particular edge joining $u$ with $v$, and $w$ is the weight of the edge. If keys is True but dat a is False, the tuples are three-tuples of the form $(u, v, k)$.

If $G$ is not a multigraph, the tuples are of the form $(u, v, w)$ if data is True or $(u, v)$ if data is False.

Return type iterator

## Examples

```
>>> from networkx.algorithms import tree
```

Find minimum spanning edges by Kruskal's algorithm

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> mst = tree.minimum_spanning_edges(G, algorithm='kruskal', data=False)
>>> edgelist = list(mst)
>>> sorted(edgelist)
[(0, 1), (1, 2), (2, 3)]
```

Find minimum spanning edges by Prim's algorithm

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> mst = tree.minimum_spanning_edges(G, algorithm='prim', data=F'alse)
>>> edgelist = list(mst)
>>> sorted(edgelist)
[(0, 1), (1, 2), (2, 3)]
```


## Notes

For Borůvka's algorithm, each edge must have a weight attribute, and each edge weight must be distinct.
For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.
Modified code from David Eppstein, April 2006 http://www.ics.uci.edu/~eppstein/PADS/

## maximum_spanning_edges

maximum_spanning_edges ( $G$, algorithm='kruskal', weight='weight', data=True)
Generate edges in a maximum spanning forest of an undirected weighted graph.
A maximum spanning tree is a subgraph of the graph (a tree) with the maximum possible sum of edge weights. A spanning forest is a union of the spanning trees for each connected component of the graph.

## Parameters

- G (undirected Graph) - An undirected graph. If G is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- algorithm (string) - The algorithm to use when finding a maximum spanning tree. Valid choices are 'kruskal', 'prim', or 'boruvka'. The default is 'kruskal'.
- weight (string) - Edge data key to use for weight (default 'weight').
- keys (bool) - Whether to yield edge key in multigraphs in addition to the edge. If G is not a multigraph, this is ignored.
- data (bool, optional) - If True yield the edge data along with the edge.


## Returns

edges - An iterator over tuples representing edges in a maximum spanning tree of G .
If $G$ is a multigraph and both keys and data are True, then the tuples are four-tuples of the form $(u, v, k, w)$, where $(u, v)$ is an edge, $k$ is the edge key identifying the particular edge joining $u$ with $v$, and $w$ is the weight of the edge. If keys is True but data is False, the tuples are three-tuples of the form $(u, v, k)$.

If $G$ is not a multigraph, the tuples are of the form $(u, v, w)$ if data is True or $(u, v)$ if data is False.

## Return type iterator

## Examples

```
>>> from networkx.algorithms import tree
```

Find maximum spanning edges by Kruskal's algorithm

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> mst = tree.maximum_spanning_edges(G, algorithm='kruskal', data=False)
>>> edgelist = list(mst)
>>> sorted(edgelist)
[(0, 1), (0, 3), (1, 2)]
```

Find maximum spanning edges by Prim's algorithm

```
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0,3,weight=2) # assign weight 2 to edge 0-3
>>> mst = tree.maximum_spanning_edges(G, algorithm='prim', data=False)
>>> edgelist = list(mst)
>>> sorted(edgelist)
[(0, 1), (0, 3), (3, 2)]
```


## Notes

For Borůvka's algorithm, each edge must have a weight attribute, and each edge weight must be distinct.
For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.
Modified code from David Eppstein, April 2006 http://www.ics.uci.edu/~eppstein/PADS/

### 4.47 Triads

Functions for analyzing triads of a graph.
triadic_census $(\mathrm{G}) \quad$ Determines the triadic census of a directed graph.

### 4.47.1 triadic_census

## triadic_census $(G)$

Determines the triadic census of a directed graph.
The triadic census is a count of how many of the 16 possible types of triads are present in a directed graph.
Parameters G (digraph) - A NetworkX DiGraph
Returns census - Dictionary with triad names as keys and number of occurrences as values.
Return type dict

## Notes

This algorithm has complexity $O(m)$ where $m$ is the number of edges in the graph.

## See also:

triad_graph()

## References

### 4.48 Vitality

Vitality measures.
closeness_vitality(G[, node, weight, ...]) Returns the closeness vitality for nodes in the graph.

### 4.48.1 closeness_vitality

closeness_vitality ( $G$, node=None, weight=None, wiener_index=None)
Returns the closeness vitality for nodes in the graph.
The closeness vitality of a node, defined in Section 3.6.2 of [1], is the change in the sum of distances between all node pairs when excluding that node.

## Parameters

- G (NetworkX graph) - A strongly-connected graph.
- weight (string) - The name of the edge attribute used as weight. This is passed directly to the wiener_index () function.
- node (object) - If specified, only the closeness vitality for this node will be returned. Otherwise, a dictionary mappping each node to its closeness vitality will be returned.
Other Parameters wiener_index (number) - If you have already computed the Wiener index of the graph G , you can provide that value here. Otherwise, it will be computed for you.


## Returns

If node is None, this function returnes a dictionary with nodes as keys and closeness vitality as the value. Otherwise, it returns only the closeness vitality for the specified node.

The closeness vitality of a node may be negative infinity if removing that node would disconnect the graph.
Return type dictionary or float

## Examples

```
>>> G = nx.cycle_graph(3)
>>> nx.closeness_vitality(G)
{0: 2.0, 1: 2.0, 2: 2.0}
```


## See also:

```
closeness_centrality()
```


## References

### 4.49 Voronoi cells

Functions for computing the Voronoi cells of a graph.

### 4.49.1 voronoi_cells

## voronoi_cells ( $G$, center_nodes, weight='weight')

Returns the Voronoi cells centered at center_nodes with respect to the shortest-path distance metric.
If $C$ is a set of nodes in the graph and $c$ is an element of $C$, the Voronoi cell centered at a node $c$ is the set of all nodes $v$ that are closer to $c$ than to any other center node in $C$ with respect to the shortest-path distance metric. ${ }^{1}$
For directed graphs, this will compute the "outward" Voronoi cells, as defined in ${ }^{1}$, in which distance is measured from the center nodes to the target node. For the "inward" Voronoi cells, use the DiGraph.reverse() method to reverse the orientation of the edges before invoking this function on the directed graph.

## Parameters

- G (NetworkX graph)
- center_nodes (set) - A nonempty set of nodes in the graph G that represent the center of the Voronoi cells.
- weight (string or function) - The edge attribute (or an arbitrary function) representing the weight of an edge. This keyword argument is as described in the documentation for multi_source_dijkstra_path(), for example.
Returns A mapping from center node to set of all nodes in the graph closer to that center node than to any other center node. The keys of the dictionary are the element of center_nodes, and the values of the dictionary form a partition of the nodes of $G$.


## Return type dictionary

## Examples

To get only the partition of the graph induced by the Voronoi cells, take the collection of all values in the returned dictionary:

```
>>> G = nx.path_graph(6)
>>> center_nodes = {0, 3}
>>> cells = nx.voronoi__cells(G, center_nodes)
>>> partition = set(map(frozenset, cells.values()))
>>> sorted(map(sorted, partition))
[[0, 1], [2, 3, 4, 5]]
```

Raises ValueError - If center_nodes is empty.

## References

### 4.50 Wiener index

Functions related to the Wiener index of a graph.

[^90]
### 4.50.1 wiener_index

```
wiener_index (G, weight=None)
```

Returns the Wiener index of the given graph.
The Wiener index of a graph is the sum of the shortest-path distances between each pair of reachable nodes. For pairs of nodes in undirected graphs, only one orientation of the pair is counted.

## Parameters

- G (NetworkX graph)
- weight (object) - The edge attribute to use as distance when computing shortest-path distances. This is passed directly to the networkx. shortest_path_length() function.

Returns The Wiener index of the graph G.
Return type float
Raises NetworkXError - If the graph $G$ is not connected.

## Notes

If a pair of nodes is not reachable, the distance is assumed to be infinity. This means that for graphs that are not strongly-connected, this function returns inf.

The Wiener index is not usually defined for directed graphs, however this function uses the natural generalization of the Wiener index to directed graphs.

## Examples

The Wiener index of the (unweighted) complete graph on $n$ nodes equals the number of pairs of the $n$ nodes, since each pair of nodes is at distance one:

```
>>> import networkx as nx
>>> n = 10
>>> G = nx.complete_graph(n)
>>> nx.wiener_index(G) == n * (n - 1) / 2
True
```

Graphs that are not strongly-connected have infinite Wiener index:

```
>>> G = nx.empty_graph(2)
>>> nx.wiener_index(G)
inf
```


## CHAPTER 5

## Functions

Functional interface to graph methods and assorted utilities.

### 5.1 Graph

| degree(G[, nbunch, weight]) | Return degree of single node or of nbunch of nodes. |
| :---: | :---: |
| degree_histogram(G) | Return a list of the frequency of each degree value. |
| density(G) | Return the density of a graph. |
| info(G[, n]) | Print short summary of information for the graph G or the node $n$. |
| create_empty_copy(G[, with_data]) | Return a copy of the graph G with all of the edges removed. |
| is_directed(G) | Return True if graph is directed. |
| add_star(G, nodes, \*\*attr) | Add a star to Graph G. |
| add_path(G, nodes, \*\*attr) | Add a path to the Graph G. |
| add_cycle(G, nodes, \*\*attr) | Add a cycle to the Graph G. |

### 5.1.1 degree

## degree ( $G$, nbunch=None, weight=None)

Return degree of single node or of nbunch of nodes. If nbunch is ommitted, then return degrees of all nodes.

### 5.1.2 degree_histogram

degree_histogram $(G)$
Return a list of the frequency of each degree value.
Parameters G (Networkx graph) - A graph
Returns hist - A list of frequencies of degrees. The degree values are the index in the list.
Return type list

## Notes

Note: the bins are width one, hence len(list) can be large (Order(number_of_edges))

### 5.1.3 density

## density $(G)$

Return the density of a graph.
The density for undirected graphs is

$$
d=\frac{2 m}{n(n-1)}
$$

and for directed graphs is

$$
d=\frac{m}{n(n-1)},
$$

where n is the number of nodes and m is the number of edges in G .

## Notes

The density is 0 for a graph without edges and 1 for a complete graph. The density of multigraphs can be higher than 1 .

Self loops are counted in the total number of edges so graphs with self loops can have density higher than 1.

### 5.1.4 info

info ( $G, n=$ None)
Print short summary of information for the graph $G$ or the node $n$.

## Parameters

- G (Networkx graph) - A graph
- n (node (any hashable)) - A node in the graph G


### 5.1.5 create_empty_copy

create_empty_copy ( $G$, with_data=True)
Return a copy of the graph $G$ with all of the edges removed.

## Parameters

- G (graph) - A NetworkX graph
- with_data $($ bool $($ default=True $)$ ) - Propagate Graph and Nodes data to the new graph.

See also:

```
    empty_graph()
```


### 5.1.6 is_directed

is_directed $(G)$
Return True if graph is directed.

### 5.1.7 add_star

add_star ( $G$, nodes, **attr)
Add a star to Graph G.
The first node in nodes is the middle of the star. It is connected to all other nodes.

## Parameters

- nodes (iterable container) - A container of nodes.
- attr (keyword arguments, optional (default= no attributes)) - Attributes to add to every edge in star.

See also:

```
add_path(),add_cycle()
```


## Examples

```
>>> G = nx.Graph()
>>> nx.add_star(G, [0, 1, 2, 3])
>> nx.add_star(G, [10, 11, 12], weight=2)
```


### 5.1.8 add_path

add_path ( $G$, nodes, **attr)
Add a path to the Graph G.

## Parameters

- nodes (iterable container) - A container of nodes. A path will be constructed from the nodes (in order) and added to the graph.
- attr (keyword arguments, optional (default= no attributes)) - Attributes to add to every edge in path.


## See also:

```
add_star(),add_cycle()
```


## Examples

```
>>> G = nx.Graph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> nx.add_path(G, [10, 11, 12], weight=7)
```


### 5.1.9 add_cycle

add_cycle (G, nodes, **attr)
Add a cycle to the Graph G.

## Parameters

- nodes (iterable container) - A container of nodes. A cycle will be constructed from the nodes (in order) and added to the graph.
- attr (keyword arguments, optional (default= no attributes)) - Attributes to add to every edge in cycle.
See also:

```
add_path(),add_star()
```

Examples

```
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_cycle(G, [0, 1, 2, 3])
>>> nx.add_cycle(G, [10, 11, 12], weight=7)
```


### 5.2 Nodes

| nodes $(\mathrm{G})$ | Return an iterator over the graph nodes. |
| :--- | :--- |
| number_of_nodes $(\mathrm{G})$ | Return the number of nodes in the graph. |
| all_neighbors(graph, node) | Returns all of the neighbors of a node in the graph. |
| non_neighbors(graph, node) | Returns the non-neighbors of the node in the graph. |
| common_neighbors(G, u, v) | Return the common neighbors of two nodes in a graph. |

### 5.2.1 nodes

## nodes ( $G$ )

Return an iterator over the graph nodes.

### 5.2.2 number_of_nodes

```
number_of_nodes (G)
```

Return the number of nodes in the graph.

### 5.2.3 all_neighbors

all_neighbors (graph, node)
Returns all of the neighbors of a node in the graph.
If the graph is directed returns predecessors as well as successors.

## Parameters

- graph (NetworkX graph) - Graph to find neighbors.
- node (node) - The node whose neighbors will be returned.

Returns neighbors - Iterator of neighbors
Return type iterator

### 5.2.4 non_neighbors

## non_neighbors (graph, node)

Returns the non-neighbors of the node in the graph.

## Parameters

- graph (NetworkX graph) - Graph to find neighbors.
- node (node) - The node whose neighbors will be returned.

Returns non_neighbors - Iterator of nodes in the graph that are not neighbors of the node.
Return type iterator

### 5.2.5 common_neighbors

common_neighbors ( $G, u, v$ )
Return the common neighbors of two nodes in a graph.

## Parameters

- G (graph) - A NetworkX undirected graph.
- u, $\mathbf{v}$ (nodes) - Nodes in the graph.

Returns cnbors - Iterator of common neighbors of $u$ and $v$ in the graph.
Return type iterator
Raises NetworkXError - If $u$ or $v$ is not a node in the graph.

Examples

```
>>> G = nx.complete_graph(5)
>>> sorted(nx.common_neighbors(G, 0, 1))
[2, 3, 4]
```


### 5.3 Edges

| edges $(\mathrm{G}[$, nbunch $])$ | Return iterator over edges incident to nodes in nbunch. |
| :--- | :--- |
| number_of_edges $(\mathbf{G})$ | Return the number of edges in the graph. |
| non_edges(graph | Returns the non-existent edges in the graph. |

### 5.3.1 edges

edges ( $G$, nbunch $=$ None)
Return iterator over edges incident to nodes in nbunch.
Return all edges if nbunch is unspecified or nbunch=None.
For digraphs, edges=out_edges

### 5.3.2 number_of_edges

```
number_of_edges (G)
```

Return the number of edges in the graph.

### 5.3.3 non_edges

non_edges (graph)
Returns the non-existent edges in the graph.
Parameters graph (NetworkX graph.) - Graph to find non-existent edges.
Returns non_edges - Iterator of edges that are not in the graph.
Return type iterator

### 5.4 Attributes

| set_node_attributes(G, name, values) | Sets node attributes from a given value or dictionary of val- <br> ues. |
| :--- | :--- |
| get_node_attributes(G, name) | Get node attributes from graph |
| set_edge_attributes(G, name, values) | Sets edge attributes from a given value or dictionary of val- <br> ues. |
| get_edge_attributes(G, name) | Get edge attributes from graph |

### 5.4.1 set_node_attributes

set_node_attributes ( $G$, name, values)
Sets node attributes from a given value or dictionary of values.

## Parameters

- G (NetworkX Graph)
- name (string) - Name of the node attribute to set.
- values (dict) - Dictionary of attribute values keyed by node. If values is not a dictionary, then it is treated as a single attribute value that is then applied to every node in G . This means that if you provide a mutable object, like a list, updates to that object will be reflected in the node attribute for each node.


## Examples

After computing some property of the nodes of a graph, you may want to assign a node attribute to store the value of that property for each node:

```
>>> G = nx.path_graph(3)
>>> bb = nx.betweenness_centrality(G) # this is a dictionary
>>> nx.set_node_attributes(G, 'betweenness', bb)
>>> G.node[1]['betweenness']
1.0
```

If you provide a list as the third argument, updates to the list will be reflected in the node attribute for each node:

```
>>> labels = []
>>> nx.set_node_attributes(G, 'labels', labels)
>>> labels.append('foo')
>>> G.node[0]['labels']
['foo']
>>> G.node[1]['labels']
['foo']
>>> G.node[2]['labels']
['foo']
```


### 5.4.2 get_node_attributes

get_node_attributes ( $G$, name)
Get node attributes from graph

## Parameters

- G (NetworkX Graph)
- name (string) - Attribute name


## Returns

Return type Dictionary of attributes keyed by node.

## Examples

```
>>> G=nx.Graph()
>>> G.add_nodes_from([1,2,3],color='red')
>>> color=nx.get_node_attributes(G,'color')
>>> color[1]
'red'
```


### 5.4.3 set_edge_attributes

set_edge_attributes ( $G$, name, values)
Sets edge attributes from a given value or dictionary of values.

## Parameters

- G (NetworkX Graph)
- name (string) - Name of the edge attribute to set.
- values (dict) - Dictionary of attribute values keyed by edge (tuple). For multigraphs, the tuples must be of the form ( $u, v$, key), where $u$ and $v$ are nodes and key is the key corresponding to the edge. For non-multigraphs, the keys must be tuples of the form $(u, v)$.

If values is not a dictionary, then it is treated as a single attribute value that is then applied to every edge in G. This means that if you provide a mutable object, like a list, updates to that object will be reflected in the edge attribute for each edge.

## Examples

After computing some property of the nodes of a graph, you may want to assign a node attribute to store the value of that property for each node:

```
>>> G = nx.path_graph(3)
>>> bb = nx.edge_betweenness_centrality(G, normalized=False)
>>> nx.set_edge_attributes(G, 'betweenness', bb)
>>> G.edge[1][2]['betweenness']
2.0
```

If you provide a list as the third argument, updates to the list will be reflected in the edge attribute for each node:

```
>>> labels = []
>>> nx.set_edge_attributes(G, 'labels', labels)
>>> labels.append('foo')
>>> G.edge[0][1]['labels']
['foo']
>>> G.edge[1][2]['labels']
['foo']
```


### 5.4.4 get_edge_attributes

get_edge_attributes ( $G$, name)
Get edge attributes from graph

## Parameters

- G (NetworkX Graph)
- name (string) - Attribute name


## Returns

- Dictionary of attributes keyed by edge. For (di)graphs, the keys are
- 2-tuples of the form $((u, v)$. For multi(di)graphs, the keys are 3-tuples of $)$
- the form $((u, v, k e y)$.

Examples

```
>>> G=nx.Graph()
>>> nx.add_path(G, [1, 2, 3], color='red')
>>> color=nx.get_edge_attributes(G, 'color')
>>> color[(1, 2)]
'red'
```


### 5.5 Freezing graph structure

| freeze(G) | Modify graph to prevent further change by adding or re- <br> moving nodes or edges. |
| :--- | :--- |
| is_frozen $(\mathrm{G})$ | Return True if graph is frozen. |

### 5.5.1 freeze

## freeze ( $G$ )

Modify graph to prevent further change by adding or removing nodes or edges.
Node and edge data can still be modified.
Parameters G (graph) - A NetworkX graph

## Examples

```
>>> G=nx.path_graph(4)
>>> G=nx.freeze(G)
>>> try:
... G.add_edge (4,5)
... except nx.NetworkXError as e:
... print(str(e))
Frozen graph can't be modified
```


## Notes

To "unfreeze" a graph you must make a copy by creating a new graph object:

```
>>> graph = nx.path_graph(4)
>>> frozen_graph = nx.freeze(graph)
>>> unfrozen_graph = nx.Graph(frozen_graph)
>>> nx.is_frozen(unfrozen_graph)
False
```


## See also:

```
is_frozen()
```


### 5.5.2 is_frozen

is_frozen ( $G$ )
Return True if graph is frozen.
Parameters G (graph) - A NetworkX graph

## See also:

freeze()

## Graph generators

### 6.1 Atlas

Generators for the small graph atlas.

| graph_atlas(i) | Returns graph number i from the Graph Atlas. |
| :--- | :--- |
| graph_atlas_g() | Return the list of all graphs with up to seven nodes named <br> in the Graph Atlas. |

### 6.1.1 graph_atlas

graph_atlas (i)
Returns graph number i from the Graph Atlas.
For more information, see graph_atlas_g().
Parameters i (int) - The index of the graph from the atlas to get. The graph at index 0 is assumed to be the null graph.
Returns A list of Graph objects, the one at index $i$ corresponding to the graph $i$ in the Graph Atlas.
Return type list
See also:

```
    graph_atlas_g()
```


## Notes

The time required by this function increases linearly with the argument $i$, since it reads a large file sequentially in order to generate the graph.

## References

### 6.1.2 graph_atlas_g

graph_atlas_g()
Return the list of all graphs with up to seven nodes named in the Graph Atlas.

The graphs are listed in increasing order by
1.number of nodes,
2.number of edges,
3.degree sequence (for example $111223<112222$ ),
4.number of automorphisms,
in that order, with three exceptions as described in the Notes section below. This causes the list to correspond with the index of the graphs in the Graph Atlas [atlas], with the first graph, $\mathrm{G}[0$ ], being the null graph.

Returns A list of Graph objects, the one at index $i$ corresponding to the graph $i$ in the Graph Atlas.
Return type list

## See also:

```
graph_atlas()
```


## Notes

This function may be expensive in both time and space, since it reads a large file sequentially in order to populate the list.

Although the NetworkX atlas functions match the order of graphs given in the "Atlas of Graphs" book, there are (at least) three errors in the ordering described in the book. The following three pairs of nodes violate the lexicographically nondecreasing sorted degree sequence rule:
-graphs 55 and 56 with degree sequences 001111 and 000112,
-graphs 1007 and 1008 with degree sequences 3333444 and 3333336,
-graphs 1012 and 1213 with degree sequences 1244555 and 1244456.

## References

### 6.2 Classic

Generators for some classic graphs.
The typical graph generator is called as follows:

```
>>> G=nx.complete_graph(100)
```

returning the complete graph on n nodes labeled $0, . ., 99$ as a simple graph. Except for empty_graph, all the generators in this module return a Graph class (i.e. a simple, undirected graph).

| balanced_tree(r, h[, create_using]) | Return the perfectly balanced r-ary tree of height h. |
| :--- | :--- |
| barbell_graph(m1, m2[, create_using]) | Return the Barbell Graph: two complete graphs connected <br> by a path. |
| complete_graph(n[, create_using]) | Return the complete graph K_n with n nodes. |
| complete_multipartite_graph(\*subset_sizes) | Returns the complete multipartite graph with the specified <br> subset sizes. |
| circular_ladder_graph(n[, create_using]) | Return the circular ladder graph CL_n of length n. |
| cycle_graph(n[, create_using]) | Return the cycle graph C_n of cyclicly connected nodes. |
|  |  |

Table 6.2 - continued from previous page

| dorogovtsev_goltsev_mendes_graph(n[, ...]) | Return the hierarchically constructed Dorogovtsev-Goltsev-Mendes graph. |
| :---: | :---: |
| empty_graph([n, create_using]) | Return the empty graph with n nodes and zero edges. |
| grid_2d_graph(m, n[, periodic, create_using]) | Return the 2d grid graph of mxn nodes |
| grid_graph(dim[, periodic]) | Return the n -dimensional grid graph. |
| hypercube_graph(n) | Return the n -dimensional hypercube. |
| ladder_graph(n[, create_using]) | Return the Ladder graph of length n . |
| lollipop_graph(m, n[, create_using]) | Return the Lollipop Graph; K _m connected to P_n. |
| null_graph([create_using]) | Return the Null graph with no nodes or edges. |
| path_graph(n[, create_using]) | Return the Path graph P_n of linearly connected nodes. |
| star_graph(n[, create_using]) | Return the star graph |
| trivial_graph([create_using]) | Return the Trivial graph with one node (with label 0 ) and no edges. |
| turan_graph(n, r) | Return the Turan Graph |
| wheel_graph(n[, create_using]) | Return the wheel graph |

### 6.2.1 balanced_tree

balanced_tree ( $r, h$, create_using=None)
Return the perfectly balanced $r$-ary tree of height $h$.

## Parameters

- $\mathbf{r}($ int $)$ - Branching factor of the tree; each node will have $r$ children.
- $\mathbf{h}$ (int) - Height of the tree.
- create_using (Graph, optional (default None)) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.

Returns G - A balanced r -ary tree of height h .
Return type NetworkX graph

## Notes

This is the rooted tree where all leaves are at distance h from the root. The root has degree r and all other internal nodes have degree $r+1$.
Node labels are integers, starting from zero.
A balanced tree is also known as a complete $r$-ary tree.

### 6.2.2 barbell_graph

barbell_graph ( $m 1, m 2$, create_using=None)
Return the Barbell Graph: two complete graphs connected by a path.
For $\mathrm{m} 1>1$ and $\mathrm{m} 2>=0$.
Two identical complete graphs $K_{\_}\{m 1\}$ form the left and right bells, and are connected by a path $P_{-}\{m 2\}$.
The $2 \star \mathrm{~m} 1+\mathrm{m} 2$ nodes are numbered $0, \ldots, m 1-1$ for the left barbell, $\mathrm{m} 1, \ldots, m 1+m 2-1$ for the path, and $\mathrm{m} 1+\mathrm{m} 2, \ldots, 2 \star \mathrm{~m} 1+\mathrm{m} 2-1$ for the right barbell.

The 3 subgraphs are joined via the edges $(m 1-1, m 1)$ and $(m 1+m 2-1, m 1+m 2)$. If $m 2=0$, this is merely two complete graphs joined together.

This graph is an extremal example in David Aldous and Jim Fill's e-text on Random Walks on Graphs.

### 6.2.3 complete_graph

complete_graph ( $n$, create_using=None)
Return the complete graph $\mathrm{K} \_\mathrm{n}$ with n nodes.

## Parameters

- $\mathbf{n}$ (int or iterable container of nodes) - If n is an integer, nodes are from range( n ). If n is a container of nodes, those nodes appear in the graph.
- create_using (Graph, optional (default None)) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


## Examples

```
>>> G = nx.complete_graph(9)
>>> len(G)
9
>>> G.size()
36
>>> G = nx.complete_graph(range(11,14))
>>> list(G.nodes())
[11, 12, 13]
>>> G = nx.complete_graph(4, nx.DiGraph())
>>> G.is_directed()
True
```


### 6.2.4 complete_multipartite_graph

```
complete_multipartite_graph(*subset_sizes)
```

Returns the complete multipartite graph with the specified subset sizes.
Parameters subset_sizes (tuple of integers or tuple of node iterables) - The arguments can either all be integer number of nodes or they can all be iterables of nodes. If integers, they represent the number of vertices in each subset of the multipartite graph. If iterables, each is used to create the nodes for that subset. The length of subset_sizes is the number of subsets.

## Returns

G - Returns the complete multipartite graph with the specified subsets.
For each node, the node attribute 'subset' is an integer indicating which subset contains the node.

## Return type NetworkX Graph

## Examples

Creating a complete tripartite graph, with subsets of one, two, and three vertices, respectively.

```
>>> import networkx as nx
>>> G = nx.complete_multipartite_graph(1, 2, 3)
>>> [G.node[u]['subset'] for u in G]
[0, 1, 1, 2, 2, 2]
>>> list(G.edges(0))
[(0, 1), (0, 2), (0, 3), (0, 4), (0, 5)]
>>> list(G.edges(2))
[(2, 0), (2, 3), (2, 4), (2, 5)]
>>> list(G.edges(4))
[(4, 0), (4, 1), (4, 2)]
```

```
>>> G = nx.complete_multipartite_graph('a', 'bc', 'def')
>>> [G.node[u]['subset'] for u in sorted(G)]
[0, 1, 1, 2, 2, 2]
```


## Notes

This function generalizes several other graph generator functions.
-If no subset sizes are given, this returns the null graph.
-If a single subset size n is given, this returns the empty graph on n nodes.
-If two subset sizes $m$ and $n$ are given, this returns the complete bipartite graph on $m+n$ nodes.

- If subset sizes 1 and $n$ are given, this returns the star graph on $n+1$ nodes.


## See also:

```
complete_bipartite_graph()
```


### 6.2.5 circular_ladder_graph

circular_ladder_graph ( $n$, create_using=None)
Return the circular ladder graph $\mathrm{CL} \_\mathrm{n}$ of length n .
CL_n consists of two concentric $n$-cycles in which each of the $n$ pairs of concentric nodes are joined by an edge.
Node labels are the integers 0 to $\mathrm{n}-1$

### 6.2.6 cycle_graph

cycle_graph ( $n$, create_using=None)
Return the cycle graph C_n of cyclicly connected nodes.
C_n is a path with its two end-nodes connected.

## Parameters

- $\mathbf{n}$ (int or iterable container of nodes) - If $n$ is an integer, nodes are from range ( $n$ ). If $n$ is a container of nodes, those nodes appear in the graph.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


## Notes

If create_using is directed, the direction is in increasing order.

### 6.2.7 dorogovtsev_goltsev_mendes_graph

dorogovtsev_goltsev_mendes_graph (n, create_using=None)
Return the hierarchically constructed Dorogovtsev-Goltsev-Mendes graph.
n is the generation. See: arXiv:/cond-mat/0112143 by Dorogovtsev, Goltsev and Mendes.

### 6.2.8 empty_graph

empty_graph ( $n=0$, create_using=None)
Return the empty graph with n nodes and zero edges.

## Parameters

- $\mathbf{n}$ (int or iterable container of nodes $($ default $=0)$ ) - If n is an integer, nodes are from range ( n ). If n is a container of nodes, those nodes appear in the graph.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.
- For example
- >>> G=nx.empty_graph(10)
- >>> G.number_of_nodes()
- 10
- >>> G.number_of_edges()
- 0
- >>> G=nx.empty_graph("ABC")
- >>> G.number_of_nodes()
- 3
- >>> sorted(G)
- ['A', ‘B', ‘C’]


## Notes

The variable create_using should point to a "graph"-like object that will be cleared (nodes and edges will be removed) and refitted as an empty "graph" with nodes specified in $n$. This capability is useful for specifying the class-nature of the resulting empty "graph" (i.e. Graph, DiGraph, MyWeirdGraphClass, etc.).

The variable create_using has two main uses: Firstly, the variable create_using can be used to create an empty digraph, multigraph, etc. For example,

```
>>> n=10
>>> G=nx.empty_graph(n, create_using=nx.DiGraph())
```

will create an empty digraph on n nodes.
Secondly, one can pass an existing graph (digraph, multigraph, etc.) via create_using. For example, if G is an existing graph (resp. digraph, multigraph, etc.), then empty_graph(n, create_using=G) will empty $G$ (i.e. delete all nodes and edges using G.clear()) and then add n nodes and zero edges, and return the modified graph.

See also create_empty_copy(G).

### 6.2.9 grid_2d_graph

grid_2d_graph ( $m$, $n$, periodic=False, create_using=None)
Return the 2 d grid graph of mxn nodes
The grid graph has each node connected to its four nearest neighbors.

## Parameters

- m, n (int or iterable container of nodes (default $=0)$ ) - If an integer, nodes are from range ( $n$ ). If a container, those become the coordinate of the node.
- periodic $($ bool $($ default $=$ False $))$ - If True will connect boundary nodes in periodic fashion.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


### 6.2.10 grid_graph

grid_graph (dim, periodic=False)
Return the n -dimensional grid graph.
'dim' is a tuple or list with the size in each dimension or an iterable of nodes for each dimension. The dimension of the grid_graph is the length of the tuple or list 'dim'.
E.g. $G=$ grid_graph $(\operatorname{dim}=[2,3])$ produces a $2 \times 3$ grid graph.
E.g. $G=$ grid_graph $(\operatorname{dim}=[\operatorname{range}(7,9)$, range $(3,6)])$ produces a $2 \times 3$ grid graph.

If periodic=True then join grid edges with periodic boundary conditions.

### 6.2.11 hypercube_graph

## hypercube_graph ( $n$ )

Return the n-dimensional hypercube.
Node labels are the integers 0 to $2 * * \mathrm{n}-1$.

### 6.2.12 ladder_graph

ladder_graph ( $n$, create_using=None)
Return the Ladder graph of length $n$.
This is two rows of n nodes, with each pair connected by a single edge.
Node labels are the integers 0 to $2 * \mathrm{n}-1$.

### 6.2.13 Iollipop_graph

lollipop_graph ( $m, n$, create_using=None)
Return the Lollipop Graph; $\mathrm{K} \_m$ connected to $P$ _n.
This is the Barbell Graph without the right barbell.

## Parameters

- m, n (int or iterable container of nodes (default $=0)$ ) - If an integer, nodes are from range $(m)$ and range $(m, m+n)$. If a container, the entries are the coordinate of the node.

The nodes for $m$ appear in the complete graph $K \_m$ and the nodes for $n$ appear in the path P_n

- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


## Notes

The 2 subgraphs are joined via an edge ( $\mathrm{m}-1, \mathrm{~m}$ ). If $\mathrm{n}=0$, this is merely a complete graph.
(This graph is an extremal example in David Aldous and Jim Fill's etext on Random Walks on Graphs.)

### 6.2.14 null_graph

null_graph (create_using=None)
Return the Null graph with no nodes or edges.
See empty_graph for the use of create_using.

### 6.2.15 path_graph

path_graph (n, create_using=None)
Return the Path graph $\mathrm{P} \_\mathrm{n}$ of linearly connected nodes.

## Parameters

- n (int or iterable) - If an integer, node labels are 0 to $n$ with center 0 . If an iterable of nodes, the center is the first.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


### 6.2.16 star_graph

star_graph (n, create_using=None)
Return the star graph
The star graph consists of one center node connected to n outer nodes.

## Parameters

- $\mathbf{n}$ (int or iterable) - If an integer, node labels are 0 to n with center 0 . If an iterable of nodes, the center is the first.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.


## Notes

The graph has $n+1$ nodes for integer $n$. So star_graph(3) is the same as star_graph(range(4)).

### 6.2.17 trivial_graph

## trivial_graph (create_using=None)

Return the Trivial graph with one node (with label 0 ) and no edges.

### 6.2.18 turan_graph

## turan_graph ( $n, r$ )

Return the Turan Graph
The Turan Graph is a complete multipartite graph on $n$ vertices with $r$ disjoint subsets. It is the graph with the edges for any graph with $n$ vertices and $r$ disjoint subsets.

Given n and $r$, we generate a complete multipartite graph with $r-(n \bmod r)$ partitions of size $n / r$, rounded down, and $n \bmod r$ partitions of size $n / r+1$, rounded down.

## Parameters

- $\mathbf{n}($ int $)$ - The number of vertices.
- $\mathbf{r}($ int $)$ - The number of partitions. Must be less than or equal to $n$.


## Notes

Must satisfy $1<=r<=n$. The graph has $(r-1)\left(n^{2}\right) /(2 r)$ edges, rounded down.

### 6.2.19 wheel_graph

```
wheel_graph (n, create_using=None)
```

Return the wheel graph
The wheel graph consists of a hub node connected to a cycle of ( $\mathrm{n}-1$ ) nodes.

## Parameters

- n (int or iterable) - If an integer, node labels are 0 to $n$ with center 0 . If an iterable of nodes, the center is the first.
- create_using (Graph, optional (default Graph())) - If provided this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.
- Node labels are the integers 0 to $\mathbf{n}-1$.


### 6.3 Expanders

Provides explicit constructions of expander graphs.

| margulis_gabber_galil_graph(n[, <br> ate_using] $)$ | cre- | Return the Margulis-Gabber-Galil undirected MultiGraph <br> on $\mathrm{n}^{\wedge} 2$ nodes. |
| :--- | :--- | :--- |
| chordal_cycle_graph(p[, create_using]) | Return the chordal cycle graph on p nodes. |  |

### 6.3.1 margulis_gabber_galil_graph

margulis_gabber_galil_graph (n, create_using=None)
Return the Margulis-Gabber-Galil undirected MultiGraph on $\mathrm{n}^{\wedge} 2$ nodes.
The undirected MultiGraph is regular with degree 8. Nodes are integer pairs. The second-largest eigenvalue of the adjacency matrix of the graph is at most 5 sqrt $\{2\}$, regardless of $n$.

## Parameters

- $\mathbf{n}$ (int) - Determines the number of nodes in the graph: $\mathrm{n}^{\wedge} 2$.
- create_using (graph-like) - A graph-like object that receives the constructed edges. If None, then a MultiGraph instance is used.

Returns G - The constructed undirected multigraph.
Return type graph
Raises NetworkXError - If the graph is directed or not a multigraph.

### 6.3.2 chordal_cycle_graph

chordal_cycle_graph ( $p$, create_using=None)
Return the chordal cycle graph on $p$ nodes.
The returned graph is a cycle graph on p nodes with chords joining each vertex x to its inverse modulo p . This graph is a (mildly explicit) 3-regular expander ${ }^{1}$.
p must be a prime number.

## Parameters

- $\mathbf{p}$ ( a prime number) - The number of vertices in the graph. This also indicates where the chordal edges in the cycle will be created.
- create_using (graph-like) - A graph-like object that receives the constructed edges. If None, then a MultiGraph instance is used.

Returns G - The constructed undirected multigraph.
Return type graph
Raises NetworkXError - If the graph provided in create_using is directed or not a multigraph.

[^91]
## References

### 6.4 Small

Various small and named graphs, together with some compact generators.

| make_small_graph(graph_description[, ..]) | Return the small graph described by graph_description. |
| :--- | :--- |
| LCF_graph(n, shift_list, repeats[, create_using]) | Return the cubic graph specified in LCF notation. |
| bull_graph([create_using]) | Return the Bull graph. |
| chvatal_graph([create_using]) | Return the Chvátal graph. |
| cubical_graph([create_using]) | Return the 3-regular Platonic Cubical graph. |
| desargues_graph([create_using]) | Return the Desargues graph. |
| diamond_graph([create_using]) | Return the Diamond graph. |
| dodecahedral_graph([create_using]) | Return the Platonic Dodecahedral graph. |
| frucht_graph([create_using]) | Return the Frucht Graph. |
| heawood_graph([create_using]) | Return the Heawood graph, a (3,6) cage. |
| house_graph([create_using]) | Return the House graph (square with triangle on top). |
| house_x_graph([create_using]) | Return the House graph with a cross inside the house |
|  | square. |
| icosahedral_graph([create_using]) | Return the Platonic Icosahedral graph. |
| krackhardt_kite_graph([create_using]) | Return the Krackhardt Kite Social Network. |
| moebius_kantor_graph([create_using]) | Return the Moebius-Kantor graph. |
| octahedral_graph([create_using]) | Return the Platonic Octahedral graph. |
| pappus_graph() | Return the Pappus graph. |
| petersen_graph([create_using]) | Return the Petersen graph. |
| sedgewick_maze_graph([create_using]) | Return a small maze with a cycle. |
| tetrahedral_graph([create_using]) | Return the 3-regular Platonic Tetrahedral graph. |
| truncated_cube_graph([create_using]) | Return the skeleton of the truncated cube. |
| truncated_tetrahedron_graph([create_using]) | Return the skeleton of the truncated Platonic tetrahedron. |
| tutte_graph([create_using]) | Return the Tutte graph. |

### 6.4.1 make_small_graph

make_small_graph (graph_description, create_using=None)
Return the small graph described by graph_description.
graph_description is a list of the form [ltype,name, n, xlist]
Here ltype is one of "adjacencylist" or "edgelist", name is the name of the graph and $n$ the number of nodes. This constructs a graph of n nodes with integer labels $0, . ., \mathrm{n}-1$.
If ltype="adjacencylist" then xlist is an adjacency list with exactly $n$ entries, in with the $j$ 'th entry (which can be empty) specifies the nodes connected to vertex j. e.g. the "square" graph C_4 can be obtained by

```
>>> G=nx.make_small_graph(["adjacencylist","C_4",4,[[2,4],[1,3],[2,4],[1,3]]])
```

or, since we do not need to add edges twice,

```
>>> G=nx.make_small_graph(["adjacencylist","C_4",4,[[2,4],[3],[4],[]]])
```

If ltype="edgelist" then xlist is an edge list written as [[v1,w2],[v2,w2],..,,[vk,wk]], where vj and wj integers in the range $1, . .$, n e.g. the "square" graph $\mathrm{C}_{-} 4$ can be obtained by

```
>>> G=nx.make_small_graph(["edgelist","C_4",4,[[1,2],[3,4],[2,3],[4,1]]])
```

Use the create_using argument to choose the graph class/type.

### 6.4.2 LCF_graph

LCF_graph ( $n$, shift_list, repeats, create_using=None)
Return the cubic graph specified in LCF notation.
LCF notation (LCF=Lederberg-Coxeter-Fruchte) is a compressed notation used in the generation of various cubic Hamiltonian graphs of high symmetry. See, for example, dodecahedral_graph, desargues_graph, heawood_graph and pappus_graph below.
$\mathbf{n}$ (number of nodes) The starting graph is the n -cycle with nodes $0, \ldots, \mathrm{n}-1$. (The null graph is returned if $\mathrm{n}<$ 0.$)$
shift_list $=[\mathrm{s} 1, \mathrm{~s} 2, . ., \mathrm{sk}]$, a list of integer shifts mod n ,
repeats integer specifying the number of times that shifts in shift_list are successively applied to each v_current in the n-cycle to generate an edge between v_current and v_current+shift mod n.

For v1 cycling through the n-cycle a total of $k *$ repeats with shift cycling through shiftlist repeats times connect v1 with v1+shift $\bmod n$

The utility graph K_\{3,3\}

```
>>> G=nx.LCF_graph(6, [3,-3], 3)
```

The Heawood graph

```
>>> G=nx.LCF_graph(14, [5, -5], 7)
```

See http://mathworld.wolfram.com/LCFNotation.html for a description and references.

### 6.4.3 bull_graph

```
bull_graph (create_using=None)
```

Return the Bull graph.

### 6.4.4 chvatal_graph

```
chvatal_graph(create_using=None)
```

Return the Chvátal graph.

### 6.4.5 cubical_graph

cubical_graph (create_using=None)
Return the 3-regular Platonic Cubical graph.

### 6.4.6 desargues_graph

desargues_graph (create_using=None)
Return the Desargues graph.

### 6.4.7 diamond_graph

diamond_graph (create_using=None)
Return the Diamond graph.

### 6.4.8 dodecahedral_graph

dodecahedral_graph (create_using=None)
Return the Platonic Dodecahedral graph.

### 6.4.9 frucht_graph

frucht_graph (create_using=None)
Return the Frucht Graph.
The Frucht Graph is the smallest cubical graph whose automorphism group consists only of the identity element.

### 6.4.10 heawood_graph

heawood_graph (create_using=None)
Return the Heawood graph, a $(3,6)$ cage.

### 6.4.11 house_graph

house_graph (create_using=None)
Return the House graph (square with triangle on top).

### 6.4.12 house_x_graph

house_x_graph (create_using=None)
Return the House graph with a cross inside the house square.

### 6.4.13 icosahedral_graph

icosahedral_graph (create_using=None)
Return the Platonic Icosahedral graph.

### 6.4.14 krackhardt_kite_graph

krackhardt_kite_graph (create_using=None)
Return the Krackhardt Kite Social Network.
A 10 actor social network introduced by David Krackhardt to illustrate: degree, betweenness, centrality, closeness, etc. The traditional labeling is: Andre $=1$, Beverley=2, Carol=3, Diane=4, Ed=5, Fernando=6, Garth=7, Heather=8, Ike=9, Jane=10.

### 6.4.15 moebius_kantor_graph

```
moebius_kantor_graph(create_using=None)
```

Return the Moebius-Kantor graph.

### 6.4.16 octahedral_graph

```
octahedral_graph (create_using=None)
```

Return the Platonic Octahedral graph.

### 6.4.17 pappus_graph

## pappus_graph ()

Return the Pappus graph.

### 6.4.18 petersen_graph

```
petersen_graph (create_using=None)
```

Return the Petersen graph.

### 6.4.19 sedgewick_maze_graph

sedgewick_maze_graph (create_using=None)
Return a small maze with a cycle.
This is the maze used in Sedgewick,3rd Edition, Part 5, Graph Algorithms, Chapter 18, e.g. Figure 18.2 and following. Nodes are numbered $0, . ., 7$

### 6.4.20 tetrahedral_graph

tetrahedral_graph (create_using=None)
Return the 3-regular Platonic Tetrahedral graph.

### 6.4.21 truncated_cube_graph

truncated_cube_graph (create_using=None)
Return the skeleton of the truncated cube.

### 6.4.22 truncated_tetrahedron_graph

truncated_tetrahedron_graph (create_using=None)
Return the skeleton of the truncated Platonic tetrahedron.

### 6.4.23 tutte_graph

tutte_graph (create_using=None)
Return the Tutte graph.

### 6.5 Random Graphs

Generators for random graphs.

| fast_gnp_random_graph(n, p[, seed, directed]) | Returns a G_\{n,p\} random graph, also known as an Erdős-Rényi graph or a binomial graph. |
| :---: | :---: |
| gnp_random_graph(n, p[, seed, directed]) | Returns a $G \_\{n, p\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph. |
| dense_gnm_random_graph(n, m[, seed]) | Returns a G_\{ $\mathrm{n}, \mathrm{m}\}$ random graph. |
| gnm_random_graph(n, m[, seed, directed]) | Returns a $G_{-}\{n, m\}$ random graph. |
| erdos_renyi_graph(n, p[, seed, directed]) | Returns a $G \_\{n, p\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph. |
| binomial_graph(n, p[, seed, directed]) | Returns a $\mathrm{G}_{-}\{\mathrm{n}, \mathrm{p}\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph. |
| newman_watts_strogatz_graph(n, k, p[, seed]) | Return a Newman-Watts-Strogatz small-world graph. |
| watts_strogatz_graph(n, k, p[, seed]) | Return a Watts-Strogatz small-world graph. |
| connected_watts_strogatz_graph(n, k, p[, ...]) | Returns a connected Watts-Strogatz small-world graph. |
| random_regular_graph(d, n[, seed]) | Returns a random d-regular graph on n nodes. |
| barabasi_albert_graph(n, m[, seed]) | Returns a random graph according to the Barabási-Albert preferential attachment model. |
| powerlaw_cluster_graph(n, m, p[, seed]) | Holme and Kim algorithm for growing graphs with powerlaw degree distribution and approximate average clustering. |
| random_kernel_graph(n, kernel_integral[, ...]) | Return an random graph based on the specified kernel. |
| random_lobster(n, p1, p2[, seed]) | Returns a random lobster graph. |
| random_shell_graph(constructor[, seed]) | Returns a random shell graph for the constructor given. |
| random_powerlaw_tree(n[, gamma, seed, tries]) | Returns a tree with a power law degree distribution. |
| random_powerlaw_tree_sequence(n[, gamma, ...]) | Returns a degree sequence for a tree with a power law distribution. |

### 6.5.1 fast_gnp_random_graph

fast_gnp_random_graph ( $n, p$, seed=None, directed=False $)$
Returns a $\mathrm{G}_{-}\{\mathrm{n}, \mathrm{p}\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- $\mathbf{p}($ float $)$ - Probability for edge creation.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True, this function returns a directed graph.


## Notes

The $G \_\{n, p\}$ graph algorithm chooses each of the $\left[\begin{array}{ll}n & (n-1)\end{array}\right] / 2$ (undirected) or $n(n-1)$ (directed) possible edges with probability $p$.

This algorithm runs in $O(n+m)$ time, where $m$ is the expected number of edges, which equals $p \quad n \quad(n-1)$ / 2. This should be faster than gnp_random_graph () when $p$ is small and the expected number of edges is small (that is, the graph is sparse).

## See also:

```
gnp_random_graph()
```


## References

### 6.5.2 gnp_random_graph

gnp_random_graph ( $n, p$, seed=None, directed=False)
Returns a $\mathrm{G}_{-}\{\mathrm{n}, \mathrm{p}\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.
The $G_{n, p}$ model chooses each of the possible edges with probability $p$.
The functions binomial_graph() and erdos_renyi_graph() are aliases of this function.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- $\mathbf{p}($ float $)$ - Probability for edge creation.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True, this function returns a directed graph.


## See also:

```
fast_gnp_random_graph()
```


## Notes

This algorithm runs in $O\left(n^{\wedge} 2\right)$ time. For sparse graphs (that is, for small values of $p$ ), fast_gnp_random_graph() is a faster algorithm.

## References

### 6.5.3 dense_gnm_random_graph

## dense_gnm_random_graph $(n, m$, seed=None)

Returns a $G_{-}\{n, m\}$ random graph.
In the $G_{-}\{n, m\}$ model, a graph is chosen uniformly at random from the set of all graphs with $n$ nodes and $m$ edges.

This algorithm should be faster than gnm_random_graph() for dense graphs.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- m (int) - The number of edges.
- seed (int, optional) - Seed for random number generator (default=None).


## See also:

gnm_random_graph()

## Notes

Algorithm by Keith M. Briggs Mar 31, 2006. Inspired by Knuth's Algorithm S (Selection sampling technique), in section 3.4.2 of ${ }^{1}$.

## References

### 6.5.4 gnm_random_graph

gnm_random_graph ( $n, m$, seed=None, directed=False)
Returns a $G \_\{n, m\}$ random graph.
In the $G_{-}\{n, m\}$ model, a graph is chosen uniformly at random from the set of all graphs with $n$ nodes and $m$ edges.

This algorithm should be faster than dense_gnm_random_graph () for sparse graphs.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes.
- m (int) - The number of edges.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True return a directed graph


## See also:

```
dense_gnm_random_graph()
```


### 6.5.5 erdos_renyi_graph

erdos_renyi_graph ( $n, p$, seed=None, directed=False)
Returns a $G \_\{n, p\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.
The $G_{n, p}$ model chooses each of the possible edges with probability $p$.
The functions binomial_graph() and erdos_renyi_graph() are aliases of this function.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- $\mathbf{p}$ (float) - Probability for edge creation.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True, this function returns a directed graph.

See also:

```
fast_gnp_random_graph()
```


## Notes

This algorithm runs in $O\left(n^{\wedge} 2\right)$ time. For sparse graphs (that is, for small values of $p$ ), fast_gnp_random_graph () is a faster algorithm.

[^92]
## References

### 6.5.6 binomial_graph

binomial_graph ( $n, p$, seed=None, directed=False)
Returns a $G \_\{n, p\}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.
The $G_{n, p}$ model chooses each of the possible edges with probability $p$.
The functions binomial_graph() and erdos_renyi_graph() are aliases of this function.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes.
- $\mathbf{p}($ float $)$ - Probability for edge creation.
- seed (int, optional) - Seed for random number generator (default=None).
- directed (bool, optional (default=False)) - If True, this function returns a directed graph.


## See also:

```
fast_gnp_random_graph()
```


## Notes

This algorithm runs in $O\left(n^{\wedge} 2\right)$ time. For sparse graphs (that is, for small values of $p$ ), fast_gnp_random_graph() is a faster algorithm.

## References

### 6.5.7 newman_watts_strogatz_graph

newman_watts_strogatz_graph ( $n, k, p$, seed=None)
Return a Newman-Watts-Strogatz small-world graph.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- $\mathbf{k}$ (int) - Each node is joined with its k nearest neighbors in a ring topology.
- $\mathbf{p}($ float $)$ - The probability of adding a new edge for each edge.
- seed (int, optional) - The seed for the random number generator (the default is None).


## Notes

First create a ring over n nodes. Then each node in the ring is connected with its k nearest neighbors (or $\mathrm{k}-1$ neighbors if $k$ is odd). Then shortcuts are created by adding new edges as follows: for each edge ( $u, v$ ) in the underlying " n -ring with k nearest neighbors" with probability $p$ add a new edge ( $\mathrm{u}, \mathrm{w}$ ) with randomly-chosen existing node $w$. In contrast with watts_strogatz_graph (), no edges are removed.

## See also:

```
watts_strogatz_graph()
```


## References

### 6.5.8 watts_strogatz_graph

watts_strogatz_graph $(n, k, p$, seed=None)
Return a Watts-Strogatz small-world graph.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes
- $\mathbf{k}($ int $)$ - Each node is joined with its $k$ nearest neighbors in a ring topology.
- $\mathbf{p}$ (float) - The probability of rewiring each edge
- seed (int, optional) - Seed for random number generator (default=None)

See also:

```
newman_watts_strogatz_graph(), connected_watts_strogatz_graph()
```


## Notes

First create a ring over n nodes. Then each node in the ring is joined to its k nearest neighbors (or $\mathrm{k}-1$ neighbors if $k$ is odd). Then shortcuts are created by replacing some edges as follows: for each edge ( $u, v$ ) in the underlying " $n$-ring with $k$ nearest neighbors" with probability $p$ replace it with a new edge ( $u, w$ ) with uniformly random choice of existing node w .
In contrast with newman_watts_strogatz_graph(), the random rewiring does not increase the number of edges. The rewired graph is not guaranteed to be connected as in connected_watts_strogatz_graph().

## References

### 6.5.9 connected_watts_strogatz_graph

connected_watts_strogatz_graph ( $n, k, p$, tries $=100$, seed $=$ None $)$
Returns a connected Watts-Strogatz small-world graph.
Attempts to generate a connected graph by repeated generation of Watts-Strogatz small-world graphs. An exception is raised if the maximum number of tries is exceeded.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes
- $\mathbf{k}($ int $)$ - Each node is joined with its k nearest neighbors in a ring topology.
- $\mathbf{p}$ (float) - The probability of rewiring each edge
- tries (int) - Number of attempts to generate a connected graph.
- seed (int, optional) - The seed for random number generator.


## See also:

```
newman_watts_strogatz_graph(),watts_strogatz_graph()
```


### 6.5.10 random_regular_graph

random_regular_graph $(d, n$, seed=None)
Returns a random d-regular graph on n nodes.
The resulting graph has no self-loops or parallel edges.

## Parameters

- d (int) - The degree of each node.
- $\mathbf{n}$ (integer) - The number of nodes. The value of $n * d$ must be even.
- seed (hashable object) - The seed for random number generator.


## Notes

The nodes are numbered from 0 to $\mathrm{n}-1$.
Kim and Vu's paper ${ }^{2}$ shows that this algorithm samples in an asymptotically uniform way from the space of random graphs when $d=O\left(n^{\wedge}\{1 / 3\right.$-epsilon $)$.

Raises NetworkXError - If $n * d$ is odd or d is greater than or equal to n .

## References

### 6.5.11 barabasi_albert_graph

barabasi_albert_graph ( $n, m$, seed=None)
Returns a random graph according to the Barabási-Albert preferential attachment model.
A graph of $n$ nodes is grown by attaching new nodes each with $m$ edges that are preferentially attached to existing nodes with high degree.

## Parameters

- $\mathbf{n}($ int $)$ - Number of nodes
- m (int) - Number of edges to attach from a new node to existing nodes
- seed (int, optional) - Seed for random number generator (default=None).

Returns G
Return type Graph
Raises NetworkXError - If m does not satisfy $1<=m<n$.

## References

### 6.5.12 powerlaw_cluster_graph

powerlaw_cluster_graph ( $n, m, p$, seed=None)
Holme and Kim algorithm for growing graphs with powerlaw degree distribution and approximate average clustering.

[^93]
## Parameters

- $\mathbf{n}$ (int) - the number of nodes
- $\mathbf{m}$ (int) - the number of random edges to add for each new node
- $\mathbf{p}$ (float, ) - Probability of adding a triangle after adding a random edge
- seed (int, optional) - Seed for random number generator (default=None).


## Notes

The average clustering has a hard time getting above a certain cutoff that depends on $m$. This cutoff is often quite low. The transitivity (fraction of triangles to possible triangles) seems to decrease with network size.
It is essentially the Barabási-Albert (BA) growth model with an extra step that each random edge is followed by a chance of making an edge to one of its neighbors too (and thus a triangle).
This algorithm improves on BA in the sense that it enables a higher average clustering to be attained if desired.
It seems possible to have a disconnected graph with this algorithm since the initial $m$ nodes may not be all linked to a new node on the first iteration like the BA model.

Raises NetworkXError - If m does not satisfy $1<=m<=n$ or $p$ does not satisfy $0<=p$ <= 1 .

## References

### 6.5.13 random_kernel_graph

random_kernel_graph ( $n$, kernel_integral, kernel_root=None, seed=None)
Return an random graph based on the specified kernel.
The algorithm chooses each of the $[\mathrm{n}(\mathrm{n}-1)] / 2$ possible edges with probability specified by a kernel kappa $(x, y)^{1}$. The kernel kappa ( $x, y$ ) must be a symmetric (in $x, y$ ), non-negative, bounded function.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes
- kernal_integral (function) - Function that returns the definite integral of the kernel kappa( $x, y), F(y, a, b):=$ int_a^b kappa $(x, y) d x$
- kernel_root (function (optional)) - Function that returns the root b of the equation $F(y, a, b)=r$. If None, the root is found using scipy.optimize.brentq() (this requires SciPy).
- seed (int, optional) - Seed for random number generator (default=None)


## Notes

The kernel is specified through its definite integral which must be provided as one of the arguments. If the integral and root of the kernel integral can be found in $O$ (1) time then this algorithm runs in time $O(n+m)$ where m is the expected number of edges ${ }^{2}$.

[^94]The nodes are set to integers from 0 to $\mathrm{n}-1$.

## Examples

Generate an Erdős-Rényi random graph $G(n, c / n)$, with kernel kappa $(x, y)=c$ where $c$ is the mean expected degree.

```
>>> def integral(u, w, z):
... return C* (z-w)
>>> def root(u, w, r):
... return r/c+w
>>> c=1
>>> graph = random_kernel_graph(1000, integral, root)
```


## See also:

```
gnp_random_graph(), expected_degree_graph()
```


## References

### 6.5.14 random_lobster

random_lobster ( $n, p 1, p 2$, seed=None)
Returns a random lobster graph.
A lobster is a tree that reduces to a caterpillar when pruning all leaf nodes. A caterpillar is a tree that reduces to a path graph when pruning all leaf nodes; setting p2 to zero produces a caterillar.

## Parameters

- $\mathbf{n}$ (int) - The expected number of nodes in the backbone
- p1 (float) - Probability of adding an edge to the backbone
- p2 (float) - Probability of adding an edge one level beyond backbone
- seed (int, optional) - Seed for random number generator (default=None).


### 6.5.15 random_shell_graph

random_shell_graph (constructor, seed=None)
Returns a random shell graph for the constructor given.

## Parameters

- constructor (list of three-tuples) - Represents the parameters for a shell, starting at the center shell. Each element of the list must be of the form ( $n, m, d$ ), where $n$ is the number of nodes in the shell, $m$ is the number of edges in the shell, and $d$ is the ratio of inter-shell (next) edges to intra-shell edges. If $d$ is zero, there will be no intra-shell edges, and if $d$ is one there will be all possible intra-shell edges.
- seed (int, optional) - Seed for random number generator (default=None).


## Examples

```
>>> constructor = [(10, 20, 0.8), (20, 40, 0.8)]
>>> G = nx.random_shell_graph(constructor)
```


### 6.5.16 random_powerlaw_tree

random_powerlaw_tree ( $n$, gamma $=3$, seed $=$ None, tries $=100$ )
Returns a tree with a power law degree distribution.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes.
- gamma (float) - Exponent of the power law.
- seed (int, optional) - Seed for random number generator (default=None).
- tries (int) - Number of attempts to adjust the sequence to make it a tree.

Raises NetworkXError - If no valid sequence is found within the maximum number of attempts.

## Notes

A trial power law degree sequence is chosen and then elements are swapped with new elements from a powerlaw distribution until the sequence makes a tree (by checking, for example, that the number of edges is one smaller than the number of nodes).

### 6.5.17 random_powerlaw_tree_sequence

random_powerlaw_tree_sequence ( $n$, gamma $=3$, seed $=$ None, tries $=100$ )
Returns a degree sequence for a tree with a power law distribution.

## Parameters

- $\mathbf{n}$ (int, ) - The number of nodes.
- gamma (float) - Exponent of the power law.
- seed (int, optional) - Seed for random number generator (default=None).
- tries (int) - Number of attempts to adjust the sequence to make it a tree.

Raises NetworkXError - If no valid sequence is found within the maximum number of attempts.

## Notes

A trial power law degree sequence is chosen and then elements are swapped with new elements from a power law distribution until the sequence makes a tree (by checking, for example, that the number of edges is one smaller than the number of nodes).

### 6.6 Duplication Divergence

Functions for generating graphs based on the "duplication" method.
These graph generators start with a small initial graph then duplicate nodes and (partially) duplicate their edges. These functions are generally inspired by biological networks.

| duplication_divergence_graph(n, p[, seed]) | Returns an undirected graph using the duplication- <br> divergence model. |
| :--- | :--- |
| partial_duplication_graph( $\mathrm{N}, \mathrm{n}, \mathrm{p}, \mathrm{q}[$, seed $])$ | Return a random graph using the partial duplication model. |

### 6.6.1 duplication_divergence_graph

duplication_divergence_graph ( $n, p$, seed=None)
Returns an undirected graph using the duplication-divergence model.
A graph of $n$ nodes is created by duplicating the initial nodes and retaining edges incident to the original nodes with a retention probability $p$.

## Parameters

- $\mathbf{n}($ int $)$ - The desired number of nodes in the graph.
- $\mathbf{p}$ (float) - The probability for retaining the edge of the replicated node.
- seed (int, optional) - A seed for the random number generator of random (default=None).


## Returns G

Return type Graph
Raises NetworkXError - If p is not a valid probability. If n is less than 2 .

## Notes

This algorithm appears in [1].
This implementation disallows the possibility of generating disconnected graphs.

## References

### 6.6.2 partial_duplication_graph

partial_duplication_graph ( $N, n, p, q$, seed=None)
Return a random graph using the partial duplication model.

## Parameters

- $\mathbf{N}($ int $)$ - The total number of nodes in the final graph.
- $\mathbf{n}($ int $)$ - The number of nodes in the initial clique.
- $\mathbf{p}$ (float) - The probability of joining each neighbor of a node to the duplicate node. Must be a number in the between zero and one, inclusive.
- $\mathbf{q}$ (float) - The probability of joining the source node to the duplicate node. Must be a number in the between zero and one, inclusive.
- seed (int, optional) - Seed for random number generator (default=None).


## Notes

A graph of nodes is grown by creating a fully connected graph of size $n$. The following procedure is then repeated until a total of N nodes have been reached.
1.A random node, $u$, is picked and a new node, $v$, is created.
2.For each neighbor of $u$ an edge from the neighbor to $v$ is created with probability p .
3.An edge from $u$ to $v$ is created with probability q .

This algorithm appears in [1].
This implementation allows the possibility of generating disconnected graphs.

## References

### 6.7 Degree Sequence

Generate graphs with a given degree sequence or expected degree sequence.

| configuration_model(deg_sequence[, ..]) | Return a random graph with the given degree sequence. |
| :--- | :--- |
| directed_configuration_model(..., ..]) | Return a directed_random graph with the given degree se- <br> quences. |
| expected_degree_graph(w[, seed, selfloops]) | Return a random graph with given expected degrees. |
| havel_hakimi_graph(deg_sequence[, create_using]) | Return a simple graph with given degree sequence con- <br> structed using the Havel-Hakimi algorithm. |
| directed_havel_hakimi_graph(in_deg_sequence,, | Return a directed graph with the given degree sequences. |
| $\ldots .)$. | Make a tree for the given degree sequence. |
| degree_sequence_tree(deg_sequence[,...]) | Return a simple random graph with the given degree se- <br> quence. |
| random_degree_sequence_graph(sequence[, ...]) |  |

### 6.7.1 configuration_model

configuration_model (deg_sequence, create_using=None, seed=None)
Return a random graph with the given degree sequence.
The configuration model generates a random pseudograph (graph with parallel edges and self loops) by randomly assigning edges to match the given degree sequence.

## Parameters

- deg_sequence (list of integers) - Each list entry corresponds to the degree of a node.
- create_using (graph, optional (default MultiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - Seed for random number generator.

Returns $\mathbf{G}$ - A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence. $^{\text {sequ }}$

## Return type MultiGraph

Raises NetworkXError - If the degree sequence does not have an even sum.

## See also:

```
is_valid__degree_sequence()
```


## Notes

As described by Newman ${ }^{1}$.
A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self loops and parallel edges. An exception is raised if the degree sequence does not have an even sum.

This configuration model construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn't have the exact degree sequence specified.

The density of self-loops and parallel edges tends to decrease as the number of nodes increases. However, typically the number of self-loops will approach a Poisson distribution with a nonzero mean, and similarly for the number of parallel edges. Consider a node with k stubs. The probability of being joined to another stub of the same node is basically $(\mathrm{k}-1) / \mathrm{N}$ where k is the degree and N is the number of nodes. So the probability of a self-loop scales like $\mathrm{c} / \mathrm{N}$ for some constant c . As N grows, this means we expect c self-loops. Similarly for parallel edges.

## References

## Examples

```
>>> from networkx.utils import powerlaw_sequence
>>> z=nx.utils.create_degree_sequence(100,powerlaw_sequence)
>>> G=nx.configuration_model(z)
```

To remove parallel edges:

```
>>> G=nx.Graph(G)
```

To remove self loops:

```
>>> G.remove_edges_from(G.selfloop_edges())
```


### 6.7.2 directed_configuration_model

directed_configuration_model (in_degree_sequence, out_degree_sequence, create_using=None, seed=None)
Return a directed_random graph with the given degree sequences.
The configuration model generates a random directed pseudograph (graph with parallel edges and self loops) by randomly assigning edges to match the given degree sequences.

## Parameters

- in_degree_sequence (list of integers) - Each list entry corresponds to the in-degree of a node.

[^95]- out_degree_sequence (list of integers) - Each list entry corresponds to the out-degree of a node.
- create_using (graph, optional (default MultiDiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - Seed for random number generator.

Returns G - A graph with the specified degree sequences. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence.

## Return type MultiDiGraph

Raises NetworkXError - If the degree sequences do not have the same sum.

## See also:

```
configuration_model()
```


## Notes

Algorithm as described by Newman ${ }^{1}$.
A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self loops and parallel edges. An exception is raised if the degree sequences does not have the same sum.

This configuration model construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn't have the exact degree sequence specified. This "finite-size effect" decreases as the size of the graph increases.

## References

## Examples

```
>>> D=nx.DiGraph([(0,1),(1,2),(2,3)]) # directed path graph
>>> din=list(d for n, d in D.in_degree())
>>> dout=list(d for n, d in D.out_degree())
>>> din.append(1)
>>> dout[0]=2
>>> D=nx.directed_configuration_model(din,dout)
```

To remove parallel edges:

```
>>> D=nx.DiGraph(D)
```

To remove self loops:

```
>>> D.remove_edges_from(D.selfloop_edges())
```

[^96]
### 6.7.3 expected_degree_graph

## expected_degree_graph ( $w$, seed=None, selfloops=True)

Return a random graph with given expected degrees.
Given a sequence of expected degrees $W=\left(w \_0, w \_1,1 \operatorname{dot} s, w \_\{n-1\}\right)$ of length $n$ this algorithm assigns an edge between node $u$ and node $v$ with probability

$$
p_{u v}=\frac{w_{u} w_{v}}{\sum_{k} w_{k}}
$$

## Parameters

- $\mathbf{w}$ (list) - The list of expected degrees.
- selfloops (bool (default=True)) - Set to False to remove the possibility of self-loop edges.
- seed (hashable object, optional) - The seed for the random number generator.


## Returns

Return type Graph

## Examples

>>> $\mathrm{z}=[10$ for $i$ in range (100)]
>>> G=nx.expected_degree_graph(z)

## Notes

The nodes have integer labels corresponding to index of expected degrees input sequence.
The complexity of this algorithm is mathcal $\{O\}(n+m)$ where $n$ is the number of nodes and $m$ is the expected number of edges.

The model in ${ }^{1}$ includes the possibility of self-loop edges. Set selfloops=False to produce a graph without self loops.

For finite graphs this model doesn't produce exactly the given expected degree sequence. Instead the expected degrees are as follows.
For the case without self loops (selfloops=False),

$$
E[\operatorname{deg}(u)]=\sum_{v \neq u} p_{u v}=w_{u}\left(1-\frac{w_{u}}{\sum_{k} w_{k}}\right)
$$

NetworkX uses the standard convention that a self-loop edge counts 2 in the degree of a node, so with self loops (selfloops=True),

$$
E[\operatorname{deg}(u)]=\sum_{v \neq u} p_{u v}+2 p_{u u}=w_{u}\left(1+\frac{w_{u}}{\sum_{k} w_{k}}\right)
$$

[^97]
## References

### 6.7.4 havel_hakimi_graph

havel_hakimi_graph (deg_sequence, create_using=None)
Return a simple graph with given degree sequence constructed using the Havel-Hakimi algorithm.

## Parameters

- deg_sequence (list of integers) - Each integer corresponds to the degree of a node (need not be sorted).
- create_using (graph, optional (default Graph)) - Return graph of this type. The instance will be cleared. Directed graphs are not allowed.

Raises NetworkXException - For a non-graphical degree sequence (i.e. one not realizable by some simple graph).

## Notes

The Havel-Hakimi algorithm constructs a simple graph by successively connecting the node of highest degree to other nodes of highest degree, resorting remaining nodes by degree, and repeating the process. The resulting graph has a high degree-associativity. Nodes are labeled $1, .$. , len(deg_sequence), corresponding to their position in deg_sequence.
The basic algorithm is from Hakimi ${ }^{1}$ and was generalized by Kleitman and Wang ${ }^{2}$.

## References

### 6.7.5 directed_havel_hakimi_graph

directed_havel_hakimi_graph (in_deg_sequence, out_deg_sequence, create_using=None)
Return a directed graph with the given degree sequences.

## Parameters

- in_deg_sequence (list of integers) - Each list entry corresponds to the in-degree of a node.
- out_deg_sequence (list of integers) - Each list entry corresponds to the out-degree of a node.
- create_using (graph, optional (default DiGraph)) - Return graph of this type. The instance will be cleared.

Returns $\mathbf{G}$ - A graph with the specified degree sequences. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence
Return type DiGraph
Raises NetworkXError - If the degree sequences are not digraphical.

## See also:

configuration_model()

[^98]
## Notes

Algorithm as described by Kleitman and Wang ${ }^{1}$.

## References

### 6.7.6 degree_sequence_tree

degree_sequence_tree (deg_sequence, create_using=None)
Make a tree for the given degree sequence.
A tree has \#nodes-\#edges=1 so the degree sequence must have len(deg_sequence)-sum(deg_sequence) $/ 2=1$

### 6.7.7 random_degree_sequence_graph

random_degree_sequence_graph (sequence, seed=None, tries=10)
Return a simple random graph with the given degree sequence.
If the maximum degree $d \_m$ in the sequence $\operatorname{is} O\left(m^{\wedge}\{1 / 4\}\right)$ then the algorithm produces almost uniform random graphs in $O$ ( m d_m) time where $m$ is the number of edges.

## Parameters

- sequence (list of integers) - Sequence of degrees
- seed (hashable object, optional) - Seed for random number generator
- tries (int, optional) - Maximum number of tries to create a graph

Returns G - A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in the sequence.

Return type Graph

## Raises

- NetworkXUnfeasible - If the degree sequence is not graphical.
- NetworkXError - If a graph is not produced in specified number of tries


## See also:

is_valid_degree_sequence(), configuration_model()

## Notes

The generator algorithm ${ }^{1}$ is not guaranteed to produce a graph.

[^99]
## References

## Examples

```
>>> sequence = [1, 2, 2, 3]
>>> G = nx.random_degree_sequence_graph(sequence)
>>> sorted(d for n, d in G.degree())
[1, 2, 2, 3]
```


### 6.8 Random Clustered

Generate graphs with given degree and triangle sequence.
random_clustered_graph(joint_degree_sequence) Generate a random graph with the given joint independent edge degree and triangle degree sequence.

### 6.8.1 random_clustered_graph

random_clustered_graph (joint_degree_sequence, create_using=None, seed=None)
Generate a random graph with the given joint independent edge degree and triangle degree sequence.
This uses a configuration model-like approach to generate a random graph (with parallel edges and self-loops) by randomly assigning edges to match the given joint degree sequence.

The joint degree sequence is a list of pairs of integers of the form $\left[\left(d \_\{1, i\}, d \_\{1, t\}\right)\right.$, $\left.\operatorname{dotsc},\left(d \_\{n, i\}, d \_\{n, t\}\right)\right]$. According to this list, vertex $u$ is $a$ member of $d \_\{u, t\}$ triangles and has $d \_\{u, i\}$ other edges. The number $d \_\{u, t\}$ is the triangle degree of $u$ and the number $d \_\{u, i\}$ is the independent edge degree.

## Parameters

- joint_degree_sequence (list of integer pairs) - Each list entry corresponds to the independent edge degree and triangle degree of a node.
- create_using (graph, optional (default MultiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - The seed for the random number generator.

Returns G - A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence.

## Return type MultiGraph

Raises NetworkXError - If the independent edge degree sequence sum is not even or the triangle degree sequence sum is not divisible by 3 .

## Notes

As described by Miller ${ }^{1}$ (see also Newman ${ }^{2}$ for an equivalent description).

[^100]A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self loops and parallel edges. An exception is raised if the independent degree sequence does not have an even sum or the triangle degree sequence sum is not divisible by 3 .

This configuration model-like construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn't have the exact degree sequence specified. This "finite-size effect" decreases as the size of the graph increases.

## References

## Examples

```
>>> deg = [(1, 0), (1, 0), (1, 0), (2, 0), (1, 0), (2, 1), (0, 1), (0, 1)]
>>> G = nx.random_clustered_graph(deg)
```

To remove parallel edges:

```
>>> G = nx.Graph(G)
```

To remove self loops:

```
>>> G.remove_edges_from(G.selfloop_edges())
```


### 6.9 Directed

Generators for some directed graphs, including growing network (GN) graphs and scale-free graphs.

| gn_graph(n[, kernel, create_using, seed]) | Return the growing network (GN) digraph with n nodes. |
| :--- | :--- |
| gnr_graph(n, p[, create_using, seed]) | Return the growing network with redirection (GNR) di- <br> graph with n nodes and redirection probability p. |
| gnc_graph(n[, create_using, seed]) | Return the growing network with copying (GNC) digraph <br> with n nodes. |
| random_k_out_graph(n, k, alpha[, ...]) | Returns a random k-out graph with preferential attachment. |
| scale_free_graph(n[, alpha, beta, gamma, ...]) | Returns a scale-free directed graph. |

### 6.9.1 gn_graph

gn_graph ( $n$, kernel=None, create_using=None, seed=None)
Return the growing network (GN) digraph with n nodes.
The GN graph is built by adding nodes one at a time with a link to one previously added node. The target node for the link is chosen with probability based on degree. The default attachment kernel is a linear function of the degree of a node.

The graph is always a (directed) tree.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes for the generated graph.
- kernel (function) - The attachment kernel.
- create_using (graph, optional (default DiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - The seed for the random number generator.


## Examples

To create the undirected GN graph, use the to_directed () method:

```
>>> D = nx.gn_graph(10) # the GN graph
>>> G = D.to_undirected() # the undirected version
```

To specify an attachment kernel, use the kernel keyword argument:

```
>>> D = nx.gn_graph(10, kernel=lambda x: x ** 1.5) # A_k = k^1.5
```


## References

### 6.9.2 gnr_graph

gnr_graph $(n, p$, create_using=None, seed $=$ None $)$
Return the growing network with redirection (GNR) digraph with n nodes and redirection probability p .
The GNR graph is built by adding nodes one at a time with a link to one previously added node. The previous target node is chosen uniformly at random. With probabiliy p the link is instead "redirected" to the successor node of the target.

The graph is always a (directed) tree.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes for the generated graph.
- $\mathbf{p}$ (float) - The redirection probability.
- create_using (graph, optional (default DiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - The seed for the random number generator.


## Examples

To create the undirected GNR graph, use the to_directed () method:

```
>>> D = nx.gnr_graph(10, 0.5) # the GNR graph
>>> G = D.to_undirected() # the undirected version
```


## References

### 6.9.3 gnc_graph

gnc_graph ( $n$, create_using=None, seed=None)
Return the growing network with copying (GNC) digraph with n nodes.

The GNC graph is built by adding nodes one at a time with a link to one previously added node (chosen uniformly at random) and to all of that node's successors.

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes for the generated graph.
- create_using (graph, optional (default DiGraph)) - Return graph of this type. The instance will be cleared.
- seed (hashable object, optional) - The seed for the random number generator.


## References

### 6.9.4 random_k_out_graph

random_k_out_graph ( $n, k$, alpha, self_loops=True, seed=None)
Returns a random $k$-out graph with preferential attachment.
A random $k$-out graph with preferential attachment is a multidigraph generated by the following algorithm.
1.Begin with an empty digraph, and initially set each node to have weight alpha.
2.Choose a node u with out-degree less than k uniformly at random.
3. Choose a node v from with probability proportional to its weight.
4. Add a directed edge from $u$ to $v$, and increase the weight of $v$ by one.
5.If each node has out-degree k , halt, otherwise repeat from step 2.

For more information on this model of random graph, see [1].

## Parameters

- $\mathbf{n}($ int $)$ - The number of nodes in the returned graph.
- $\mathbf{k}$ (int) - The out-degree of each node in the returned graph.
- alpha (float) - A positive float representing the initial weight of each vertex. A higher number means that in step 3 above, nodes will be chosen more like a true uniformly random sample, and a lower number means that nodes are more likely to be chosen as their in-degree increases. If this parameter is not positive, a ValueError is raised.
- self_loops (bool) - If True, self-loops are allowed when generating the graph.
- seed (int) - If provided, this is used as the seed for the random number generator.

Returns A k-out-regular multidigraph generated according to the above algorithm.
Return type MultiDiGraph
Raises ValueError - If alpha is not positive.

## Notes

The returned multidigraph may not be strongly connected, or even weakly connected.

## References

[1]: Peterson, Nicholas R., and Boris Pittel. "Distance between two random k-out digraphs, with and without preferential attachment." arXiv preprint arXiv:1311.5961 (2013). [http://arxiv.org/abs/1311.5961](http://arxiv.org/abs/1311.5961)

### 6.9.5 scale_free_graph

scale_free_graph $(n$, alpha $=0.41$, beta=0.54, gamma=0.05, delta_in $=0.2$, delta_out=0, create_using=None, seed=None)
Returns a scale-free directed graph.

## Parameters

- n (integer) - Number of nodes in graph
- alpha (float) - Probability for adding a new node connected to an existing node chosen randomly according to the in-degree distribution.
- beta $($ float $)$ - Probability for adding an edge between two existing nodes. One existing node is chosen randomly according the in-degree distribution and the other chosen randomly according to the out-degree distribution.
- gamma (float) - Probability for adding a new node connected to an existing node chosen randomly according to the out-degree distribution.
- delta_in (float) - Bias for choosing ndoes from in-degree distribution.
- delta_out (float) - Bias for choosing ndoes from out-degree distribution.
- create_using (graph, optional (default MultiDiGraph)) - Use this graph instance to start the process (default=3-cycle).
- seed (integer, optional) - Seed for random number generator


## Examples

Create a scale-free graph on one hundred nodes:

```
>>> G = nx.scale_free_graph(100)
```


## Notes

The sum of alpha, beta, and gamma must be 1 .

## References

### 6.10 Geometric

Generators for geometric graphs.

| random_geometric_graph(n, radius[, dim, ...]) | Returns a random geometric graph in the unit cube. |
| :--- | :--- |
| geographical_threshold_graph(n, theta[, ...]) | Returns a geographical threshold graph. |
|  | Continued on next page |

Table 6.10 - continued from previous page

| waxman_graph(n[, alpha, beta, L, domain, metric]) | Return a Waxman random graph. |
| :--- | :--- |
| navigable_small_world_graph(n[, p, q, r, ...]) | Return a navigable small-world graph. |

### 6.10.1 random_geometric_graph

random_geometric_graph ( $n$, radius, $\operatorname{dim}=2$, pos $=$ None, metric $=$ None)
Returns a random geometric graph in the unit cube.
The random geometric graph model places $n$ nodes uniformly at random in the unit cube. Two nodes are joined by an edge if the distance between the nodes is at most radius.

## Parameters

- $\mathbf{n}$ (int or iterable) - Number of nodes or iterable of nodes
- radius (float) - Distance threshold value
- dim (int, optional) - Dimension of graph
- pos (dict, optional) - A dictionary keyed by node with node positions as values.
- metric (function) - A metric on vectors of numbers (represented as lists or tuples). This must be a function that accepts two lists (or tuples) as input and yields a number as output. The function must also satisfy the four requirements of a metric. Specifically, if $d$ is the function and $x, y$, and $z$ are vectors in the graph, then $d$ must satisfy

1. $d^{*}(* x, y) 0$,
2. $d^{*}(* x, y)=0$ if and only if $x=y$,
3. $d^{*}\left({ }^{*} x, y\right)=d^{*}\left({ }^{*} y, x\right)$,
4. $d^{*}\left({ }^{*} x, z\right) d^{*}\left({ }^{*} x, y\right)+d^{*}\left({ }^{*} y, z\right)$.

If this argument is not specified, the Euclidean distance metric is used.
Returns A random geometric graph, undirected and without self-loops. Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as provided by the pos keyword argument or, if pos was not provided, as generated by this function.

## Return type Graph

## Examples

Create a random geometric graph on twenty nodes where nodes are joined by an edge if their distance is at most 0.1:
>>> G = nx.random_geometric_graph (20, 0.1)
Specify an alternate distance metric using the metric keyword argument. For example, to use the "taxicab metric" instead of the default Euclidean metric:

```
>>> dist = lambda x, y: sum(abs(a - b) for a, b in zip(x, y))
>>> G = nx.random_geometric_graph(10, 0.1, metric=dist)
```


## Notes

This uses an $O\left(n^{\wedge} 2\right)$ algorithm to build the graph. A faster algorithm is possible using $k$-d trees.
The pos keyword argument can be used to specify node positions so you can create an arbitrary distribution and domain for positions.

For example, to use a 2D Gaussian distribution of node positions with mean $(0,0)$ and standard deviation 2:

```
>>> import random
>>> n = 20
>>> p = {i: (random.gauss(0, 2), random.gauss(0, 2)) for i in range(n)}
>>> G = nx.random_geometric_graph(n, 0.2, pos=p)
```


## References

### 6.10.2 geographical_threshold_graph

geographical_threshold_graph ( $n$, theta, alpha=2, dim $=2$, pos $=$ None, weight $=$ None, metric $=$ None )
Returns a geographical threshold graph.
The geographical threshold graph model places $n$ nodes uniformly at random in a rectangular domain. Each node u is assigned a weight $w_{u}$. Two nodes u and v are joined by an edge if

$$
w_{u}+w_{v} \geq \theta r^{\alpha}
$$

where $r$ is the distance between u and v , and $\theta, \alpha$ are parameters.

## Parameters

- $\mathbf{n}$ (int or iterable) - Number of nodes or iterable of nodes
- theta (float) - Threshold value
- alpha (float, optional) - Exponent of distance function
- dim (int, optional) - Dimension of graph
- pos (dict) - Node positions as a dictionary of tuples keyed by node.
- weight (dict) - Node weights as a dictionary of numbers keyed by node.
- metric (function) - A metric on vectors of numbers (represented as lists or tuples). This must be a function that accepts two lists (or tuples) as input and yields a number as output. The function must also satisfy the four requirements of a metric. Specifically, if $d$ is the function and $x, y$, and $z$ are vectors in the graph, then $d$ must satisfy

1. $d^{*}(* x, y) 0$,
2. $d^{*}(* x, y)=0$ if and only if $x=y$,
3. $d^{*}(* x, y)=d^{*}(* y, x)$,
4. $d^{*}(* x, z) d^{*}(* x, y)+d^{*}\left({ }^{*} y, z\right)$.

If this argument is not specified, the Euclidean distance metric is used.

## Returns

A random geographic threshold graph, undirected and without self-loops.

Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as provided by the pos keyword argument or, if pos was not provided, as generated by this function. Similarly, each node has a node attribute 'weight' that stores the weight of that node as provided or as generated.

Return type Graph

## Examples

Specify an alternate distance metric using the metric keyword argument. For example, to use the "taxicab metric" instead of the default Euclidean metric:

```
>>> dist = lambda x, y: sum(abs(a - b) for a, b in zip(x, y))
>>> G = nx.geographical_threshold_graph(10, 0.1, metric=dist)
```


## Notes

If weights are not specified they are assigned to nodes by drawing randomly from the exponential distribution with rate parameter $\lambda=1$. To specify weights from a different distribution, use the weight keyword argument:

```
>>> import random
>>> n = 20
>>> w = {i: random.expovariate(5.0) for i in range(n)}
>>> G = nx.geographical_threshold_graph(20, 50, weight=w)
```

If node positions are not specified they are randomly assigned from the uniform distribution.

## References

### 6.10.3 waxman_graph

waxman_graph ( $n$, alpha=0.4, beta $=0.1, L=$ None, domain $=(0,0,1,1)$, metric $=$ None $)$
Return a Waxman random graph.
The Waxman random graph model places $n$ nodes uniformly at random in a rectangular domain. Each pair of nodes at distance $d$ is joined by an edge with probability

$$
p=\alpha \exp (-d / \beta L)
$$

This function implements both Waxman models, using the $L$ keyword argument.
-Waxman-1: if $L$ is not specified, it is set to be the maximum distance between any pair of nodes.
-Waxman-2: if $L$ is specified, the distance between a pair of nodes is chosen uniformly at random from the interval [ $0, \mathrm{~L}$ ].

## Parameters

- $\mathbf{n}$ (int or iterable) - Number of nodes or iterable of nodes
- alpha (float) - Model parameter
- beta (float) - Model parameter
- L (float, optional) - Maximum distance between nodes. If not specified, the actual distance is calculated.
- domain (four-tuple of numbers, optional) - Domain size, given as a tuple of the form (x_min,y_min, x_max,y_max).
- metric (function) - A metric on vectors of numbers (represented as lists or tuples). This must be a function that accepts two lists (or tuples) as input and yields a number as output. The function must also satisfy the four requirements of a metric. Specifically, if $d$ is the function and $x, y$, and $z$ are vectors in the graph, then $d$ must satisfy

1. $d^{*}\left({ }^{*} x, y\right) 0$,
2. $d^{*}(* x, y)=0$ if and only if $x=y$,
3. $d^{*}(* x, y)=d^{*}(* y, x)$,
4. $d^{*}(* x, z) d^{*}(* x, y)+d^{*}\left({ }^{*} y, z\right)$.

If this argument is not specified, the Euclidean distance metric is used.
Returns A random Waxman graph, undirected and without self-loops. Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as generated by this function.

## Return type Graph

## Examples

Specify an alternate distance metric using the metric keyword argument. For example, to use the "taxicab metric" instead of the default Euclidean metric:

```
>>> dist = lambda x, y: sum(abs(a - b) for a, b in zip(x, y))
>>> G = nx.waxman_graph(10, 0.5, 0.1, metric=dist)
```


## References

### 6.10.4 navigable_small_world_graph

navigable_small_world_graph ( $n, p=1, q=1, r=2$, dim $=2$, seed $=$ None)
Return a navigable small-world graph.
A navigable small-world graph is a directed grid with additional long-range connections that are chosen randomly.
[...] we begin with a set of nodes [...] that are identified with the set of lattice points in an $n \times n$ square, $\{(i, j): i \in\{1,2, \ldots, n\}, j \in\{1,2, \ldots, n\}\}$, and we define the lattice distance between two nodes $(i, j)$ and $(k, l)$ to be the number of "lattice steps" separating them: $d((i, j),(k, l))$ $=|k-i|+|l-j|$.

For a universal constant $p>=1$, the node $u$ has a directed edge to every other node within lattice distance $p$ - these are its local contacts. For universal constants $q>=0$ and $r>=0$ we also construct directed edges from $u$ to $q$ other nodes (the long-range contacts) using independent random trials; the i`th directed edge from ` $u$ has endpoint $v$ with probability proportional to $[d(u, v)]^{\wedge}\{-r\}$.

## Parameters

[^101]- n (int) - The number of nodes.
- $\mathbf{p}($ int $)$ - The diameter of short range connections. Each node is joined with every other node within this lattice distance.
- $\mathbf{q}$ (int) - The number of long-range connections for each node.
- r (float) - Exponent for decaying probability of connections. The probability of connecting to a node at lattice distance $d$ is $1 / d^{\wedge} r$.
- $\operatorname{dim}(i n t)$ - Dimension of grid
- seed (int, optional) - Seed for random number generator (default=None).


## References

### 6.11 Line Graph

Functions for generating line graphs.
line_graph(G[, create_using]) Returns the line graph of the graph or digraph G.

### 6.11.1 line_graph

line_graph ( $G$, create_using=None)
Returns the line graph of the graph or digraph G.
The line graph of a graph $G$ has a node for each edge in $G$ and an edge joining those nodes if the two edges in $G$ share a common node. For directed graphs, nodes are adjacent exactly when the edges they represent form a directed path of length two.
The nodes of the line graph are 2-tuples of nodes in the original graph (or 3-tuples for multigraphs, with the key of the edge as the third element).

For information about self-loops and more discussion, see the Notes section below.
Parameters G (graph) - A NetworkX Graph, DiGraph, MultiGraph, or MultiDigraph.
Returns $\mathbf{L}$ - The line graph of G.
Return type graph

## Examples

```
>>> import networkx as nx
>>> G = nx.star_graph (3)
>>> L = nx.line_graph(G)
>>> print(sorted(map(sorted, L.edges()))) # makes a 3-clique, K3
[[(0, 1), (0, 2)], [(0, 1), (0, 3)], [(0, 2), (0, 3)]]
```


## Notes

Graph, node, and edge data are not propagated to the new graph. For undirected graphs, the nodes in G must be sortable, otherwise the constructed line graph may not be correct.

## Self-loops in undirected graphs

For an undirected graph $G$ without multiple edges, each edge can be written as a set $\{u, v\}$. Its line graph $L$ has the edges of $G$ as its nodes. If $x$ and $y$ are two nodes in $L$, then $\{x, y\}$ is an edge in $L$ if and only if the intersection of $x$ and $y$ is nonempty. Thus, the set of all edges is determined by the set of all pairwise intersections of edges in G .

Trivially, every edge in G would have a nonzero intersection with itself, and so every node in $L$ should have a self-loop. This is not so interesting, and the original context of line graphs was with simple graphs, which had no self-loops or multiple edges. The line graph was also meant to be a simple graph and thus, self-loops in L are not part of the standard definition of a line graph. In a pairwise intersection matrix, this is analogous to excluding the diagonal entries from the line graph definition.

Self-loops and multiple edges in $G$ add nodes to $L$ in a natural way, and do not require any fundamental changes to the definition. It might be argued that the self-loops we excluded before should now be included. However, the self-loops are still "trivial" in some sense and thus, are usually excluded.

## Self-loops in directed graphs

For a directed graph $G$ without multiple edges, each edge can be written as a tuple ( $u, v$ ). Its line graph $L$ has the edges of $G$ as its nodes. If $x$ and $y$ are two nodes in $L$, then $(x, y)$ is an edge in $L$ if and only if the tail of $x$ matches the head of $y$, for example, if $x=(a, b)$ and $y=(b, c)$ for some vertices $a, b$, and $c$ in $G$.

Due to the directed nature of the edges, it is no longer the case that every edge in $G$ should have a self-loop in L. Now, the only time self-loops arise is if a node in $G$ itself has a self-loop. So such self-loops are no longer "trivial" but instead, represent essential features of the topology of G. For this reason, the historical development of line digraphs is such that self-loops are included. When the graph $G$ has multiple edges, once again only superficial changes are required to the definition.

## References

-Harary, Frank, and Norman, Robert Z., "Some properties of line digraphs", Rend. Circ. Mat. Palermo, II. Ser. 9 (1960), 161-168.
-Hemminger, R. L.; Beineke, L. W. (1978), "Line graphs and line digraphs", in Beineke, L. W.; Wilson, R. J., Selected Topics in Graph Theory, Academic Press Inc., pp. 271-305.

### 6.12 Ego Graph

Ego graph.
ego_graph(G, n[, radius, center, ...])
Returns induced subgraph of neighbors centered at node n within a given radius.

### 6.12.1 ego_graph

ego_graph $(G, n$, radius $=1$, center $=$ True, undirected $=$ False, distance $=$ None ) Returns induced subgraph of neighbors centered at node $n$ within a given radius.

## Parameters

- G (graph) - A NetworkX Graph or DiGraph
- $\mathbf{n}$ (node) - A single node
- radius (number, optional) - Include all neighbors of distance<=radius from n .
- center (bool, optional) - If False, do not include center node in graph
- undirected (bool, optional) - If True use both in- and out-neighbors of directed graphs.
- distance (key, optional) - Use specified edge data key as distance. For example, setting distance='weight' will use the edge weight to measure the distance from the node $n$.


## Notes

For directed graphs D this produces the "out" neighborhood or successors. If you want the neighborhood of predecessors first reverse the graph with D.reverse(). If you want both directions use the keyword argument undirected=True.

Node, edge, and graph attributes are copied to the returned subgraph.

### 6.13 Stochastic

Functions for generating stochastic graphs from a given weighted directed graph.

| stochastic_graph(G[, copy, weight $])$ | Returns a right-stochastic representation of directed graph <br>  <br> G. |
| :--- | :--- |

### 6.13.1 stochastic_graph

stochastic_graph (G, copy=True, weight='weight')
Returns a right-stochastic representation of directed graph G.
A right-stochastic graph is a weighted digraph in which for each node, the sum of the weights of all the out-edges of that node is 1 . If the graph is already weighted (for example, via a 'weight' edge attribute), the reweighting takes that into account.

## Parameters

- G (directed graph) - A DiGraph or MultiDiGraph.
- copy (boolean, optional) - If this is True, then this function returns a new graph with the stochastic reweighting. Otherwise, the original graph is modified in-place (and also returned, for convenience).
- weight (edge attribute key (optional, default='weight')) - Edge attribute key used for reading the existing weight and setting the new weight. If no attribute with this key is found for an edge, then the edge weight is assumed to be 1 . If an edge has a weight, it must be a a positive number.


### 6.14 Intersection

Generators for random intersection graphs.
uniform_random_intersection_graph(n, m, p[, Return a uniform random intersection graph. ...])

Table 6.14 - continued from previous page

| k_random_intersection_graph(n, m, k) | Return a intersection graph with randomly chosen attribute <br> sets for each node that are of equal size $(\mathrm{k})$. |
| :--- | :--- |
| general_random_intersection_graph(n, m, p) | Return a random intersection graph with independent prob- <br> abilities for connections between node and attribute sets. |

### 6.14.1 uniform_random_intersection_graph

uniform_random_intersection_graph ( $n, m, p$, seed=None)
Return a uniform random intersection graph.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes in the first bipartite set (nodes)
- m (int) - The number of nodes in the second bipartite set (attributes)
- p (float) - Probability of connecting nodes between bipartite sets
- seed (int, optional) - Seed for random number generator (default=None).

See also:
gnp_random_graph()

## References

### 6.14.2 k_random_intersection_graph

k_random_intersection_graph ( $n, m, k$ )
Return a intersection graph with randomly chosen attribute sets for each node that are of equal size (k).

## Parameters

- $\mathbf{n}$ (int) - The number of nodes in the first bipartite set (nodes)
- $\mathbf{m}$ (int) - The number of nodes in the second bipartite set (attributes)
- $\mathbf{k}$ (float) - Size of attribute set to assign to each node.
- seed (int, optional) - Seed for random number generator (default=None).

See also:
gnp_random_graph(), uniform_random_intersection_graph()

## References

### 6.14.3 general_random_intersection_graph

general_random_intersection_graph ( $n, m, p$ )
Return a random intersection graph with independent probabilities for connections between node and attribute sets.

## Parameters

- $\mathbf{n}$ (int) - The number of nodes in the first bipartite set (nodes)
- $\mathbf{m}$ (int) - The number of nodes in the second bipartite set (attributes)
- $\mathbf{p}$ (list of floats of length $m$ ) - Probabilities for connecting nodes to each attribute
- seed (int, optional) - Seed for random number generator (default=None).

See also:
gnp_random_graph(), uniform_random_intersection_graph()

References

### 6.15 Social Networks

Famous social networks.

| karate_club_graph () | Return Zachary's Karate Club graph. |
| :--- | :--- |
| davis_southern_women_graph () | Return Davis Southern women social network. |
| florentine_families_graph () | Return Florentine families graph. |

### 6.15.1 karate_club_graph

karate_club_graph()
Return Zachary’s Karate Club graph.
Each node in the returned graph has a node attribute 'club' that indicates the name of the club to which the member represented by that node belongs, either 'Mr. Hi' or 'Officer'.

## Examples

To get the name of the club to which a node belongs:

```
>>> import networkx as nx
>>> G = nx.karate_club_graph()
>>> G.node[5]['club']
'Mr. Hi'
>>> G.node[9]['club']
'Officer'
```


## References

### 6.15.2 davis_southern_women_graph

davis_southern_women_graph ()
Return Davis Southern women social network.
This is a bipartite graph.

## References

### 6.15.3 florentine_families_graph

## florentine_families_graph()

Return Florentine families graph.

## References

### 6.16 Community

Generators for classes of graphs used in studying social networks.

| caveman_graph(l, k) | Returns a caveman graph of l cliques of size k. |
| :--- | :--- |
| connected_caveman_graph $(\mathrm{l}, \mathrm{k})$ | Returns a connected caveman graph of l cliques of size k. |
| relaxed_caveman_graph(l, $\mathrm{k}, \mathrm{p}[$, seed $])$ | Return a relaxed caveman graph. |
| random_partition_graph(sizes, p_in, p_out[, ..]) | Return the random partition graph with a partition of sizes. |
| planted_partition_graph(l, k, p_in, p_out[, ...]) | Return the planted l-partition graph. |
| gaussian_random_partition_graph(n, s, v, ...) | Generate a Gaussian random partition graph. |
| ring_of_cliques(num_cliques, clique_size) | Defines a "ring of cliques" graph. |

### 6.16.1 caveman_graph

## caveman_graph $(l, k)$

Returns a caveman graph of 1 cliques of size $k$.

## Parameters

- l (int) - Number of cliques
- $\mathbf{k}($ int $)$ - Size of cliques

Returns G - caveman graph
Return type NetworkX Graph

## Notes

This returns an undirected graph, it can be converted to a directed graph using nx.to_directed(), or a multigraph using nx.MultiGraph (nx.caveman_graph ( $1, k$ )). Only the undirected version is described in ${ }^{1}$ and it is unclear which of the directed generalizations is most useful.

## Examples

```
>>> G = nx.caveman_graph(3, 3)
```


## See also:

> connected_caveman_graph()

[^102]
## References

### 6.16.2 connected_caveman_graph

connected_caveman_graph $(l, k)$
Returns a connected caveman graph of 1 cliques of size k .
The connected caveman graph is formed by creating $n$ cliques of size $k$, then a single edge in each clique is rewired to a node in an adjacent clique.

## Parameters

- $\mathbf{l}($ int $)$ - number of cliques
- $\mathbf{k}($ int $)$ - size of cliques

Returns G - connected caveman graph
Return type NetworkX Graph

## Notes

This returns an undirected graph, it can be converted to a directed graph using nx.to_directed (), or a multigraph using nx.MultiGraph (nx.caveman_graph (l,k)). Only the undirected version is described in ${ }^{1}$ and it is unclear which of the directed generalizations is most useful.

## Examples

```
>>> G = nx.connected_caveman_graph(3, 3)
```


## References

### 6.16.3 relaxed_caveman_graph

```
relaxed_caveman_graph (l, k, p, seed=None)
```

Return a relaxed caveman graph.
A relaxed caveman graph starts with 1 cliques of size $k$. Edges are then randomly rewired with probability $p$ to link different cliques.

## Parameters

- l (int) - Number of groups
- $\mathbf{k}($ int $)$ - Size of cliques
- p (float) - Probabilty of rewiring each edge.
- seed (int,optional) - Seed for random number generator(default=None)

Returns G-Relaxed Caveman Graph
Return type NetworkX Graph
Raises NetworkXError: - If p is not in $[0,1]$

[^103]
## Examples

```
>>> G = nx.relaxed_caveman_graph(2, 3, 0.1, seed=42)
```


## References

### 6.16.4 random_partition_graph

random_partition_graph (sizes, p_in, p_out, seed=None, directed=False)
Return the random partition graph with a partition of sizes.
A partition graph is a graph of communities with sizes defined by s in sizes. Nodes in the same group are connected with probability p_in and nodes of different groups are connected with probability p_out.

## Parameters

- sizes (list of ints) - Sizes of groups
- p_in (float) - probability of edges with in groups
- p_out (float) - probability of edges between groups
- directed (boolean optional, default=False) - Whether to create a directed graph
- seed (int optional, default None) - A seed for the random number generator

Returns $\mathbf{G}$ - random partition graph of size sum(gs)
Return type NetworkX Graph or DiGraph
Raises NetworkXError - If p_in or p_out is not in [0,1]

## Examples

```
>>> G = nx.random_partition_graph([10,10,10],.25,.01)
>>> len(G)
30
>>> partition = G.graph['partition']
>>> len(partition)
3
```


## Notes

This is a generalization of the planted-1-partition described in ${ }^{1}$. It allows for the creation of groups of any size. The partition is store as a graph attribute 'partition'.

[^104]
## References

### 6.16.5 planted_partition_graph

planted_partition_graph $\left(l, k, p \_i n, p \_o u t\right.$, seed $=$ None, directed $=$ False $)$
Return the planted l-partition graph.
This model partitions a graph with $n=1 * k$ vertices in 1 groups with $k$ vertices each. Vertices of the same group are linked with a probability p _in, and vertices of different groups are linked with probability p_out.

## Parameters

- $\mathbf{l}($ int $)$ - Number of groups
- $\mathbf{k}($ int $)$ - Number of vertices in each group
- p_in (float) - probability of connecting vertices within a group
- p_out (float) - probability of connected vertices between groups
- seed (int,optional) - Seed for random number generator(default=None)
- directed (bool,optional (default=False)) - If True return a directed graph

Returns G - planted 1-partition graph
Return type NetworkX Graph or DiGraph
Raises NetworkXError: - If p_in,p_out are not in [0, 1$]$ or

## Examples

```
>>> G = nx.planted_partition_graph(4, 3, 0.5, 0.1,seed=42)
```


## See also:

```
random_partition_model()
```


## References

### 6.16.6 gaussian_random_partition_graph

gaussian_random_partition_graph ( $n, s, v, p \_i n, p \_o u t$, directed $=$ False, seed $=$ None )
Generate a Gaussian random partition graph.
A Gaussian random partition graph is created by creating k partitions each with a size drawn from a normal distribution with mean $s$ and variance $s / v$. Nodes are connected within clusters with probability p_in and between clusters with probability p_out[1]

## Parameters

- $\mathbf{n}($ int $)$ - Number of nodes in the graph
- s (float) - Mean cluster size
- $\mathbf{v}($ float $)$ - Shape parameter. The variance of cluster size distribution is $\mathrm{s} / \mathrm{v}$.
- p_in (float) - Probabilty of intra cluster connection.
- p_out (float) - Probability of inter cluster connection.
- directed (boolean, optional default=False) - Whether to create a directed graph or not
- seed (int) - Seed value for random number generator

Returns G - gaussian random partition graph
Return type NetworkX Graph or DiGraph
Raises NetworkXError - If $s$ is > n If p_in or p_out is not in $[0,1]$

## Notes

Note the number of partitions is dependent on $\mathrm{s}, \mathrm{v}$ and n , and that the last partition may be considerably smaller, as it is sized to simply fill out the nodes [1]
See also:

```
random_partition_graph()
```

Examples

```
>>> G = nx.gaussian_random_partition_graph(100,10,10,.25,.1)
>>> len(G)
100
```


## References

### 6.16.7 ring_of_cliques

ring_of_cliques (num_cliques, clique_size)
Defines a "ring of cliques" graph.
A ring of cliques graph is consisting of cliques, connected through single links. Each clique is a complete graph.

## Parameters

- num_cliques (int) - Number of cliques
- clique_size (int) - Size of cliques

Returns G - ring of cliques graph
Return type NetworkX Graph
Raises NetworkXError - If the number of cliques is lower than 2 or if the size of cliques is smaller than 2.

## Examples

>>> G = nx.ring_of_cliques (8, 4)

## See also:

## Notes

The connected_caveman_graph graph removes a link from each clique to connect it with the next clique. Instead, the ring_of_cliques graph simply adds the link without removing any link from the cliques.

### 6.17 Non Isomorphic Trees

Implementation of the Wright, Richmond, Odlyzko and McKay (WROM) algorithm for the enumeration of all nonisomorphic free trees of a given order. Rooted trees are represented by level sequences, i.e., lists in which the i-th element specifies the distance of vertex $i$ to the root.

| nonisomorphic_trees(order[, create]) | Returns a list of nonisomporphic trees |
| :--- | :--- |
| number_of_nonisomorphic_trees(order) | Returns the number of nonisomorphic trees |

### 6.17.1 nonisomorphic_trees

nonisomorphic_trees (order, create $=$ 'graph')
Returns a list of nonisomporphic trees

## Parameters

- order (int) - order of the desired tree(s)
- create (graph or matrix (default="Graph)) - If graph is selected a list of trees will be returned, if matrix is selected a list of adjancency matrix will be returned


## Returns

- G (List of NetworkX Graphs)
- M (List of Adjacency matrices)


## References

### 6.17.2 number_of_nonisomorphic_trees

number_of_nonisomorphic_trees (order)
Returns the number of nonisomorphic trees
Parameters order (int) - order of the desired tree(s)
Returns length
Return type Number of nonisomorphic graphs for the given order

## References

### 6.18 Triads

Functions that generate the triad graphs, that is, the possible digraphs on three nodes.
triad_graph(triad_name) Returns the triad graph with the given name.

### 6.18.1 triad_graph

## triad_graph (triad_name)

Returns the triad graph with the given name.
Each string in the following tuple is a valid triad name:

```
('003', '012', '102', '021D', '021U', '021C', '111D', '111U',
'030T', '030C', '201', '120D', '120U', '120C', '210', '300')
```

Each triad name corresponds to one of the possible valid digraph on three nodes.
Parameters triad_name (string) - The name of a triad, as described above.
Returns The digraph on three nodes with the given name. The nodes of the graph are the singlecharacter strings 'a', 'b', and 'c'.

Return type DiGraph
Raises ValueError - If triad_name is not the name of a triad.

## See also:

```
triadic_census()
```


### 6.19 Joint Degree Sequence

Generate graphs with a given joint degree

| is_valid_joint_degree(joint_degrees) | Checks whether the given joint degree dictionary is realiz- <br> able as a simple graph. |
| :--- | :--- |
| joint_degree_graph(joint_degrees[, seed]) | Generates a random simple graph with the given joint de- <br> gree dictionary. |

### 6.19.1 is_valid_joint_degree

is_valid_joint_degree (joint_degrees)
Checks whether the given joint degree dictionary is realizable as a simple graph.
A joint degree dictionary is a dictionary of dictionaries, in which entry joint_degrees [ $k$ ] [ 1 ] is an integer representing the number of edges joining nodes of degree $k$ with nodes of degree $l$. Such a dictionary is realizable as a simple graph if and only if the following conditions are satisfied.

- each entry must be an integer,
-the total number of nodes of degree $k$, computed by sum(joint_degrees [k].values()) / k, must be an integer,
-the total number of edges joining nodes of degree $\boldsymbol{k}$ with nodes of degree $l$ cannot exceed the total number of possible edges,
-each diagonal entry joint_degrees [k][k] must be even (this is a convention assumed by the joint_degree_graph() function).

Parameters joint_degrees (dictionary of dictionary of integers) - A joint degree dictionary in which entry joint_degrees [k][l] is the number of edges joining nodes of degree $k$ with nodes of degree $l$.

Returns Whether the given joint degree dictionary is realizable as a simple graph.
Return type bool

## References

### 6.19.2 joint_degree_graph

joint_degree_graph (joint_degrees, seed=None)
Generates a random simple graph with the given joint degree dictionary.

## Parameters

- joint_degrees (dictionary of dictionary of integers) - A joint degree dictionary in which entry joint_degrees[k][l] is the number of edges joining nodes of degree $k$ with nodes of degree $l$.
- seed (hashable object, optional) - Seed for random number generator.

Returns G - A graph with the specified joint degree dictionary.
Return type Graph
Raises NetworkXError - If joint_degrees dictionary is not realizable.

## Notes

In each iteration of the "while loop" the algorithm picks two disconnected nodes $v$ and $w$, of degree $k$ and $l$ correspondingly, for which joint_degrees [k] [l] has not reached its target yet. It then adds edge ( $v, w$ ) and increases the number of edges in graph $G$ by one.

The intelligence of the algorithm lies in the fact that it is always possible to add an edge between such disconnected nodes $v$ and $w$, even if one or both nodes do not have free stubs. That is made possible by executing a "neighbor switch", an edge rewiring move that releases a free stub while keeping the joint degree of G the same.

The algorithm continues for E (number of edges) iterations of the "while loop", at the which point all entries of the given joint_degrees [k][l] have reached their target values and the construction is complete.

## References

Examples

```
>>> import networkx as nx
>>> joint_degrees = {1: {4: 1},
... 2: {2: 2, 3: 2, 4: 2},
\cdots 3: {2: 2, 4: 1},
... 4: {1: 1, 2: 2, 3: 1}}
>>> G=nx.joint_degree_graph(joint_degrees)
>>>
```


## Linear algebra

### 7.1 Graph Matrix

Adjacency matrix and incidence matrix of graphs.

| adjacency_matrix $(\mathrm{G}[$, nodelist, weight $])$ | Return adjacency matrix of G. |
| :--- | :--- |
| incidence_matrix $(\mathrm{G}[$, nodelist, edgelist, ...]) | Return incidence matrix of G. |

### 7.1.1 adjacency_matrix

adjacency_matrix (G, nodelist=None, weight='weight')
Return adjacency matrix of G.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- weight (string or None, optional (default='weight')) - The edge data key used to provide each value in the matrix. If None, then each edge has weight 1.

Returns A - Adjacency matrix representation of $G$.
Return type SciPy sparse matrix

## Notes

For directed graphs, entry $\mathrm{i}, \mathrm{j}$ corresponds to an edge from i to j .
If you want a pure Python adjacency matrix representation try networkx.convert.to_dict_of_dicts which will return a dictionary-of-dictionaries format that can be addressed as a sparse matrix.

For MultiGraph/MultiDiGraph with parallel edges the weights are summed. See to_numpy_matrix for other options.

The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the edge weight attribute (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Scipy sparse matrix can be modified as follows:

```
>>> import scipy as sp
>>> G = nx.Graph([(1,1)])
>>> A = nx.adjacency_matrix(G)
>>> print(A.todense())
[[1]]
>>> A.setdiag(A.diagonal()*2)
>>> print(A.todense())
[[2]]
```


## See also:

```
to_numpy_matrix(),to_scipy_sparse_matrix(),to_dict_of_dicts()
```


### 7.1.2 incidence_matrix

incidence_matrix (G, nodelist=None, edgelist=None, oriented=False, weight=None)
Return incidence matrix of $G$.
The incidence matrix assigns each row to a node and each column to an edge. For a standard incidence matrix a 1 appears wherever a row's node is incident on the column's edge. For an oriented incidence matrix each edge is assigned an orientation (arbitrarily for undirected and aligning to direction for directed). A -1 appears for the tail of an edge and 1 for the head of the edge. The elements are zero otherwise.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list, optional (default= all nodes in $G$ )) - The rows are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- edgelist (list, optional (default= all edges in $G$ ) ) - The columns are ordered according to the edges in edgelist. If edgelist is None, then the ordering is produced by G.edges().
- oriented (bool, optional (default=False)) - If True, matrix elements are +1 or -1 for the head or tail node respectively of each edge. If False,+1 occurs at both nodes.
- weight (string or None, optional (default=None)) - The edge data key used to provide each value in the matrix. If None, then each edge has weight 1 . Edge weights, if used, should be positive so that the orientation can provide the sign.

Returns A - The incidence matrix of G.
Return type SciPy sparse matrix

## Notes

For MultiGraph/MultiDiGraph, the edges in edgelist should be (u,v,key) 3-tuples.
"Networks are the best discrete model for so many problems in applied mathematics" ${ }^{1}$.

[^105]
## References

### 7.2 Laplacian Matrix

Laplacian matrix of graphs.

| laplacian_matrix(G[, nodelist, weight $])$ | Return the Laplacian matrix of G. |
| :--- | :--- |
| normalized_laplacian_matrix(G[, nodelist, ...]) | Return the normalized Laplacian matrix of G. |
| directed_laplacian_matrix(G[, nodelist, ...]) | Return the directed Laplacian matrix of G. |

### 7.2.1 laplacian_matrix

laplacian_matrix ( $G$, nodelist=None, weight='weight')
Return the Laplacian matrix of G.
The graph Laplacian is the matrix $\mathrm{L}=\mathrm{D}-\mathrm{A}$, where A is the adjacency matrix and D is the diagonal matrix of node degrees.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- weight (string or None, optional (default='weight')) - The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

Returns L - The Laplacian matrix of G.
Return type SciPy sparse matrix

## Notes

For MultiGraph/MultiDiGraph, the edges weights are summed.
See also:

```
to_numpy_matrix(),normalized_laplacian_matrix()
```


### 7.2.2 normalized_laplacian_matrix

## normalized_laplacian_matrix (G, nodelist=None, weight='weight')

Return the normalized Laplacian matrix of G.
The normalized graph Laplacian is the matrix

$$
N=D^{-1 / 2} L D^{-1 / 2}
$$

where $L$ is the graph Laplacian and $D$ is the diagonal matrix of node degrees.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- weight (string or None, optional (default='weight')) - The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

Returns $\mathbf{N}$ - The normalized Laplacian matrix of G.
Return type NumPy matrix

## Notes

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.
If the Graph contains selfloops, $D$ is defined as $\operatorname{diag}(\operatorname{sum}(A, 1))$, where $A$ is the adjacency matrix ${ }^{2}$.

## See also:

laplacian_matrix()

## References

### 7.2.3 directed_laplacian_matrix

directed_laplacian_matrix ( $G$, nodelist=None, weight='weight', walk_type=None, alpha=0.95)
Return the directed Laplacian matrix of $G$.
The graph directed Laplacian is the matrix

$$
L=I-\left(\Phi^{1 / 2} P \Phi^{-1 / 2}+\Phi^{-1 / 2} P^{T} \Phi^{1 / 2}\right) / 2
$$

where $I$ is the identity matrix, $P$ is the transition matrix of the graph, and Phi a matrix with the Perron vector of $P$ in the diagonal and zeros elsewhere.

Depending on the value of walk_type, $P$ can be the transition matrix induced by a random walk, a lazy random walk, or a random walk with teleportation (PageRank).

## Parameters

- G (DiGraph) - A NetworkX graph
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- weight (string or None, optional (default='weight')) - The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.
- walk_type (string or None, optional (default=None)) - If None, P is selected depending on the properties of the graph. Otherwise is one of 'random', 'lazy', or 'pagerank'
- alpha (real) - (1-alpha) is the teleportation probability used with pagerank

Returns L - Normalized Laplacian of G.
Return type NumPy array

## Raises

- NetworkXError - If NumPy cannot be imported

[^106]- NetworkXNot Implemnted - If G is not a DiGraph


## Notes

Only implemented for DiGraphs
See also:

```
laplacian_matrix()
```


## References

### 7.3 Spectrum

Eigenvalue spectrum of graphs.

| laplacian_spectrum $(G[$, weight $])$ | Return eigenvalues of the Laplacian of G |
| :--- | :--- |
| adjacency_spectrum $(G[$, weight $])$ | Return eigenvalues of the adjacency matrix of G. |

### 7.3.1 laplacian_spectrum

laplacian_spectrum ( $G$, weight='weight')
Return eigenvalues of the Laplacian of $G$

## Parameters

- G (graph) - A NetworkX graph
- weight (string or None, optional (default='weight')) - The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

Returns evals - Eigenvalues
Return type NumPy array

## Notes

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.
See also:
laplacian_matrix()

### 7.3.2 adjacency_spectrum

adjacency_spectrum ( $G$, weight='weight')
Return eigenvalues of the adjacency matrix of $G$.

## Parameters

- G (graph) - A NetworkX graph
- weight (string or None, optional (default='weight')) - The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

Returns evals - Eigenvalues
Return type NumPy array

## Notes

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.
See also:

```
adjacency_matrix()
```


### 7.4 Algebraic Connectivity

Algebraic connectivity and Fiedler vectors of undirected graphs.

| algebraic_connectivity $(\mathrm{G}[$, weight, ...]) | Return the algebraic connectivity of an undirected graph. |
| :--- | :--- |
| fiedler_vector $(\mathrm{G}[$, weight, normalized, tol, ...]) | Return the Fiedler vector of a connected undirected graph. |
| spectral_ordering $(\mathrm{G}[$, weight, normalized, ...]) | Compute the spectral_ordering of a graph. |

### 7.4.1 algebraic_connectivity

algebraic_connectivity ( $G$, weight='weight', normalized $=$ False, tol $=1 e-08$, method='tracemin')
Return the algebraic connectivity of an undirected graph.
The algebraic connectivity of a connected undirected graph is the second smallest eigenvalue of its Laplacian matrix.

## Parameters

- G (NetworkX graph) - An undirected graph.
- weight (object, optional) - The data key used to determine the weight of each edge. If None, then each edge has unit weight. Default value: None.
- normalized (bool, optional) - Whether the normalized Laplacian matrix is used. Default value: False.
- tol (float, optional) - Tolerance of relative residual in eigenvalue computation. Default value: 1e-8.
- method (string, optional) - Method of eigenvalue computation. It should be one of 'tracemin' (TraceMIN), 'lanczos' (Lanczos iteration) and 'lobpcg' (LOBPCG). Default value: 'tracemin'.

The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

| Value | Solver |
| :--- | :--- |
| 'tracemin_pcg'' | Preconditioned conjugate gradient method |
| 'tracemin_chol' | Cholesky factorization |
| 'tracemin_lu' | LU factorization |

Returns algebraic_connectivity - Algebraic connectivity.
Return type float

## Raises

- NetworkXNot Implemented - If G is directed.
- NetworkXError - If G has less than two nodes.


## Notes

Edge weights are interpreted by their absolute values. For MultiGraph's, weights of parallel edges are summed. Zero-weighted edges are ignored.

To use Cholesky factorization in the TraceMIN algorithm, the scikits.sparse package must be installed.

## See also:

```
laplacian_matrix()
```


### 7.4.2 fiedler_vector

fiedler_vector ( $G$, weight $=$ 'weight', normalized $=$ False, tol $=1 e-08$, method='tracemin')
Return the Fiedler vector of a connected undirected graph.
The Fiedler vector of a connected undirected graph is the eigenvector corresponding to the second smallest eigenvalue of the Laplacian matrix of of the graph.

## Parameters

- G (NetworkX graph) - An undirected graph.
- weight (object, optional) - The data key used to determine the weight of each edge. If None, then each edge has unit weight. Default value: None.
- normalized (bool, optional) - Whether the normalized Laplacian matrix is used. Default value: False.
- tol (float, optional) - Tolerance of relative residual in eigenvalue computation. Default value: 1e-8.
- method (string, optional) - Method of eigenvalue computation. It should be one of 'tracemin' (TraceMIN), 'lanczos' (Lanczos iteration) and 'lobpcg' (LOBPCG). Default value: 'tracemin'.
The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

| Value | Solver |
| :--- | :--- |
| 'tracemin_pcg' | Preconditioned conjugate gradient method |
| 'tracemin_chol' | Cholesky factorization |
| 'tracemin_lu' | LU factorization |

Returns fiedler_vector - Fiedler vector.
Return type NumPy array of floats.

## Raises

- NetworkXNot Implemented - If G is directed.
- NetworkXError - If G has less than two nodes or is not connected.


## Notes

Edge weights are interpreted by their absolute values. For MultiGraph's, weights of parallel edges are summed. Zero-weighted edges are ignored.

To use Cholesky factorization in the TraceMIN algorithm, the scikits.sparse package must be installed.

## See also:

```
laplacian_matrix()
```


### 7.4.3 spectral_ordering

```
spectral_ordering ( }G\mathrm{ , weight='weight', normalized=False,tol=le-08, method='tracemin')
```

Compute the spectral_ordering of a graph.
The spectral ordering of a graph is an ordering of its nodes where nodes in the same weakly connected components appear contiguous and ordered by their corresponding elements in the Fiedler vector of the component.

## Parameters

- G (NetworkX graph) - A graph.
- weight (object, optional) - The data key used to determine the weight of each edge. If None, then each edge has unit weight. Default value: None.
- normalized (bool, optional) - Whether the normalized Laplacian matrix is used. Default value: False.
- tol (float, optional) - Tolerance of relative residual in eigenvalue computation. Default value: 1e-8.
- method (string, optional) - Method of eigenvalue computation. It should be one of 'tracemin' (TraceMIN), 'lanczos' (Lanczos iteration) and 'lobpcg' (LOBPCG). Default value: 'tracemin'.

The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

| Value | Solver |
| :--- | :--- |
| 'tracemin_pcg' | Preconditioned conjugate gradient method |
| 'tracemin_chol' | Cholesky factorization |
| 'tracemin_lu' | LU factorization |

Returns spectral_ordering - Spectral ordering of nodes.
Return type NumPy array of floats.
Raises NetworkXError - If G is empty.

## Notes

Edge weights are interpreted by their absolute values. For MultiGraph's, weights of parallel edges are summed. Zero-weighted edges are ignored.

To use Cholesky factorization in the TraceMIN algorithm, the scikits.sparse package must be installed.

## See also:

```
laplacian_matrix()
```


### 7.5 Attribute Matrices

Functions for constructing matrix-like objects from graph attributes.

| attr_matrix $(\mathbf{G}[$, edge_attr, node_attr, ...]) | Returns a NumPy matrix using attributes from G. |
| :--- | :--- |
| attr_sparse_matrix(G[, edge_attr, ...]) | Returns a SciPy sparse matrix using attributes from G. |

### 7.5.1 attr_matrix

attr_matrix (G, edge_attr=None, node_attr=None, normalized $=$ False, rc_order=None, dtype=None, order=None)
Returns a NumPy matrix using attributes from G.
If only G is passed in, then the adjacency matrix is constructed.
Let A be a discrete set of values for the node attribute node_attr. Then the elements of A represent the rows and columns of the constructed matrix. Now, iterate through every edge $e=(u, v)$ in $G$ and consider the value of the edge attribute edge_attr. If ua and va are the values of the node attribute node_attr for $u$ and $v$, respectively, then the value of the edge attribute is added to the matrix element at (ua, va).

## Parameters

- G (graph) - The NetworkX graph used to construct the NumPy matrix.
- edge_attr (str, optional) - Each element of the matrix represents a running total of the specified edge attribute for edges whose node attributes correspond to the rows/cols of the matirx. The attribute must be present for all edges in the graph. If no attribute is specified, then we just count the number of edges whose node attributes correspond to the matrix element.
- node_attr (str, optional) - Each row and column in the matrix represents a particular value of the node attribute. The attribute must be present for all nodes in the graph. Note, the values of this attribute should be reliably hashable. So, float values are not recommended. If no attribute is specified, then the rows and columns will be the nodes of the graph.
- normalized (bool, optional) - If True, then each row is normalized by the summation of its values.
- rc_order (list, optional) - A list of the node attribute values. This list specifies the ordering of rows and columns of the array. If no ordering is provided, then the ordering will be random (and also, a return value).


## Other Parameters

- dtype (NumPy data-type, optional) - A valid NumPy dtype used to initialize the array. Keep in mind certain dtypes can yield unexpected results if the array is to be normalized. The parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.
- order ( $\left\{{ }^{\prime} C\right.$ ', ' $F$ ' $\}$, optional) - Whether to store multidimensional data in C- or Fortrancontiguous (row- or column-wise) order in memory. This parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.


## Returns

- M (NumPy matrix) - The attribute matrix.
- ordering (list) - If rc_order was specified, then only the matrix is returned. However, if rc_order was None, then the ordering used to construct the matrix is returned as well.


## Examples

Construct an adjacency matrix:

```
>>> G = nx.Graph()
>>> G.add_edge(0,1,thickness=1,weight=3)
>>> G.add_edge(0,2,thickness=2)
>>> G.add_edge(1,2,thickness=3)
>>> nx.attr_matrix(G, rc_order=[0,1,2])
matrix([[ 0., 1., 1.],
    [ 1., 0., 1.],
    [ 1., 1., 0.]])
```

Alternatively, we can obtain the matrix describing edge thickness.

```
>>> nx.attr_matrix(G, edge_attr='thickness', rc_order=[0,1,2])
matrix([[ 0., 1., 2.],
    [ 1., 0., 3.],
    [2., 3., 0.]])
```

We can also color the nodes and ask for the probability distribution over all edges ( $u, v$ ) describing:
$\operatorname{Pr}(\mathrm{v}$ has color $\mathrm{Y} \mid \mathrm{u}$ has color X )

```
>>> G.node[0]['color'] = 'red'
>>> G.node[1]['color'] = 'red'
>>> G.node[2]['color'] = 'blue'
>>> rc = ['red', 'blue']
>>> nx.attr_matrix(G, node_attr='color', normalized=True, rc_order=rc)
matrix([[ 0.33333333, 0.66666667],
    [ 1. , 0. ] ])
```

For example, the above tells us that for all edges ( $\mathbf{u}, \mathrm{v}$ ):
$\operatorname{Pr}(\mathrm{v}$ is red $\mid \mathrm{u}$ is red $)=1 / 3 \operatorname{Pr}(\mathrm{v}$ is blue $\mid \mathrm{u}$ is red $)=2 / 3$
$\operatorname{Pr}(\mathrm{v}$ is red l is blue $)=1 \operatorname{Pr}(\mathrm{v}$ is blue l u is blue $)=0$
Finally, we can obtain the total weights listed by the node colors.

```
>>> nx.attr_matrix(G, edge_attr='weight', node_attr='color', rc_order=rc)
matrix([[ 3., 2.],
    [ 2., 0.]])
```

Thus, the total weight over all edges ( $u, v$ ) with $u$ and $v$ having colors:
(red, red) is 3 \# the sole contribution is from edge ( 0,1 ) (red, blue) is 2 \# contributions from edges $(0,2)$ and ( 1,2 ) (blue, red) is 2 \# same as (red, blue) since graph is undirected (blue, blue) is 0 \# there are no edges with blue endpoints

### 7.5.2 attr_sparse_matrix

attr_sparse_matrix(G, edge_attr=None, node_attr=None, normalized=False, rc_order=None, dtype $=$ None)
Returns a SciPy sparse matrix using attributes from G.
If only G is passed in, then the adjacency matrix is constructed.

Let A be a discrete set of values for the node attribute node_attr. Then the elements of A represent the rows and columns of the constructed matrix. Now, iterate through every edge $e=(u, v)$ in $G$ and consider the value of the edge attribute edge_attr. If ua and va are the values of the node attribute node_attr for $u$ and $v$, respectively, then the value of the edge attribute is added to the matrix element at (ua, va).

## Parameters

- G (graph) - The NetworkX graph used to construct the NumPy matrix.
- edge_attr (str, optional) - Each element of the matrix represents a running total of the specified edge attribute for edges whose node attributes correspond to the rows/cols of the matirx. The attribute must be present for all edges in the graph. If no attribute is specified, then we just count the number of edges whose node attributes correspond to the matrix element.
- node_attr (str, optional) - Each row and column in the matrix represents a particular value of the node attribute. The attribute must be present for all nodes in the graph. Note, the values of this attribute should be reliably hashable. So, float values are not recommended. If no attribute is specified, then the rows and columns will be the nodes of the graph.
- normalized (bool, optional) - If True, then each row is normalized by the summation of its values.
- rc_order (list, optional) - A list of the node attribute values. This list specifies the ordering of rows and columns of the array. If no ordering is provided, then the ordering will be random (and also, a return value).

Other Parameters dtype (NumPy data-type, optional) - A valid NumPy dtype used to initialize the array. Keep in mind certain dtypes can yield unexpected results if the array is to be normalized. The parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.

## Returns

- M (SciPy sparse matrix) - The attribute matrix.
- ordering (list) - If rc_order was specified, then only the matrix is returned. However, if rc_order was None, then the ordering used to construct the matrix is returned as well.


## Examples

Construct an adjacency matrix:

```
>>> G = nx.Graph()
>>> G.add_edge(0,1,thickness=1,weight=3)
>>> G.add_edge(0,2,thickness=2)
>>> G.add_edge(1,2,thickness=3)
>>> M = nx.attr_sparse_matrix(G, rc_order=[0,1,2])
>>> M.todense()
matrix([[ 0., 1., 1.],
    [ 1., 0., 1.],
    [ 1., 1., 0.]])
```

Alternatively, we can obtain the matrix describing edge thickness.

```
>>> M = nx.attr_sparse_matrix(G, edge_attr='thickness', rc_order=[0,1,2])
>>> M.todense()
matrix([[ 0., 1., 2.],
    [ 1., 0., 3.],
    [2., 3., 0.]])
```

We can also color the nodes and ask for the probability distribution over all edges (u,v) describing:
$\operatorname{Pr}(\mathrm{v}$ has color $\mathrm{Y} \mid \mathrm{u}$ has color X$)$

```
>>> G.node[0]['color'] = 'red'
>>> G.node[1]['color'] = 'red'
>>> G.node[2]['color'] = 'blue'
>>> rc = ['red', 'blue']
>>> M = nx.attr_sparse_matrix(G, node_attr='color',
|}\mathrm{ normalized=True, rc_order=rc)
>>> M.todense()
matrix([[ 0.33333333, 0.66666667],
    [ 1. , 0. ]])
```

For example, the above tells us that for all edges (u,v):
$\operatorname{Pr}(\mathrm{v}$ is red l u is red $)=1 / 3 \operatorname{Pr}(\mathrm{v}$ is blue l u is red $)=2 / 3$
$\operatorname{Pr}(\mathrm{v}$ is red l u is blue $)=1 \operatorname{Pr}(\mathrm{v}$ is blue l u is blue $)=0$
Finally, we can obtain the total weights listed by the node colors.

```
>>> M = nx.attr_sparse_matrix(G, edge_attr='weight',
\hookrightarrow node_attr='color', rc_order=rc)
>>> M.todense()
matrix([[ 3., 2.],
    [ 2., 0.]])
```

Thus, the total weight over all edges ( $u, v$ ) with $u$ and $v$ having colors:
(red, red) is 3 \# the sole contribution is from edge ( 0,1 ) (red, blue) is 2 \# contributions from edges $(0,2)$ and $(1,2)$ (blue, red) is 2 \# same as (red, blue) since graph is undirected (blue, blue) is 0 \# there are no edges with blue endpoints

## Converting to and from other data formats

### 8.1 To NetworkX Graph

Functions to convert NetworkX graphs to and from other formats.
The preferred way of converting data to a NetworkX graph is through the graph constuctor. The constructor calls the to_networkx_graph() function which attempts to guess the input type and convert it automatically.

## Examples

Create a graph with a single edge from a dictionary of dictionaries

```
>>> d={0: {1: 1}} # dict-of-dicts single edge (0,1)
>>> G=nx.Graph(d)
```


## See also:

```
nx_agraph, nx_pydot
```

to_networkx_graph(data[, create_using, ...]) Make a NetworkX graph from a known data structure.

### 8.1.1 to_networkx_graph

to_networkx_graph (data, create_using=None, multigraph_input=False)
Make a NetworkX graph from a known data structure.
The preferred way to call this is automatically from the class constructor

```
>>> d={0: {1: {'weight':1}}} # dict-of-dicts single edge (0,1)
```

$\ggg G=n x$. Graph (d)
instead of the equivalent

```
>>> G=nx.from_dict_of_dicts(d)
```


## Parameters

- data (object to be converted) -

Current known types are: any NetworkX graph dict-of-dicts dist-of-lists list of edges numpy matrix numpy ndarray scipy sparse matrix pygraphviz agraph

- create_using (NetworkX graph) - Use specified graph for result. Otherwise a new graph is created.
- multigraph_input (bool (default False)) - If True and data is a dict_of_dicts, try to create a multigraph assuming dict_of_dict_of_lists. If data and create_using are both multigraphs then create a multigraph from a multigraph.


### 8.2 Dictionaries

| to_dict_of_dicts $(\mathrm{G}[$, nodelist, edge_data $])$ | Return adjacency representation of graph as a dictionary of <br> dictionaries. |
| :--- | :--- |
| from_dict_of_dicts $(\mathrm{d}[$, create_using,...$])$ | Return a graph from a dictionary of dictionaries. |

### 8.2.1 to_dict_of_dicts

```
to_dict_of_dicts (G, nodelist=None, edge_data=None)
```

Return adjacency representation of graph as a dictionary of dictionaries.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list) - Use only nodes specified in nodelist
- edge_data (list, optional) - If provided, the value of the dictionary will be set to edge_data for all edges. This is useful to make an adjacency matrix type representation with 1 as the edge data. If edgedata is None, the edgedata in $G$ is used to fill the values. If $G$ is a multigraph, the edgedata is a dict for each pair (u,v).


### 8.2.2 from_dict_of_dicts

from_dict_of_dicts (d, create_using=None, multigraph_input=False)
Return a graph from a dictionary of dictionaries.

## Parameters

- d (dictionary of dictionaries) - A dictionary of dictionaries adjacency representation.
- create_using (NetworkX graph) - Use specified graph for result. Otherwise a new graph is created.
- multigraph_input (bool (default False)) - When True, the values of the inner dict are assumed to be containers of edge data for multiple edges. Otherwise this routine assumes the edge data are singletons.


## Examples

```
>>> dod= {0: {1:{'weight':1}}} # single edge (0,1)
>>> G=nx.from_dict_of_dicts(dod)
```

or $\ggg \mathrm{G}=\mathrm{nx}$.Graph(dod) \# use Graph constructor

### 8.3 Lists

| to_dict_of_lists(G[, nodelist]) | Return adjacency representation of graph as a dictionary of <br> lists. |
| :--- | :--- |
| from_dict_of_lists(d[, create_using]) | Return a graph from a dictionary of lists. |
| to_edgelist(G[, nodelist]) | Return a list of edges in the graph. |
| from_edgelist(edgelist[, create_using]) | Return a graph from a list of edges. |

### 8.3.1 to_dict_of_lists

to_dict_of_lists ( $G$, nodelist=None)
Return adjacency representation of graph as a dictionary of lists.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list) - Use only nodes specified in nodelist


## Notes

Completely ignores edge data for MultiGraph and MultiDiGraph.

### 8.3.2 from_dict_of_lists

from_dict_of_lists (d, create_using=None)
Return a graph from a dictionary of lists.

## Parameters

- d (dictionary of lists) - A dictionary of lists adjacency representation.
- create_using (NetworkX graph) - Use specified graph for result. Otherwise a new graph is created.

Examples

```
>>> dol= {0:[1]} # single edge (0,1)
>>> G=nx.from_dict_of_lists(dol)
```

or $\ggg$ G=nx.Graph(dol) \# use Graph constructor

### 8.3.3 to_edgelist

to_edgelist ( $G$, nodelist=None)
Return a list of edges in the graph.

## Parameters

- G (graph) - A NetworkX graph
- nodelist (list) - Use only nodes specified in nodelist


### 8.3.4 from_edgelist

from_edgelist (edgelist, create_using=None)
Return a graph from a list of edges.

## Parameters

- edgelist (list or iterator) - Edge tuples
- create_using (NetworkX graph) - Use specified graph for result. Otherwise a new graph is created.


## Examples

```
>>> edgelist= [(0,1)] # single edge (0,1)
>>> G=nx.from_edgelist(edgelist)
```

or $\ggg$ G=nx.Graph(edgelist) \# use Graph constructor

### 8.4 Numpy

Functions to convert NetworkX graphs to and from numpy/scipy matrices.
The preferred way of converting data to a NetworkX graph is through the graph constuctor. The constructor calls the to_networkx_graph() function which attempts to guess the input type and convert it automatically.

## Examples

Create a 10 node random graph from a numpy matrix

```
>>> import numpy
>>> a = numpy.reshape(numpy.random.random_integers(0,1,size=100),(10,10))
>>> D = nx.DiGraph(a)
```

or equivalently

```
>>> D = nx.to_networkx_graph(a,create_using=nx.DiGraph())
```


## See also:

```
nx_agraph, nx_pydot
```

| to_numpy_matrix $(\mathrm{G}[$, nodelist, dtype, order,...$])$ | Return the graph adjacency matrix as a NumPy matrix. |
| :--- | :--- |
| to_numpy_recarray $(\mathrm{G}[$, nodelist, dtype, order $])$ | Return the graph adjacency matrix as a NumPy recarray. |
| from_numpy_matrix $(\mathrm{A}[$, parallel_edges,...$])$ | Return a graph from numpy matrix. |

### 8.4.1 to_numpy_matrix

to_numpy_matrix (G, nodelist $=$ None, dtype $=$ None, order $=$ None, multigraph_weight $=<$ built-in function sum>, weight='weight', nonedge=0.0)
Return the graph adjacency matrix as a NumPy matrix.

## Parameters

- G (graph) - The NetworkX graph used to construct the NumPy matrix.
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- dtype (NumPy data type, optional) - A valid single NumPy data type used to initialize the array. This must be a simple type such as int or numpy.float64 and not a compound data type (see to_numpy_recarray) If None, then the NumPy default is used.
- order ( $\left\{C^{\prime} C^{\prime}, ~ ' F\right.$ ' $\}$, optional) - Whether to store multidimensional data in C- or Fortrancontiguous (row- or column-wise) order in memory. If None, then the NumPy default is used.
- multigraph_weight (\{sum, min, max\}, optional) - An operator that determines how weights in multigraphs are handled. The default is to sum the weights of the multiple edges.
- weight (string or None optional (default $=$ 'weight')) - The edge attribute that holds the numerical value used for the edge weight. If an edge does not have that attribute, then the value 1 is used instead.
- nonedge (float (default $=0.0)$ ) - The matrix values corresponding to nonedges are typically set to zero. However, this could be undesirable if there are matrix values corresponding to actual edges that also have the value zero. If so, one might prefer nonedges to have some other value, such as nan.

Returns M - Graph adjacency matrix
Return type NumPy matrix

## See also:

```
to_numpy_recarray(), from_numpy_matrix()
```


## Notes

The matrix entries are assigned to the weight edge attribute. When an edge does not have a weight attribute, the value of the entry is set to the number 1 . For multiple (parallel) edges, the values of the entries are determined by the multigraph_weight parameter. The default is to sum the weight attributes for each of the parallel edges.

When nodelist does not contain every node in $G$, the matrix is built from the subgraph of $G$ that is induced by the nodes in nodelist.
The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Numpy matrix can be modified as follows:

```
>>> import numpy as np
>>> G = nx.Graph([(1, 1)])
>>> A = nx.to_numpy_matrix(G)
>>> A
matrix([[ 1.]])
>>> A.A[np.diag_indices_from(A)] *= 2
>>> A
matrix([[ 2.]])
```


## Examples

```
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0,1,weight=2)
0
>>> G.add_edge (1,0)
0
>>> G.add_edge (2,2,weight=3)
0
>>> G.add_edge (2, 2)
1
>>> nx.to_numpy_matrix(G, nodelist=[0,1,2])
matrix([[ 0., 2., 0.],
    [ 1., 0., 0.],
    [ 0., 0., 4.]])
```


### 8.4.2 to_numpy_recarray

to_numpy_recarray ( $G$, nodelist=None, dtype=None, order=None)
Return the graph adjacency matrix as a NumPy recarray.

## Parameters

- G (graph) - The NetworkX graph used to construct the NumPy matrix.
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- dtype (NumPy data-type, optional) - A valid NumPy named dtype used to initialize the NumPy recarray. The data type names are assumed to be keys in the graph edge attribute dictionary.
- order ( $\left\{C^{\prime}\right.$ ', ' $F$ ' \}, optional) - Whether to store multidimensional data in C- or Fortrancontiguous (row- or column-wise) order in memory. If None, then the NumPy default is used.

Returns M - The graph with specified edge data as a Numpy recarray
Return type NumPy recarray

## Notes

When nodelist does not contain every node in $G$, the matrix is built from the subgraph of $G$ that is induced by the nodes in nodelist.

## Examples

```
>>> G = nx.Graph()
>>> G.add_edge(1,2,weight=7.0, cost=5)
>>> A=nx.to_numpy_recarray(G,dtype=[('weight',float),('cost',int)])
>>> print(A.weight)
[[ 0. 7.]
    [ 7. 0.] ]
>>> print(A.cost)
```

```
[[\begin{array}{ll}{0}&{5}\end{array}]
    [5 0]]
```


### 8.4.3 from_numpy_matrix

```
from_numpy_matrix(A, parallel_edges=False,create_using=None)
```

Return a graph from numpy matrix.
The numpy matrix is interpreted as an adjacency matrix for the graph.

## Parameters

- A (numpy matrix) - An adjacency matrix representation of a graph
- parallel_edges (Boolean) - If this is True, create_using is a multigraph, and $A$ is an integer matrix, then entry $(i, j)$ in the matrix is interpreted as the number of parallel edges joining vertices $i$ and $j$ in the graph. If it is False, then the entries in the adjacency matrix are interpreted as the weight of a single edge joining the vertices.
- create_using (NetworkX graph) - Use specified graph for result. The default is Graph()


## Notes

If create_using is an instance of networkx. MultiGraph or networkx.MultidiGraph, parallel_edges is True, and the entries of A are of type int, then this function returns a multigraph (of the same type as create_using) with parallel edges.

If create_using is an undirected multigraph, then only the edges indicated by the upper triangle of the matrix A will be added to the graph.

If the numpy matrix has a single data type for each matrix entry it will be converted to an appropriate Python data type.

If the numpy matrix has a user-specified compound data type the names of the data fields will be used as attribute keys in the resulting NetworkX graph.

## See also:

to_numpy_matrix(), to_numpy_recarray()

## Examples

Simple integer weights on edges:

```
>>> import numpy
>>> A=numpy.matrix([[1, 1], [2, 1]])
>>> G=nx.from_numpy_matrix(A)
```

If create_using is a multigraph and the matrix has only integer entries, the entries will be interpreted as weighted edges joining the vertices (without creating parallel edges):

```
>>> import numpy
>>> A = numpy.matrix([[1, 1], [1, 2]])
>>> G = nx.from_numpy_matrix(A, create_using = nx.MultiGraph())
>>> G[1][1]
{0: {'weight': 2}}
```

If create_using is a multigraph and the matrix has only integer entries but parallel_edges is True, then the entries will be interpreted as the number of parallel edges joining those two vertices:

```
>>> import numpy
>>> A = numpy.matrix([[1, 1], [1, 2]])
>>> temp = nx.MultiGraph()
>>> G = nx.from_numpy_matrix(A, parallel_edges = True, create_using = temp)
>>> G[1][1]
{0: {'weight': 1}, 1: {'weight': 1}}
```

User defined compound data type on edges:

```
>>> import numpy
>>> dt = [('weight', float), ('cost', int)]
>>> A = numpy.matrix([[(1.0, 2)]], dtype = dt)
>>> G = nx.from_numpy_matrix(A)
>>> list(G.edges())
[(0, 0)]
>>> G[0][0]['cost']
2
>>> G[0][0]['weight']
1.0
```


### 8.5 Scipy

| to_scipy_sparse_matrix $(\mathrm{G}[$, nodelist, dtype, ...]) | Return the graph adjacency matrix as a SciPy sparse ma- <br> trix. |
| :--- | :--- |
| from_scipy_sparse_matrix $(\mathrm{A}[, \ldots])$ | Creates a new graph from an adjacency matrix given as a <br>  <br>  |

### 8.5.1 to_scipy_sparse_matrix <br> to_scipy_sparse_matrix (G, nodelist=None, dtype=None, weight='weight', format='csr') <br> Return the graph adjacency matrix as a SciPy sparse matrix.

## Parameters

- G (graph) - The NetworkX graph used to construct the NumPy matrix.
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- dtype (NumPy data-type, optional) - A valid NumPy dtype used to initialize the array. If None, then the NumPy default is used.
- weight (string or None optional (default='weight')) - The edge attribute that holds the numerical value used for the edge weight. If None then all edge weights are 1.
- format (str in \{'bsr', 'csr', 'csc', 'coo', 'lil', 'dia', 'dok'\}) - The type of the matrix to be returned (default 'csr'). For some algorithms different implementations of sparse matrices can perform better. See ${ }^{1}$ for details.
Returns M - Graph adjacency matrix.

[^107]Return type SciPy sparse matrix

## Notes

The matrix entries are populated using the edge attribute held in parameter weight. When an edge does not have that attribute, the value of the entry is 1 .

For multiple edges the matrix values are the sums of the edge weights.
When nodelist does not contain every node in $G$, the matrix is built from the subgraph of $G$ that is induced by the nodes in nodelist.

Uses coo_matrix format. To convert to other formats specify the format= keyword.
The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Scipy sparse matrix can be modified as follows:

```
>>> import scipy as sp
>>> G = nx.Graph([(1,1)])
>>> A = nx.to_scipy_sparse_matrix(G)
>>> print(A.todense())
[[1]]
>>> A.setdiag(A.diagonal()*2)
>>> print(A.todense())
[[2]]
```


## Examples

```
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0,1,weight=2)
0
>>> G.add_edge (1,0)
0
>>> G.add_edge(2,2,weight=3)
0
>>> G.add_edge (2,2)
1
>>> S = nx.to_scipy_sparse_matrix(G, nodelist=[0,1,2])
>>> print(S.todense())
[[[0}
    [1 0 0)
    [0}00 4]
```


## References

### 8.5.2 from_scipy_sparse_matrix

from_scipy_sparse_matrix (A, parallel_edges=False, create_using=None, edge_attribute='weight')
Creates a new graph from an adjacency matrix given as a SciPy sparse matrix.

## Parameters

- A (scipy sparse matrix) - An adjacency matrix representation of a graph
- parallel_edges (Boolean) - If this is True, create_using is a multigraph, and A is an integer matrix, then entry $(i, j)$ in the matrix is interpreted as the number of parallel edges joining vertices $i$ and $j$ in the graph. If it is False, then the entries in the adjacency matrix are interpreted as the weight of a single edge joining the vertices.
- create_using (NetworkX graph) - Use specified graph for result. The default is Graph()
- edge_attribute (string) - Name of edge attribute to store matrix numeric value. The data will have the same type as the matrix entry (int, float, (real,imag)).


## Notes

If create_using is an instance of networkx.MultiGraph or networkx.MultiDiGraph, parallel_edges is True, and the entries of A are of type int, then this function returns a multigraph (of the same type as create_using) with parallel edges. In this case, edge_attribute will be ignored.
If create_using is an undirected multigraph, then only the edges indicated by the upper triangle of the matrix A will be added to the graph.

## Examples

```
>>> import scipy.sparse
>>> A = scipy.sparse.eye(2,2,1)
>>> G = nx.from_scipy_sparse_matrix(A)
```

If create_using is a multigraph and the matrix has only integer entries, the entries will be interpreted as weighted edges joining the vertices (without creating parallel edges):

```
>>> import scipy
>>> A = scipy.sparse.csr_matrix([[1, 1], [1, 2]])
>>> G = nx.from_scipy_sparse_matrix(A, create_using=nx.MultiGraph())
>>> G[1][1]
{0: {'weight': 2}}
```

If create_using is a multigraph and the matrix has only integer entries but parallel_edges is True, then the entries will be interpreted as the number of parallel edges joining those two vertices:

```
>>> import scipy
>>> A = scipy.sparse.csr_matrix([[1, 1], [1, 2]])
>>> G = nx.from_scipy_sparse_matrix(A, parallel_edges=True,
... create_using=nx.MultiGraph())
>>> G[1][1]
{0: {'weight': 1}, 1: {'weight': 1}}
```


### 8.6 Pandas

| to_pandas_dataframe $(\mathrm{G}[$, nodelist, dtype, ...]) | Return the graph adjacency matrix as a Pandas DataFrame. |
| :--- | :--- |
| from_pandas_dataframe $($ df, source, target $[, \ldots])$ | Return a graph from Pandas DataFrame. |

### 8.6.1 to_pandas_dataframe

to_pandas_dataframe ( $G$, nodelist=None, dtype $=$ None, order $=$ None, multigraph_weight $=<$ built-in function sum>, weight $=$ 'weight', nonedge $=0.0$ )
Return the graph adjacency matrix as a Pandas DataFrame.

## Parameters

- G (graph) - The NetworkX graph used to construct the Pandas DataFrame.
- nodelist (list, optional) - The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- multigraph_weight (\{sum, min, max \}, optional) - An operator that determines how weights in multigraphs are handled. The default is to sum the weights of the multiple edges.
- weight (string or None, optional) - The edge attribute that holds the numerical value used for the edge weight. If an edge does not have that attribute, then the value 1 is used instead.
- nonedge (float, optional) - The matrix values corresponding to nonedges are typically set to zero. However, this could be undesirable if there are matrix values corresponding to actual edges that also have the value zero. If so, one might prefer nonedges to have some other value, such as nan.
Returns df - Graph adjacency matrix
Return type Pandas DataFrame


## Notes

The DataFrame entries are assigned to the weight edge attribute. When an edge does not have a weight attribute, the value of the entry is set to the number 1 . For multiple (parallel) edges, the values of the entries are determined by the 'multigraph_weight' parameter. The default is to sum the weight attributes for each of the parallel edges.
When nodelist does not contain every node in $G$, the matrix is built from the subgraph of $G$ that is induced by the nodes in nodelist.
The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Pandas DataFrame can be modified as follows:

```
>>> import pandas as pd
>>> import numpy as np
>>> G = nx.Graph([(1,1)])
>>> df = nx.to_pandas_dataframe(G, dtype=int)
>>> df
1 1
>>> df.values[np.diag_indices_from(df)] *= 2
>>> df
    1
1 2
```


## Examples

```
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0,1,weight=2)
```

```
0
>>> G.add_edge (1,0)
0
>>> G.add_edge(2,2,weight=3)
0
>>> G.add_edge (2, 2)
1
>>> nx.to_pandas_dataframe(G, nodelist=[0,1,2], dtype=int)
    0 1 2
0}02
1 0}
0}00
```


### 8.6.2 from_pandas_dataframe

from_pandas_dataframe (df, source, target, edge_attr=None, create_using=None)
Return a graph from Pandas DataFrame.
The Pandas DataFrame should contain at least two columns of node names and zero or more columns of node attributes. Each row will be processed as one edge instance.

Note: This function iterates over DataFrame.values, which is not guaranteed to retain the data type across columns in the row. This is only a problem if your row is entirely numeric and a mix of ints and floats. In that case, all values will be returned as floats. See the DataFrame.iterrows documentation for an example.

## Parameters

- df (Pandas DataFrame) - An edge list representation of a graph
- source (str or int) - A valid column name (string or iteger) for the source nodes (for the directed case).
- target (str or int) - A valid column name (string or iteger) for the target nodes (for the directed case).
- edge_attr (str or int, iterable, True) - A valid column name (str or integer) or list of column names that will be used to retrieve items from the row and add them to the graph as edge attributes. If True, all of the remaining columns will be added.
- create_using (NetworkX graph) - Use specified graph for result. The default is Graph()


## See also:

```
to_pandas_dataframe()
```


## Examples

Simple integer weights on edges:

```
>>> import pandas as pd
>>> import numpy as np
>>> r = np.random.RandomState(seed=5)
>>> ints = r.random_integers(1, 10, size=(3,2))
>>> a = ['A', 'B', 'C']
>>> b = ['D', 'A', 'E']
>>> df = pd.DataFrame(ints, columns=['weight', 'cost'])
>>> df[0] = a
>>> df['b'] = b
```

```
>>> df
    weight cost 0 b
0 4 7 A D
1 7 1 1 B A
2 10 9 C E
>>> G=nx.from_pandas_dataframe(df, 0, 'b', ['weight', 'cost'])
>>> G['E']['C']['weight']
10
>>> G['E']['C']['cost']
9
```


## Reading and writing graphs

### 9.1 Adjacency List

### 9.1.1 Adjacency List

Read and write NetworkX graphs as adjacency lists.
Adjacency list format is useful for graphs without data associated with nodes or edges and for nodes that can be meaningfully represented as strings.

## Format

The adjacency list format consists of lines with node labels. The first label in a line is the source node. Further labels in the line are considered target nodes and are added to the graph along with an edge between the source node and target node.
The graph with edges $a-b, a-c, d-e$ can be represented as the following adjacency list (anything following the \# in a line is a comment):

```
a b c # source target target
d e
```

| read_adjlist(path[, comments, delimiter, ...]) | Read graph in adjacency list format from path. |
| :--- | :--- |
| write_adjlist(G, path[, comments, ...]) | Write graph G in single-line adjacency-list format to path. |
| parse_adjlist(lines[, comments, delimiter, ...]) | Parse lines of a graph adjacency list representation. |
| generate_adjlist(G[, delimiter]) | Generate a single line of the graph G in adjacency list for- |
|  | mat. |

### 9.1.2 read_adjlist

read_adjlist (path, comments='\#', delimiter=None, create_using=None, nodetype $=$ None, encoding='utf8')
Read graph in adjacency list format from path.

## Parameters

- path (string or file) - Filename or file handle to read. Filenames ending in .gz or .bz2 will be uncompressed.
- create_using (NetworkX graph container) - Use given NetworkX graph for holding nodes
or edges.
- nodetype (Python type, optional) - Convert nodes to this type.
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels. The default is whitespace.

Returns G - The graph corresponding to the lines in adjacency list format.
Return type NetworkX graph

## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_adjlist(G, "test.adjlist")
>>> G=nx.read_adjlist("test.adjlist")
```

The path can be a filehandle or a string with the name of the file. If a filehandle is provided, it has to be opened in 'rb' mode.

```
>>> fh=open("test.adjlist", 'rb')
>>> G=nx.read_adjlist(fh)
```

Filenames ending in .gz or .bz2 will be compressed.

```
>>> nx.write_adjlist(G,"test.adjlist.gz")
>>> G=nx.read_adjlist("test.adjlist.gz")
```

The optional nodetype is a function to convert node strings to nodetype.
For example

```
>>> G=nx.read_adjlist("test.adjlist", nodetype=int)
```

will attempt to convert all nodes to integer type.
Since nodes must be hashable, the function nodetype must return hashable types (e.g. int, float, str, frozenset or tuples of those, etc.)

The optional create_using parameter is a NetworkX graph container. The default is Graph(), an undirected graph. To read the data as a directed graph use

```
>>> G=nx.read_adjlist("test.adjlist", create_using=nx.DiGraph())
```


## Notes

This format does not store graph or node data.

## See also:

```
write_adjlist()
```


### 9.1.3 write_adjlist

write_adjlist ( $G$, path, comments='\#', delimiter=' ', encoding='utf- 8 ')
Write graph G in single-line adjacency-list format to path.

## Parameters

- G (NetworkX graph)
- path (string or file) - Filename or file handle for data output. Filenames ending in .gz or .bz2 will be compressed.
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels
- encoding (string, optional) - Text encoding.


## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_adjlist(G,"test.adjlist")
```

The path can be a filehandle or a string with the name of the file. If a filehandle is provided, it has to be opened in 'wb' mode.

```
>>> fh=open("test.adjlist",'wb')
>>> nx.write_adjlist(G, fh)
```


## Notes

This format does not store graph, node, or edge data.

## See also:

```
read_adjlist(),generate_adjlist()
```


### 9.1.4 parse_adjlist

parse_adjlist (lines, comments='\#', delimiter=None, create_using=None, nodetype=None)
Parse lines of a graph adjacency list representation.

## Parameters

- lines (list or iterator of strings) - Input data in adjlist format
- create_using (NetworkX graph container) - Use given NetworkX graph for holding nodes or edges.
- nodetype (Python type, optional) - Convert nodes to this type.
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels. The default is whitespace.

Returns G - The graph corresponding to the lines in adjacency list format.
Return type NetworkX graph

## Examples

```
>>> lines = ['1 2 5',
... '2 3 4',
... '3 5',
... '4',
\cdots•• '5']
>>> G = nx.parse_adjlist(lines, nodetype = int)
>>> list(G)
[1, 2, 3, 4, 5]
>>> list(G.edges())
[(1, 2), (1, 5), (2, 3), (2, 4), (3, 5)]
```


## See also:

```
read_adjlist()
```


### 9.1.5 generate_adjlist

generate_adjlist ( $G$, delimiter=' ')
Generate a single line of the graph $G$ in adjacency list format.

## Parameters

- G (NetworkX graph)
- delimiter (string, optional) - Separator for node labels

Returns lines - Lines of data in adjlist format.
Return type string

Examples

```
>>> G = nx.lollipop_graph(4, 3)
>>> for line in nx.generate_adjlist(G):
... print(line)
0 1 2 3
1 2 3
2 3
34
4 5
56
6
```


## See also:

```
write_adjlist(), read_adjlist()
```


### 9.2 Multiline Adjacency List

### 9.2.1 Multi-line Adjacency List

Read and write NetworkX graphs as multi-line adjacency lists.

The multi-line adjacency list format is useful for graphs with nodes that can be meaningfully represented as strings. With this format simple edge data can be stored but node or graph data is not.

## Format

The first label in a line is the source node label followed by the node degree d . The next d lines are target node labels and optional edge data. That pattern repeats for all nodes in the graph.

The graph with edges a-b, a-c, d-e can be represented as the following adjacency list (anything following the \# in a line is a comment):

```
example.multiline-adjlist
2
1
\begin{tabular}{ll}
\hline read_multiline_adjlist(path[, comments, ...]) & Read graph in multi-line adjacency list format from path. \\
\hline write_multiline_adjlist(G, path[, ...]) & Write the graph G in multiline adjacency list format to path \\
\hline parse_multiline_adjlist(lines[, comments, ...]) & \begin{tabular}{l} 
Parse lines of a multiline adjacency list representation of a \\
graph.
\end{tabular} \\
\hline generate_multiline_adjlist(G[, delimiter]) & \begin{tabular}{l} 
Generate a single line of the graph G in multiline adjacency \\
list format.
\end{tabular} \\
\hline
\end{tabular}
```

b
e

### 9.2.2 read_multiline_adjlist

read_multiline_adjlist (path, comments='\#', delimiter=None, create_using=None, nodetype=None, edgetype $=$ None, encoding $=$ 'utf-8')
Read graph in multi-line adjacency list format from path.

## Parameters

- path (string or file) - Filename or file handle to read. Filenames ending in .gz or .bz2 will be uncompressed.
- create_using (NetworkX graph container) - Use given NetworkX graph for holding nodes or edges.
- nodetype (Python type, optional) - Convert nodes to this type.
- edgetype (Python type, optional) - Convert edge data to this type.
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels. The default is whitespace.


## Returns G

Return type NetworkX graph

## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_multiline_adjlist(G,"test.adjlist")
>>> G=nx.read_multiline_adjlist("test.adjlist")
```

The path can be a file or a string with the name of the file. If a file s provided, it has to be opened in 'rb' mode.

```
>>> fh=open("test.adjlist", 'rb')
```

>>> G=nx.read_multiline_adjlist(fh)

Filenames ending in .gz or .bz2 will be compressed.

```
>>> nx.write_multiline_adjlist(G,"test.adjlist.gz")
>>> G=nx.read_multiline_adjlist("test.adjlist.gz")
```

The optional nodetype is a function to convert node strings to nodetype.
For example

```
>>> G=nx.read_multiline_adjlist("test.adjlist", nodetype=int)
```

will attempt to convert all nodes to integer type.
The optional edgetype is a function to convert edge data strings to edgetype.

```
>>> G=nx.read_multiline_adjlist("test.adjlist")
```

The optional create_using parameter is a NetworkX graph container. The default is Graph(), an undirected graph. To read the data as a directed graph use

```
>>> G=nx.read_multiline_adjlist("test.adjlist", create_using=nx.DiGraph())
```


## Notes

This format does not store graph, node, or edge data.

## See also:

```
write_multiline_adjlist()
```


### 9.2.3 write_multiline_adjlist

write_multiline_adjlist ( $G$, path, delimiter=' ', comments='\#', encoding='utf-8')
Write the graph $G$ in multiline adjacency list format to path

## Parameters

- G (NetworkX graph)
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels
- encoding (string, optional) - Text encoding.


## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_multiline_adjlist(G,"test.adjlist")
```

The path can be a file handle or a string with the name of the file. If a file handle is provided, it has to be opened in 'wb' mode.

```
>>> fh=open("test.adjlist",'wb')
>>> nx.write_multiline_adjlist(G,fh)
```

Filenames ending in .gz or .bz2 will be compressed.

```
>>> nx.write_multiline_adjlist(G,"test.adjlist.gz")
```


## See also:

```
read_multiline_adjlist()
```


### 9.2.4 parse_multiline_adjlist

parse_multiline_adjlist(lines, comments='\#', delimiter=None, create_using=None, nodetype $=$ None, edgetype $=$ None )
Parse lines of a multiline adjacency list representation of a graph.

## Parameters

- lines (list or iterator of strings) - Input data in multiline adjlist format
- create_using (NetworkX graph container) - Use given NetworkX graph for holding nodes or edges.
- nodetype (Python type, optional) - Convert nodes to this type.
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels. The default is whitespace.

Returns G - The graph corresponding to the lines in multiline adjacency list format.
Return type NetworkX graph

## Examples

```
>>> lines = ['1 2',
... "2 {'weight':3, 'name': 'Frodo'}",
... "3 {}",
... "2 1",
... "5 {'weight':6, 'name': 'Saruman'}"]
>>> G = nx.parse_multiline_adjlist(iter(lines), nodetype=int)
>>> list(G)
[1, 2, 3, 5]
```


### 9.2.5 generate_multiline_adjlist

generate_multiline_adjlist ( $G$, delimiter=' ')
Generate a single line of the graph $G$ in multiline adjacency list format.

## Parameters

- G (NetworkX graph)
- delimiter (string, optional) - Separator for node labels

Returns lines - Lines of data in multiline adjlist format.

Return type string

## Examples

```
>>> G = nx.lollipop_graph(4, 3)
>>> for line in nx.generate_multiline_adjlist(G):
    print(line)
3
{}
{}
{}
2
{}
{}
1
{}
1
{}
1
{ }
1
{ }
0
```


## See also:

```
write_multiline_adjlist(),read_multiline_adjlist()
```


### 9.3 Edge List

### 9.3.1 Edge Lists

Read and write NetworkX graphs as edge lists.
The multi-line adjacency list format is useful for graphs with nodes that can be meaningfully represented as strings. With the edgelist format simple edge data can be stored but node or graph data is not. There is no way of representing isolated nodes unless the node has a self-loop edge.

## Format

You can read or write three formats of edge lists with these functions.
Node pairs with no data:

```
2
```

Python dictionary as data:

```
2 {'weight':7, 'color':'green'}
```

Arbitrary data:

```
2 7 green
```

| read_edgelist(path[, comments, delimiter, ...]) | Read a graph from a list of edges. |
| :--- | :--- |
| write_edgelist(G, path[, comments, ...]) | Write graph as a list of edges. |
| read_weighted_edgelist(path[, comments, ...]) | Read a graph as list of edges with numeric weights. |
| write_weighted_edgelist(G, path[, comments, | Write graph G as a list of edges with numeric weights. |
| $\ldots$...]) |  |
| generate_edgelist(G[, delimiter, data]) | Generate a single line of the graph G in edge list format. |
| parse_edgelist(lines[, comments, delimiter, ...]) | Parse lines of an edge list representation of a graph. |

### 9.3.2 read_edgelist

read_edgelist (path, comments='\#', delimiter=None, create_using=None, nodetype=None, data=True, edgetype $=$ None, encoding $=$ 'utf-8')
Read a graph from a list of edges.

## Parameters

- path (file or string) - File or filename to read. If a file is provided, it must be opened in 'rb' mode. Filenames ending in .gz or .bz2 will be uncompressed.
- comments (string, optional) - The character used to indicate the start of a comment.
- delimiter (string, optional) - The string used to separate values. The default is whitespace.
- create_using (Graph container, optional,) - Use specified container to build graph. The default is networkx.Graph, an undirected graph.
- nodetype (int, float, str, Python type, optional) - Convert node data from strings to specified type
- data (bool or list of (label,type) tuples) - Tuples specifying dictionary key names and types for edge data
- edgetype (int, float, str, Python type, optional OBSOLETE) - Convert edge data from strings to specified type and use as 'weight'
- encoding (string, optional) - Specify which encoding to use when reading file.

Returns G - A networkx Graph or other type specified with create_using
Return type graph

## Examples

```
>>> nx.write_edgelist(nx.path_graph(4), "test.edgelist")
>>> G=nx.read_edgelist("test.edgelist")
```

```
>>> fh=open("test.edgelist", 'rb')
>>> G=nx.read_edgelist(fh)
>>> fh.close()
```

```
>>> G=nx.read_edgelist("test.edgelist", nodetype=int)
>>> G=nx.read_edgelist("test.edgelist",create_using=nx.DiGraph())
```

Edgelist with data in a list:

```
>>> textline = '1 2 3'
>>> fh = open('test.edgelist','w')
>>> d = fh.write(textline)
>>> fh.close()
>>> G = nx.read_edgelist('test.edgelist', nodetype=int, data=(('weight',float),))
>>> list(G)
[1, 2]
>>> list(G.edges(data=True))
[(1, 2, {'weight': 3.0})]
```

See parse_edgelist() for more examples of formatting.

## See also:

```
parse_edgelist()
```


## Notes

Since nodes must be hashable, the function nodetype must return hashable types (e.g. int, float, str, frozenset or tuples of those, etc.)

### 9.3.3 write_edgelist

write_edgelist (G, path, comments='\#', delimiter=' ', data=True, encoding='utf-8')
Write graph as a list of edges.

## Parameters

- G (graph) - A NetworkX graph
- path (file or string) - File or filename to write. If a file is provided, it must be opened in 'wb' mode. Filenames ending in .gz or .bz2 will be compressed.
- comments (string, optional) - The character used to indicate the start of a comment
- delimiter (string, optional) - The string used to separate values. The default is whitespace.
- data (bool or list, optional) - If False write no edge data. If True write a string representation of the edge data dictionary.. If a list (or other iterable) is provided, write the keys specified in the list.
- encoding (string, optional) - Specify which encoding to use when writing file.


## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_edgelist(G, "test.edgelist")
>>> G=nx.path_graph(4)
>>> fh=open("test.edgelist",'wb')
>>> nx.write_edgelist(G, fh)
>>> nx.write_edgelist(G, "test.edgelist.gz")
>>> nx.write_edgelist(G, "test.edgelist.gz", data=False)
```

```
>>> G=nx.Graph()
>>> G.add_edge (1,2,weight=7,color='red')
>>> nx.write_edgelist(G,'test.edgelist',data=False)
```

```
>>> nx.write_edgelist(G,'test.edgelist',data=['color'])
>>> nx.write_edgelist(G,'test.edgelist',data=['color','weight'])
```


## See also:

```
write_edgelist(), write_weighted_edgelist()
```


### 9.3.4 read_weighted_edgelist

read_weighted_edgelist (path, comments='\#', delimiter=None, create_using=None, nodetype=None, encoding='utf-8')
Read a graph as list of edges with numeric weights.

## Parameters

- path (file or string) - File or filename to read. If a file is provided, it must be opened in 'rb' mode. Filenames ending in .gz or .bz2 will be uncompressed.
- comments (string, optional) - The character used to indicate the start of a comment.
- delimiter (string, optional) - The string used to separate values. The default is whitespace.
- create_using (Graph container, optional,) - Use specified container to build graph. The default is networkx.Graph, an undirected graph.
- nodetype (int, float, str, Python type, optional) - Convert node data from strings to specified type
- encoding (string, optional) - Specify which encoding to use when reading file.

Returns G - A networkx Graph or other type specified with create_using
Return type graph

## Notes

Since nodes must be hashable, the function nodetype must return hashable types (e.g. int, float, str, frozenset or tuples of those, etc.)

Example edgelist file format.
With numeric edge data:

```
# read with
# >>> G=nx.read_weighted_edgelist(fh)
# source target data
a b 1
a c 3.14159
d e 42
```


### 9.3.5 write_weighted_edgelist

write_weighted_edgelist ( $G$, path, comments='\#', delimiter=' ', encoding='utf-8')
Write graph G as a list of edges with numeric weights.

## Parameters

- G (graph) - A NetworkX graph
- path (file or string) - File or filename to write. If a file is provided, it must be opened in 'wb' mode. Filenames ending in .gz or .bz2 will be compressed.
- comments (string, optional) - The character used to indicate the start of a comment
- delimiter (string, optional) - The string used to separate values. The default is whitespace.
- encoding (string, optional) - Specify which encoding to use when writing file.


## Examples

```
>>> G=nx.Graph()
>>> G.add_edge(1,2,weight=7)
>>> nx.write_weighted_edgelist(G, 'test.weighted.edgelist')
```


## See also:

```
read_edgelist(),write__edgelist(),write_weighted_edgelist()
```


### 9.3.6 generate_edgelist

generate_edgelist ( $G$, delimiter=' ', data=True)
Generate a single line of the graph $G$ in edge list format.

## Parameters

- G (NetworkX graph)
- delimiter (string, optional) - Separator for node labels
- data (bool or list of keys) - If False generate no edge data. If True use a dictionary representation of edge data. If a list of keys use a list of data values corresponding to the keys.

Returns lines - Lines of data in adjlist format.
Return type string

## Examples

```
>>> G = nx.lollipop_graph(4, 3)
>>> G[1][2]['weight'] = 3
>>> G[3][4]['capacity'] = 12
>>> for line in nx.generate_edgelist(G, data=False):
... print(line)
O 1
0}
O 3
12
1 3
2 3
34
45
56
```

```
>>> for line in nx.generate_edgelist(G):
... print(line)
0 1 {}
0 2 {}
0 3 {}
1 2 {'weight': 3}
1 3 {}
3 {}
4 {'capacity': 12}
5 {}
6{}
```

```
>>> for line in nx.generate_edgelist(G,data=['weight']):
    print(line)
1
2
3
2 3
3
3
4
5
6
```

See also:

```
write_adjlist(),read_adjlist()
```


### 9.3.7 parse_edgelist

parse_edgelist (lines, comments='\#', delimiter=None, create_using=None, nodetype=None, data=True)
Parse lines of an edge list representation of a graph.

## Parameters

- lines (list or iterator of strings) - Input data in edgelist format
- comments (string, optional) - Marker for comment lines
- delimiter (string, optional) - Separator for node labels
- create_using (NetworkX graph container, optional) - Use given NetworkX graph for holding nodes or edges.
- nodetype (Python type, optional) - Convert nodes to this type.
- data (bool or list of (label,type) tuples) - If False generate no edge data or if True use a dictionary representation of edge data or a list tuples specifying dictionary key names and types for edge data.

Returns G - The graph corresponding to lines
Return type NetworkX Graph

## Examples

Edgelist with no data:

```
>>> lines = ["1 2",
... "2 3",
... "3 4"]
>>> G = nx.parse_edgelist(lines, nodetype = int)
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges())
[(1, 2), (2, 3), (3, 4)]
```

Edgelist with data in Python dictionary representation:

```
>>> lines = ["1 2 {'weight':3}",
... "2 3 {'weight':27}",
... "3 4 {'weight':3.0}"]
>>> G = nx.parse_edgelist(lines, nodetype = int)
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges(data=True))
[(1, 2, {'weight': 3}), (2, 3, {'weight': 27}), (3, 4, {'weight': 3.0})]
```

Edgelist with data in a list:

```
>>> lines = ["1 2 3",
... "2 3 27",
... "3 4 3.0"]
>>> G = nx.parse_edgelist(lines, nodetype = int, data=(('weight',float),))
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges(data=True))
[(1, 2, {'weight': 3.0}), (2, 3, {'weight': 27.0}), (3, 4, {'weight': 3.0})]
```


## See also:

```
read__weighted_edgelist()
```


### 9.4 GEXF

Read and write graphs in GEXF format.
GEXF (Graph Exchange XML Format) is a language for describing complex network structures, their associated data and dynamics.

This implementation does not support mixed graphs (directed and undirected edges together).

### 9.4.1 Format

GEXF is an XML format. See http://gexf.net/format/schema.html for the specification and http://gexf.net/format/ basic.html for examples.

| read_gexf(path[, node_type, relabel, version]) | Read graph in GEXF format from path. |
| :--- | :--- |
| write_gexf(G, path[, encoding, prettyprint, ...]) | Write G in GEXF format to path. |
| relabel_gexf_graph $(\mathbf{G})$ | Relabel graph using "label" node keyword for node label. |

### 9.4.2 read_gexf

read_gexf (path, node_type=None, relabel=False, version='1.1draft')
Read graph in GEXF format from path.
"GEXF (Graph Exchange XML Format) is a language for describing complex networks structures, their associated data and dynamics" ${ }^{1}$.

## Parameters

- path (file or string) - File or file name to write. File names ending in .gz or .bz2 will be compressed.
- node_type (Python type (default: None)) - Convert node ids to this type if not None.
- relabel (bool (default: False)) - If True relabel the nodes to use the GEXF node "label" attribute instead of the node "id" attribute as the NetworkX node label.

Returns graph - If no parallel edges are found a Graph or DiGraph is returned. Otherwise a MultiGraph or MultiDiGraph is returned.
Return type NetworkX graph

## Notes

This implementation does not support mixed graphs (directed and undirected edges together).

## References

### 9.4.3 write_gexf

write_gexf (G, path, encoding='utf-8', prettyprint=True, version='1.1draft')
Write G in GEXF format to path.
"GEXF (Graph Exchange XML Format) is a language for describing complex networks structures, their associated data and dynamics" ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- path (file or string) - File or file name to write. File names ending in .gz or .bz2 will be compressed.
- encoding (string (optional)) - Encoding for text data.
- prettyprint (bool (optional)) - If True use line breaks and indenting in output XML.


## Examples

```
>>> G = nx.path_graph(4)
>>> nx.write_gexf(G, "test.gexf")
```

[^108]
## Notes

This implementation does not support mixed graphs (directed and undirected edges together).
The node id attribute is set to be the string of the node label. If you want to specify an id use set it as node data, e.g. node['a']['id']=1 to set the id of node ' $a$ ' to 1 .

## References

### 9.4.4 relabel_gexf_graph

relabel_gexf_graph ( $G$ )
Relabel graph using "label" node keyword for node label.
Parameters G (graph) - A NetworkX graph read from GEXF data
Returns H - A NetworkX graph with relabed nodes
Return type graph
Raises NetworkXError - If node labels are missing or not unique while relabel=True.

## Notes

This function relabels the nodes in a NetworkX graph with the "label" attribute. It also handles relabeling the specific GEXF node attributes "parents", and "pid".

### 9.5 GML

Read graphs in GML format.
"GML, the G>raph Modelling Language, is our proposal for a portable file format for graphs. GML's key features are portability, simple syntax, extensibility and flexibility. A GML file consists of a hierarchical key-value lists. Graphs can be annotated with arbitrary data structures. The idea for a common file format was born at the GD‘95; this proposal is the outcome of many discussions. GML is the standard file format in the Graphlet graph editor system. It has been overtaken and adapted by several other systems for drawing graphs."

See http://www.infosun.fim.uni-passau.de/Graphlet/GML/gml-tr.html

### 9.5.1 Format

See http://www.infosun.fim.uni-passau.de/Graphlet/GML/gml-tr.html for format specification.
Example graphs in GML format http://www-personal.umich.edu/~mejn/netdata/

| read_gml(path[, label, destringizer]) | Read graph in GML format from path. |
| :--- | :--- |
| write_gml(G, path[, stringizer]) | Write a graph G in GML format to the file or file handle <br> path. |
| parse_gml(lines[, label, destringizer]) | Parse GML graph from a string or iterable. |
| generate_gml(G[, stringizer]) | Generate a single entry of the graph G in GML format. |
| literal_destringizer(rep) | Convert a Python literal to the value it represents. |
| literal_stringizer(value) | Convert a value to a Python literal in GML representation. |

### 9.5.2 read_gml

read_gml (path, label='label', destringizer=None)
Read graph in GML format from path.

## Parameters

- path (filename or filehandle) - The filename or filehandle to read from.
- label (string, optional) - If not None, the parsed nodes will be renamed according to node attributes indicated by label. Default value: 'label'.
- destringizer (callable, optional) - A destringizer that recovers values stored as strings in GML. If it cannot convert a string to a value, a ValueError is raised. Default value : None.

Returns G - The parsed graph.
Return type NetworkX graph
Raises NetworkXError - If the input cannot be parsed.
See also:
write_gml(), parse_gml()

## Notes

The GML specification says that files should be ASCII encoded, with any extended ASCII characters (iso88591) appearing as HTML character entities.

## References

GML specification: http://www.infosun.fim.uni-passau.de/Graphlet/GML/gml-tr.html

## Examples

```
>>> G = nx.path_graph(4)
>>> nx.write_gml(G, 'test.gml')
>>> H = nx.read_gml('test.gml')
```


### 9.5.3 write_gml

write_gml (G, path, stringizer=None)
Write a graph G in GML format to the file or file handle path.

## Parameters

- G (NetworkX graph) - The graph to be converted to GML.
- path (filename or filehandle) - The filename or filehandle to write. Files whose names end with .gz or .bz2 will be compressed.
- stringizer (callable, optional) - A stringizer which converts non-int/non-float/non-dict values into strings. If it cannot convert a value into a string, it should raise a ValueError to indicate that. Default value: None.

Raises NetworkXError - If stringizer cannot convert a value into a string, or the value to convert is not a string while stringizer is None.

## See also:

```
read_gml(), generate_gml()
```


## Notes

Graph attributes named 'directed', 'multigraph', 'node’ or 'edge', node attributes named 'id' or 'label', edge attributes named 'source' or 'target' (or 'key' if G is a multigraph) are ignored because these attribute names are used to encode the graph structure.

## Examples

```
>>> G = nx.path_graph(4)
>>> nx.write_gml(G, "test.gml")
```

Filenames ending in .gz or .bz2 will be compressed.

```
>>> nx.write_gml(G, "test.gml.gz")
```


### 9.5.4 parse_gml

parse_gml (lines, label='label', destringizer=None)
Parse GML graph from a string or iterable.

## Parameters

- lines (string or iterable of strings) - Data in GML format.
- label (string, optional) - If not None, the parsed nodes will be renamed according to node attributes indicated by label. Default value: 'label'.
- destringizer (callable, optional) - A destringizer that recovers values stored as strings in GML. If it cannot convert a string to a value, a ValueError is raised. Default value : None.

Returns G - The parsed graph.
Return type NetworkX graph
Raises NetworkXError - If the input cannot be parsed.

## See also:

```
write_gml(),read_gml()
```


## Notes

This stores nested GML attributes as dictionaries in the NetworkX graph, node, and edge attribute structures.

## References

GML specification: http://www.infosun.fim.uni-passau.de/Graphlet/GML/gml-tr.html

### 9.5.5 generate_gml

generate_gml (G, stringizer=None)
Generate a single entry of the graph G in GML format.

## Parameters

- G (NetworkX graph) - The graph to be converted to GML.
- stringizer (callable, optional) - A stringizer which converts non-int/float/dict values into strings. If it cannot convert a value into a string, it should raise a ValueError raised to indicate that. Default value: None.

Returns lines - Lines of GML data. Newlines are not appended.
Return type generator of strings
Raises NetworkXError - If stringizer cannot convert a value into a string, or the value to convert is not a string while stringizer is None.

## Notes

Graph attributes named 'directed', 'multigraph', 'node' or 'edge', node attributes named 'id' or 'label', edge attributes named 'source' or 'target' (or 'key' if G is a multigraph) are ignored because these attribute names are used to encode the graph structure.

### 9.5.6 literal_destringizer

literal_destringizer (rep)
Convert a Python literal to the value it represents.
Parameters rep (string) - A Python literal.
Returns value - The value of the Python literal.
Return type object
Raises ValueError - If rep is not a Python literal.

### 9.5.7 literal_stringizer

literal_stringizer (value)
Convert a value to a Python literal in GML representation.
Parameters value (object) - The value to be converted to GML representation.
Returns rep - A double-quoted Python literal representing value. Unprintable characters are replaced by XML character references.

Return type string
Raises ValueError - If value cannot be converted to GML.

## Notes

Iiteral_stringizer is largely the same as repr in terms of functionality but attempts prefix unicode and bytes literals with $u$ and $b$ to provide better interoperability of data generated by Python 2 and Python 3 .

The original value can be recovered using the networkx.readwrite.gml.literal_destringizer() function.

### 9.6 Pickle

### 9.6.1 Pickled Graphs

Read and write NetworkX graphs as Python pickles.
"The pickle module implements a fundamental, but powerful algorithm for serializing and de-serializing a Python object structure. "Pickling" is the process whereby a Python object hierarchy is converted into a byte stream, and "unpickling" is the inverse operation, whereby a byte stream is converted back into an object hierarchy."

Note that NetworkX graphs can contain any hashable Python object as node (not just integers and strings). For arbitrary data types it may be difficult to represent the data as text. In that case using Python pickles to store the graph data can be used.

## Format

See http://docs.python.org/library/pickle.html

| read_gpickle(path) | Read graph object in Python pickle format. |
| :--- | :--- |
| write_gpickle(G, path[, protocol]) | Write graph in Python pickle format. |

### 9.6.2 read_gpickle

read_gpickle (path)
Read graph object in Python pickle format.
Pickles are a serialized byte stream of a Python object ${ }^{1}$. This format will preserve Python objects used as nodes or edges.

Parameters path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be uncompressed.
Returns G - A NetworkX graph
Return type graph

Examples

```
>>> G = nx.path_graph(4)
>>> nx.write_gpickle(G, "test.gpickle")
>>> G = nx.read_gpickle("test.gpickle")
```

[^109]
## References

### 9.6.3 write_gpickle

write_gpickle (G, path, protocol=2)
Write graph in Python pickle format.
Pickles are a serialized byte stream of a Python object ${ }^{1}$. This format will preserve Python objects used as nodes or edges.

## Parameters

- G (graph) - A NetworkX graph
- path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- protocol (integer) - Pickling protocol to use. Default value: pickle. HIGHEST_PROTOCOL.

Examples

```
>>> G = nx.path_graph(4)
>>> nx.write_gpickle(G, "test.gpickle")
```


## References

### 9.7 GraphML

### 9.7.1 GraphML

Read and write graphs in GraphML format.
This implementation does not support mixed graphs (directed and unidirected edges together), hyperedges, nested graphs, or ports.
"GraphML is a comprehensive and easy-to-use file format for graphs. It consists of a language core to describe the structural properties of a graph and a flexible extension mechanism to add application-specific data. Its main features include support of

- directed, undirected, and mixed graphs,
- hypergraphs,
- hierarchical graphs,
- graphical representations,
- references to external data,
- application-specific attribute data, and
- light-weight parsers.

[^110]Unlike many other file formats for graphs, GraphML does not use a custom syntax. Instead, it is based on XML and hence ideally suited as a common denominator for all kinds of services generating, archiving, or processing graphs."
http://graphml.graphdrawing.org/

## Format

GraphML is an XML format. See http://graphml.graphdrawing.org/specification.html for the specification and http: //graphml.graphdrawing.org/primer/graphml-primer.html for examples.

| read_graphml(path[, node_type]) | Read graph in GraphML format from path. |
| :--- | :--- |
| write_graphml(G, path[, encoding, ...]) | Write G in GraphML XML format to path |

### 9.7.2 read_graphml

```
read_graphml (path, node_type=<type 'str'>)
```

Read graph in GraphML format from path.

## Parameters

- path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- node_type (Python type (default: str)) - Convert node ids to this type

Returns graph - If no parallel edges are found a Graph or DiGraph is returned. Otherwise a MultiGraph or MultiDiGraph is returned.

Return type NetworkX graph

## Notes

This implementation does not support mixed graphs (directed and unidirected edges together), hypergraphs, nested graphs, or ports.
For multigraphs the GraphML edge "id" will be used as the edge key. If not specified then they "key" attribute will be used. If there is no "key" attribute a default NetworkX multigraph edge key will be provided.
Files with the yEd "yfiles" extension will can be read but the graphics information is discarded.
yEd compressed files ("file.graphmlz" extension) can be read by renaming the file to "file.graphml.gz".

### 9.7.3 write_graphml

write_graphml (G, path, encoding='utf-8', prettyprint=True, infer_numeric_types=False)
Write G in GraphML XML format to path

## Parameters

- G (graph) - A networkx graph
- infer_numeric_types (boolean) - Determine if numeric types should be generalized despite different python values. For example, if edges have both int and float 'weight' attributes, it will be inferred in GraphML that they are both floats (which translates to double in GraphML).
- path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- encoding (string (optional)) - Encoding for text data.
- prettyprint (bool (optional)) - If True use line breaks and indenting in output XML.


## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_graphml(G, "test.graphml")
```


## Notes

This implementation does not support mixed graphs (directed and unidirected edges together) hyperedges, nested graphs, or ports.

### 9.8 JSON

### 9.8.1 JSON data

Generate and parse JSON serializable data for NetworkX graphs.
These formats are suitable for use with the d3.js examples http://d3js.org/
The three formats that you can generate with NetworkX are:

- node-link like in the d3.js example http://bl.ocks.org/mbostock/4062045
- tree like in the d3.js example http://bl.ocks.org/mbostock/4063550
- adjacency like in the d3.js example http://bost.ocks.org/mike/miserables/

| node_link_data(G[, attrs]) | Return data in node-link format that is suitable for JSON <br> serialization and use in Javascript documents. |
| :--- | :--- |
| node_link_graph(data[, directed, ...]) | Return graph from node-link data format. |
| adjacency_data(G[, attrs]) | Return data in adjacency format that is suitable for JSON <br> serialization and use in Javascript documents. |
| adjacency_graph(data[, directed, ...]) | Return graph from adjacency data format. |
| tree_data(G, root[, attrs]) | Return data in tree format that is suitable for JSON serial- <br> ization and use in Javascript documents. |
| tree_graph(data[, attrs]) | Return graph from tree data format. |
| jit_data(G[, indent]) | Return data in JIT JSON format. |
| jit_graph(data) | Read a graph from JIT JSON. |

### 9.8.2 node_link_data

node_link_data ( $G$, attrs=None)
Return data in node-link format that is suitable for JSON serialization and use in Javascript documents.

## Parameters

- G (NetworkX graph)
- attrs (dict) - A dictionary that contains five keys 'source', 'target', 'name', 'key' and 'link'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value:

```
dict(source='source', target='target', name='name',
    key='key', link='links')
```

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.

Returns data - A dictionary with node-link formatted data.
Return type dict
Raises NetworkXError - If values in attrs are not unique.

## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([('A', 'B')])
>>> data1 = json_graph.node_link_data(G)
>>> H = nx.gn_graph(2)
>>> data2 = json_graph.node_link_data(H, {'link': 'edges', 'source': 'from',
\hookrightarrow'target': 'to'})
```

To serialize with json

```
>>> import json
>>> sl = json.dumps(data1)
>>> s2 = json.dumps(data2, {'link': 'edges', 'source': 'from', 'target': 'to'})
```


## Notes

Graph, node, and link attributes are stored in this format. Note that attribute keys will be converted to strings in order to comply with JSON.

Attribute 'key' is only used for multigraphs.

## See also:

```
node_link_graph(), adjacency_data(), tree_data()
```


### 9.8.3 node_link_graph

node_link_graph (data, directed=False, multigraph=True, attrs=None)
Return graph from node-link data format.

## Parameters

- data (dict) - node-link formatted graph data
- directed (bool) - If True, and direction not specified in data, return a directed graph.
- multigraph (bool) - If True, and multigraph not specified in data, return a multigraph.
- attrs (dict) - A dictionary that contains five keys 'source', 'target', 'name', 'key' and 'link'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. Default value:

```
dict(source='source', target='target', name='name', key='key', link='links')
```

Returns G - A NetworkX graph object
Return type NetworkX graph

## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([('A', 'B')])
>>> data = json_graph.node_link_data(G)
>>> H = json_graph.node_link_graph(data)
```


## Notes

Attribute 'key' is only used for multigraphs.

## See also:

```
node_link_data(), adjacency_data(), tree_data()
```


### 9.8.4 adjacency_data

```
adjacency_data (G, attrs={ 'id': 'id', 'key': 'key'})
```

Return data in adjacency format that is suitable for JSON serialization and use in Javascript documents.

## Parameters

- G (NetworkX graph)
- attrs (dict) - A dictionary that contains two keys 'id' and 'key'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: dict (id='id',key='key').

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.

Returns data - A dictionary with adjacency formatted data.
Return type dict
Raises NetworkXError - If values in attrs are not unique.

## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([ (1, 2)])
>>> data = json_graph.adjacency_data(G)
```

To serialize with json

```
>>> import json
>>> s = json.dumps(data)
```


## Notes

Graph, node, and link attributes will be written when using this format but attribute keys must be strings if you want to serialize the resulting data with JSON.

The default value of attrs will be changed in a future release of NetworkX.

## See also:

```
adjacency_graph(), node_link_data(), tree_data()
```


### 9.8.5 adjacency_graph

adjacency_graph (data, directed=False, multigraph=True, attrs=\{ 'id’: 'id', 'key': ‘key'\})
Return graph from adjacency data format.
Parameters data (dict) - Adjacency list formatted graph data

## Returns

- G (NetworkX graph) - A NetworkX graph object
- directed (bool) - If True, and direction not specified in data, return a directed graph.
- multigraph (bool) - If True, and multigraph not specified in data, return a multigraph.
- attrs (dict) - A dictionary that contains two keys 'id' and 'key'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: dict(id='id',key='key').


## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([ (1,2)])
>>> data = json_graph.adjacency_data(G)
>>> H = json_graph.adjacency_graph(data)
```


## Notes

The default value of attrs will be changed in a future release of NetworkX.

## See also:

adjacency_graph(), node_link_data(), tree_data()

### 9.8.6 tree_data

tree_data (G, root, attrs=\{'children': 'children', 'id': 'id'\})
Return data in tree format that is suitable for JSON serialization and use in Javascript documents.

## Parameters

- G (NetworkX graph) - G must be an oriented tree
- root (node) - The root of the tree
- attrs (dict) - A dictionary that contains two keys 'id' and 'children'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: dict (id='id', children='children').

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.

Returns data - A dictionary with node-link formatted data.
Return type dict
Raises NetworkXError - If values in attrs are not unique.

## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.DiGraph([(1, 2)])
>>> data = json_graph.tree_data(G,root=1)
```

To serialize with json

```
>>> import json
>>> s = json.dumps(data)
```


## Notes

Node attributes are stored in this format but keys for attributes must be strings if you want to serialize with JSON.

Graph and edge attributes are not stored.
The default value of attrs will be changed in a future release of NetworkX.

## See also:

```
tree_graph(), node_link_data(), node_link_data()
```


### 9.8.7 tree_graph

tree_graph (data, attrs=\{'children': 'children', 'id': 'id'\})
Return graph from tree data format.
Parameters data (dict) - Tree formatted graph data

## Returns

## - G (NetworkX DiGraph)

- attrs (dict) - A dictionary that contains two keys 'id' and 'children'. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: dict(id='id', children='children').


## Examples

```
>>> from networkx.readwrite import json_graph
>>> G = nx.DiGraph([(1,2)])
>>> data = json_graph.tree_data(G,root=1)
>>> H = json_graph.tree_graph(data)
```


## Notes

The default value of attrs will be changed in a future release of NetworkX.

## See also:

```
tree_graph(), node_link_data(), adjacency_data()
```


### 9.8.8 jit_data

jit_data (G, indent=None)
Return data in JIT JSON format.

## Parameters

- G (NetworkX Graph)
- indent (optional, default=None) - If indent is a non-negative integer, then JSON array elements and object members will be pretty-printed with that indent level. An indent level of 0 , or negative, will only insert newlines. None (the default) selects the most compact representation.


## Returns data

Return type JIT JSON string

### 9.8.9 jit_graph

jit_graph (data)
Read a graph from JIT JSON.
Parameters data (JSON Graph Object)
Returns G
Return type NetworkX Graph

### 9.9 LEDA

Read graphs in LEDA format.
LEDA is a C++ class library for efficient data types and algorithms.

### 9.9.1 Format

See http://www.algorithmic-solutions.info/leda_guide/graphs/leda_native_graph_fileformat.html

| read_leda(path[, encoding]) | Read graph in LEDA format from path. |
| :--- | :--- |
| parse_leda(lines) | Read graph in LEDA format from string or iterable. |

### 9.9.2 read_leda

read_leda (path, encoding='UTF-8')
Read graph in LEDA format from path.
Parameters path (file or string) - File or filename to read. Filenames ending in .gz or .bz2 will be uncompressed.

## Returns G

Return type NetworkX graph

## Examples

G=nx.read_leda('file.leda')

## References

### 9.9.3 parse_leda

parse_leda (lines)
Read graph in LEDA format from string or iterable.
Parameters lines (string or iterable) - Data in LEDA format.
Returns G
Return type NetworkX graph

## Examples

G=nx.parse_leda(string)

## References

### 9.10 YAML

### 9.10.1 YAML

Read and write NetworkX graphs in YAML format.
"YAML is a data serialization format designed for human readability and interaction with scripting languages." See http://www.yaml.org for documentation.

## Format

http://pyyaml.org/wiki/PyYAML

| read_yaml(path) | Read graph in YAML format from path. |
| :--- | :--- |
| write_yaml(G, path[, encoding]) | Write graph G in YAML format to path. |

### 9.10.2 read_yaml

read_yaml (path)
Read graph in YAML format from path.
YAML is a data serialization format designed for human readability and interaction with scripting languages ${ }^{1}$.
Parameters path (file or string) - File or filename to read. Filenames ending in .gz or .bz2 will be uncompressed.
Returns G
Return type NetworkX graph

Examples
>>> G=nx.path_graph (4)
$\ggg$ nx.write_yaml (G,'test.yaml')
$\ggg G=n x . r e a d \_y a m l(' t e s t . y a m l ')$

## References

### 9.10.3 write_yaml

write_yaml ( $G$, path, encoding ='UTF-8', **kwds)
Write graph $G$ in YAML format to path.
YAML is a data serialization format designed for human readability and interaction with scripting languages ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- encoding (string, optional) - Specify which encoding to use when writing file.

Examples
>>> G=nx.path_graph (4)
$\ggg$ nx.write_yaml(G,'test.yaml')

[^111]
## References

### 9.11 SparseGraph6

Functions for reading and writing graphs in the graph6 or sparse6 file formats.
According to the author of these formats,
graph6 and sparse6 are formats for storing undirected graphs in a compact manner, using only printable ASCII characters. Files in these formats have text type and contain one line per graph.
graph6 is suitable for small graphs, or large dense graphs. sparse6 is more space-efficient for large sparse graphs.
-graph6 and sparse6 homepage

### 9.11.1 Graph6

Functions for reading and writing graphs in the graph6 format.
The graph6 file format is suitable for small graphs or large dense graphs. For large sparse graphs, use the sparse6 format.
For more information, see the graph6 homepage.

| parse_graph 6 (string $)$ | Read a simple undirected graph in graph6 format from <br> string. |
| :--- | :--- |
| read_graph $6($ path $)$ | Read simple undirected graphs in graph6 format from path. |
| generate_graph $6(\mathrm{G}[$, nodes, header]) | Generate graph6 format string from a simple undirected <br> graph. |
| write_graph $6(\mathrm{G}$, path[, nodes, header] $)$ | Write a simple undirected graph to path in graph6 format. |

parse_graph6
parse_graph6 (string)
Read a simple undirected graph in graph6 format from string.
Parameters string (string) - Data in graph6 format

## Returns G

Return type Graph
Raises NetworkXError - If the string is unable to be parsed in graph6 format

## Examples

```
>>> G = nx.parse_graph6('A_')
>>> sorted(G.edges())
[(0, 1)]
```


## See also:

```
generate_graph6(),read_graph6(),write_graph6()
```


## References

## read_graph6

read_graph6 (path)
Read simple undirected graphs in graph6 format from path.
Parameters path (file or string) - File or filename to write.
Returns G - If the file contains multiple lines then a list of graphs is returned
Return type Graph or list of Graphs
Raises NetworkXError - If the string is unable to be parsed in graph6 format

## Examples

```
>>> nx.write_graph6(nx.Graph([(0,1)]), 'test.g6')
>>> G = nx.read_graph6('test.g6')
>>> sorted(G.edges())
[(0, 1)]
```


## See also:

```
generate_graph6(), parse_graph6(),write_graph6()
```


## References

```
generate_graph6
```

generate_graph6 ( $G$, nodes=None, header=True)

Generate graph6 format string from a simple undirected graph.

## Parameters

- G (Graph (undirected))
- nodes (list or iterable) - Nodes are labeled $0 . . . \mathrm{n}-1$ in the order provided. If None the ordering given by G.nodes() is used.
- header (bool) - If True add ' $\gg$ graph6 $\ll$ ' string to head of data

Returns s-String in graph6 format
Return type string
Raises NetworkXError - If the graph is directed or has parallel edges

## Examples

```
>>> G = nx.Graph([(0, 1)])
>>> nx.generate_graph6(G)
'>>graph6<<A_'
```


## See also:

```
read_graph6(), parse_graph6(),write_graph6()
```


## Notes

The format does not support edge or node labels, parallel edges or self loops. If self loops are present they are silently ignored.

## References

## write_graph6

write_graph6 (G, path, nodes=None, header=True)
Write a simple undirected graph to path in graph6 format.

## Parameters

- G (Graph (undirected))
- path (file or string) - File or filename to write.
- nodes (list or iterable) - Nodes are labeled $0 . . . \mathrm{n}-1$ in the order provided. If None the ordering given by G.nodes() is used.
- header (bool) - If True add ' $\gg$ graph6<<' string to head of data

Raises NetworkXError - If the graph is directed or has parallel edges

## Examples

```
>>> G = nx.Graph([(0, 1)])
>>> nx.write_graph6(G, 'test.g6')
```


## See also:

```
generate_graph6(),parse_graph6(),read_graph6()
```


## Notes

The format does not support edge or node labels, parallel edges or self loops. If self loops are present they are silently ignored.

## References

### 9.11.2 Sparse6

Functions for reading and writing graphs in the sparse6 format.
The sparse6 file format is a space-efficient format for large sparse graphs. For small graphs or large dense graphs, use the graph6 file format.

For more information, see the sparse6 homepage.

| parse_sparse6(string) | Read an undirected graph in sparse6 format from string. |
| :--- | :--- |
| read_sparse6(path) | Read an undirected graph in sparse6 format from path. |
|  | Continued on next page |

Table 9.12 - continued from previous page

| generate_sparse6(G[, nodes, header]) | Generate sparse6 format string from an undirected graph. |
| :--- | :--- |
| write_sparse6(G, path[, nodes, header]) | Write graph G to given path in sparse6 format. |

parse_sparse6
parse_sparse6 (string)
Read an undirected graph in sparse6 format from string.
Parameters string (string) - Data in sparse6 format

## Returns G

Return type Graph
Raises NetworkXError - If the string is unable to be parsed in sparse6 format

## Examples

```
>>> G = nx.parse_sparse6(':A_')
>>> sorted(G.edges())
[(0, 1), (0, 1), (0, 1)]
```

See also:

```
generate_sparse6(),read_sparse6(),write_sparse6()
```


## References

```
read_sparse6
```

read_sparse6 (path)

Read an undirected graph in sparse6 format from path.
Parameters path (file or string) - File or filename to write.
Returns $\mathbf{G}$ - If the file contains multple lines then a list of graphs is returned
Return type Graph/Multigraph or list of Graphs/MultiGraphs
Raises NetworkXError - If the string is unable to be parsed in sparse6 format

## Examples

```
>>> nx.write_sparse6(nx.Graph([(0,1), (0,1),(0,1)]), 'test.s6')
>>> G = nx.read_sparse6('test.s6')
>>> sorted(G.edges())
[(0, 1)]
```


## See also:

```
generate_sparse6(), read_sparse6(), parse_sparse6()
```


## References

## generate_sparse6

generate_sparse6 ( $G$, nodes=None, header=True)
Generate sparse6 format string from an undirected graph.

## Parameters

- G (Graph (undirected))
- nodes (list or iterable) - Nodes are labeled $0 \ldots \mathrm{n}-1$ in the order provided. If None the ordering given by G.nodes() is used.
- header (bool) - If True add '>>sparse6<<<' string to head of data

Returns $\mathbf{s}$ - String in sparse6 format
Return type string
Raises NetworkXError - If the graph is directed

## Examples

$\ggg G=n x . M u l t i G r a p h([(0,1),(0,1),(0,1)])$
>>> nx.generate_sparse6(G)
'>>sparse6<<:A_'

## See also:

```
read_sparse6(), parse_sparse6(),write_sparse6()
```


## Notes

The format does not support edge or node labels.

## References

write_sparse6
write_sparse6 ( $G$, path, nodes=None, header=True)
Write graph G to given path in sparse6 format.
Parameters

- G (Graph (undirected))
- path (file or string) - File or filename to write
- nodes (list or iterable) - Nodes are labeled $0 . . . \mathrm{n}-1$ in the order provided. If None the ordering given by G.nodes() is used.
- header (bool) - If True add ' $\gg$ sparse6<<<' string to head of data

Raises NetworkXError - If the graph is directed

## Examples

```
>>> G = nx.Graph([(0, 1), (0, 1), (0, 1)])
>>> nx.write_sparse6(G, 'test.s6')
```


## See also:

```
read_sparse6(), parse_sparse6(), generate_sparse6()
```


## Notes

The format does not support edge or node labels.

## References

### 9.12 Pajek

### 9.12.1 Pajek

Read graphs in Pajek format.
This implementation handles directed and undirected graphs including those with self loops and parallel edges.

## Format

See http://vlado.fmf.uni-lj.si/pub/networks/pajek/doc/draweps.htm for format information.

| read_pajek(path[, encoding]) | Read graph in Pajek format from path. |
| :--- | :--- |
| write_pajek(G, path[, encoding]) | Write graph in Pajek format to path. |
| parse_pajek(lines) | Parse Pajek format graph from string or iterable. |

### 9.12.2 read_pajek

read_pajek (path, encoding='UTF-8')
Read graph in Pajek format from path.
Parameters path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be uncompressed.

## Returns G

Return type NetworkX MultiGraph or MultiDiGraph.

## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_pajek(G, "test.net")
>>> G=nx.read_pajek("test.net")
```

To create a Graph instead of a MultiGraph use
$\ggg G 1=n x \cdot G r a p h(G)$

## References

See http://vlado.fmf.uni-lj.si/pub/networks/pajek/doc/draweps.htm for format information.

### 9.12.3 write_pajek

write_pajek (G, path, encoding='UTF-8')
Write graph in Pajek format to path.

## Parameters

- G (graph) - A Networkx graph
- path (file or string) - File or filename to write. Filenames ending in .gz or .bz2 will be compressed.


## Examples

```
>>> G=nx.path_graph(4)
>>> nx.write_pajek(G, "test.net")
```


## References

See http://vlado.fmf.uni-lj.si/pub/networks/pajek/doc/draweps.htm for format information.

### 9.12.4 parse_pajek

parse_pajek (lines)
Parse Pajek format graph from string or iterable.
Parameters lines (string or iterable) - Data in Pajek format.
Returns G
Return type NetworkX graph
See also:
read_pajek()

### 9.13 GIS Shapefile

### 9.13.1 Shapefile

Generates a networkx.DiGraph from point and line shapefiles.
"The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software. It is developed and regulated by Esri as a (mostly) open specification for data interoperability among Esri and other software products." See http://en.wikipedia.org/wiki/Shapefile for additional information.

| read_shp(path[, simplify, geom_attrs]) | Generates a networkx.DiGraph from shapefiles. |
| :--- | :--- |
| write_shp(G, outdir) | Writes a networkx.DiGraph to two shapefiles, edges and <br> nodes. |

### 9.13.2 read_shp

read_shp (path, simplify=True, geom_attrs=True)
Generates a networkx.DiGraph from shapefiles. Point geometries are translated into nodes, lines into edges. Coordinate tuples are used as keys. Attributes are preserved, line geometries are simplified into start and end coordinates. Accepts a single shapefile or directory of many shapefiles.
"The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software ${ }^{1}$."

## Parameters

- path (file or string) - File, directory, or filename to read.
- simplify (bool) - If True, simplify line geometries to start and end coordinates. If False, and line feature geometry has multiple segments, the non-geometric attributes for that feature will be repeated for each edge comprising that feature.
- geom_attrs (bool) - If True, include the Wkb, Wkt and Json geometry attributes with each edge.

NOTE: if these attributes are available, write_shp will use them to write the geometry. If nodes store the underlying coordinates for the edge geometry as well (as they do when they are read via this method) and they change, your geomety will be out of sync.

## Returns G

Return type NetworkX graph

## Examples

```
>>> G=nx.read_shp('test.shp')
```


## References

### 9.13.3 write_shp

write_shp (G, outdir)
Writes a networkx.DiGraph to two shapefiles, edges and nodes. Nodes and edges are expected to have a Well Known Binary (Wkb) or Well Known Text (Wkt) key in order to generate geometries. Also acceptable are nodes with a numeric tuple key ( $\mathrm{x}, \mathrm{y}$ ).
"The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software ${ }^{1}$."

Parameters outdir (directory path) - Output directory for the two shapefiles.

## Returns

[^112]Return type None

## Examples

nx.write_shp(digraph, ‘/shapefiles') \# doctest +SKIP

References

NetworkX provides basic functionality for visualizing graphs, but its main goal is to enable graph analysis rather than perform graph visualization. In the future, graph visualization functionality may be removed from NetworkX or only available as an add-on package.
Proper graph visualization is hard, and we highly recommend that people visualize their graphs with tools dedicated to that task. Notable examples of dedicated and fully-featured graph visualization tools are Cytoscape, Gephi, Graphviz and, for LaTeX typesetting, PGF/TikZ. To use these and other such tools, you should export your NetworkX graph into a format that can be read by those tools. For example, Cytoscape can read the GraphML format, and so, networkx.write_graphml (G) might be an appropriate choice.

### 10.1 Matplotlib

### 10.1.1 Matplotlib

Draw networks with matplotlib.

```
See also:
matplotlib http://matplotlib.org/
pygraphviz http://pygraphviz.github.io/
```

| draw(G[, pos, ax, hold]) | Draw the graph G with Matplotlib. |
| :---: | :---: |
| draw_networkx(G[, pos, arrows, with_labels]) | Draw the graph G using Matplotlib. |
| draw_networkx_nodes(G, pos[, nodelist, ...]) | Draw the nodes of the graph G. |
| draw_networkx_edges(G, pos[, edgelist, ...]) | Draw the edges of the graph G. |
| draw_networkx_labels(G, pos[, labels, ...]) | Draw node labels on the graph G. |
| draw_networkx_edge_labels(G, pos[, ...]) | Draw edge labels. |
| draw_circular(G, \*\*kwargs) | Draw the graph G with a circular layout. |
| draw_random(G, \*\*kwargs) | Draw the graph G with a random layout. |
| draw_spectral(G, \*\*kwargs) | Draw the graph G with a spectral layout. |
| draw_spring(G, \***kwargs) | Draw the graph G with a spring layout. |
| draw_shell(G, \*\*kwargs) | Draw networkx graph with shell layout. |

### 10.1.2 draw

draw (G, pos=None, ax=None, hold=None, **kwds)
Draw the graph G with Matplotlib.

Draw the graph as a simple representation with no node labels or edge labels and using the full Matplotlib figure area and no axis labels by default. See draw_networkx() for more full-featured drawing that allows title, axis labels etc.

## Parameters

- G (graph) - A networkx graph
- pos (dictionary, optional) - A dictionary with nodes as keys and positions as values. If not specified a spring layout positioning will be computed. See networkx. drawing. layout for functions that compute node positions.
- ax (Matplotlib Axes object, optional) - Draw the graph in specified Matplotlib axes.
- hold (bool, optional) - Set the Matplotlib hold state. If True subsequent draw commands will be added to the current axes.
- kwds (optional keywords) - See networkx.draw_networkx() for a description of optional keywords.


## Examples

```
>>> G=nx.dodecahedral_graph()
>>> nx.draw(G)
>>> nx.draw(G,pos=nx.spring_layout(G)) # use spring layout
```

See also:

```
draw_networkx(), draw_networkx_nodes(), draw_networkx_edges(),
draw_networkx_labels(), draw_networkx_edge_labels()
```


## Notes

This function has the same name as pylab.draw and pyplot.draw so beware when using

```
>>> from networkx import *
```

since you might overwrite the pylab.draw function.
With pyplot use

```
>>> import matplotlib.pyplot as plt
>>> import networkx as nx
>>> G=nx.dodecahedral_graph()
>>> nx.draw(G) # networkx draw()
>>> plt.draw() # pyplot draw()
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html

### 10.1.3 draw_networkx

draw_networkx (G, pos=None, arrows=True, with_labels=True, **kwds)
Draw the graph G using Matplotlib.
Draw the graph with Matplotlib with options for node positions, labeling, titles, and many other drawing features. See $\operatorname{draw}()$ for simple drawing without labels or axes.

## Parameters

- G (graph) - A networkx graph
- pos (dictionary, optional) - A dictionary with nodes as keys and positions as values. If not specified a spring layout positioning will be computed. See networkx. drawing. layout for functions that compute node positions.
- arrows (bool, optional (default=True)) - For directed graphs, if True draw arrowheads.
- with_labels (bool, optional (default=True)) - Set to True to draw labels on the nodes.
- ax (Matplotlib Axes object, optional) - Draw the graph in the specified Matplotlib axes.
- nodelist (list, optional (default G.nodes())) - Draw only specified nodes
- edgelist (list, optional (default=G.edges())) - Draw only specified edges
- node_size (scalar or array, optional (default=300)) - Size of nodes. If an array is specified it must be the same length as nodelist.
- node_color (color string, or array of floats, (default='r')) - Node color. Can be a single color format string, or a sequence of colors with the same length as nodelist. If numeric values are specified they will be mapped to colors using the cmap and vmin,vmax parameters. See matplotlib.scatter for more details.
- node_shape (string, optional (default='o')) - The shape of the node. Specification is as matplotlib.scatter marker, one of 'so^>v<dph8'.
- alpha (float, optional (default=1.0)) - The node and edge transparency
- cmap (Matplotlib colormap, optional (default=None)) - Colormap for mapping intensities of nodes
- vmin,vmax (float, optional (default=None)) - Minimum and maximum for node colormap scaling
- linewidths ([None | scalar | sequence]) - Line width of symbol border (default =1.0)
- width (float, optional (default=1.0)) - Line width of edges
- edge_color (color string, or array offloats (default='r')) - Edge color. Can be a single color format string, or a sequence of colors with the same length as edgelist. If numeric values are specified they will be mapped to colors using the edge_cmap and edge_vmin,edge_vmax parameters.
- edge_cmap (Matplotlib colormap, optional (default=None)) - Colormap for mapping intensities of edges
- edge_vmin,edge_vmax (floats, optional (default=None)) - Minimum and maximum for edge colormap scaling
- style (string, optional (default='solid')) - Edge line style (solidldashedldotted,dashdot)
- labels (dictionary, optional (default=None)) - Node labels in a dictionary keyed by node of text labels
- font_size (int, optional (default=12)) - Font size for text labels
- font_color (string, optional (default=' $k$ ' black)) - Font color string
- font_weight (string, optional (default='normal')) - Font weight
- font_family (string, optional (default='sans-serif')) - Font family
- label (string, optional) - Label for graph legend


## Notes

For directed graphs, "arrows" (actually just thicker stubs) are drawn at the head end. Arrows can be turned off with keyword arrows=False. Yes, it is ugly but drawing proper arrows with Matplotlib this way is tricky.

## Examples

```
>>> G=nx.dodecahedral_graph()
>>> nx.draw(G)
>> nx.draw(G,pos=nx.spring_layout(G)) # use spring layout
```

```
>>> import matplotlib.pyplot as plt
>> limits=plt.axis('off') # turn of axis
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html

## See also:

```
draw(), draw_networkx_nodes(), draw_networkx_edges(), draw_networkx_labels(),
```

draw_networkx_edge_labels()

### 10.1.4 draw_networkx_nodes

draw_networkx_nodes ( $G$, pos, nodelist=None, node_size $=300$, node_color='r', node_shape='o', alpha=1.0, cmap=None, vmin=None, vmax=None, ax=None, linewidths=None, label=None, **kwds)
Draw the nodes of the graph G.
This draws only the nodes of the graph G.

## Parameters

- G (graph) - A networkx graph
- pos (dictionary) - A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2 .
- ax (Matplotlib Axes object, optional) - Draw the graph in the specified Matplotlib axes.
- nodelist (list, optional) - Draw only specified nodes (default G.nodes())
- node_size (scalar or array) - Size of nodes (default=300). If an array is specified it must be the same length as nodelist.
- node_color (color string, or array of floats) - Node color. Can be a single color format string (default='r'), or a sequence of colors with the same length as nodelist. If numeric values are specified they will be mapped to colors using the cmap and vmin,vmax parameters. See matplotlib.scatter for more details.
- node_shape (string) - The shape of the node. Specification is as matplotlib.scatter marker, one of 'so^>v<dph8' (default='o').
- alpha (float) - The node transparency (default=1.0)
- cmap (Matplotlib colormap) - Colormap for mapping intensities of nodes (default=None)
- vmin,vmax (floats) - Minimum and maximum for node colormap scaling (default=None)
- linewidths ([None $\mid$ scalar $\mid$ sequence $])$ - Line width of symbol border $($ default $=1.0)$
- label ([Nonel string]) - Label for legend

Returns PathCollection of the nodes.
Return type matplotlib.collections.PathCollection

## Examples

```
>>> G=nx.dodecahedral_graph()
>>> nodes=nx.draw_networkx_nodes(G,pos=nx.spring_layout (G))
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html See also:

```
draw(), draw_networkx(), draw_networkx_edges(), draw_networkx_labels(),
draw_networkx_edge_labels()
```


### 10.1.5 draw_networkx_edges

draw_networkx_edges (G, pos, edgelist=None, width=1.0, edge_color='k', style='solid', alpha=1.0, edge_cmap=None, edge_vmin=None, edge_vmax=None, ax=None, arrows=True, label=None, ${ }^{* *}$ *kwds)
Draw the edges of the graph G.
This draws only the edges of the graph G.
Parameters

- G (graph) - A networkx graph
- pos (dictionary) - A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2 .
- edgelist (collection of edge tuples) - Draw only specified edges(default=G.edges())
- width (float, or array of floats) - Line width of edges (default=1.0)
- edge_color (color string, or array offloats) - Edge color. Can be a single color format string (default='r'), or a sequence of colors with the same length as edgelist. If numeric values are specified they will be mapped to colors using the edge_cmap and edge_vmin,edge_vmax parameters.
- style (string) - Edge line style (default='solid') (solidldashedldotted,dashdot)
- alpha (float) - The edge transparency (default=1.0)
- edge_ cmap (Matplotlib colormap) - Colormap for mapping intensities of edges (default=None)
- edge_vmin,edge_vmax (floats) - Minimum and maximum for edge colormap scaling (default=None)
- ax (Matplotlib Axes object, optional) - Draw the graph in the specified Matplotlib axes.
- arrows (bool, optional (default=True)) - For directed graphs, if True draw arrowheads.
- label ([Nonel string]) - Label for legend

Returns LineCollection of the edges
Return type matplotlib.collection.LineCollection

## Notes

For directed graphs, "arrows" (actually just thicker stubs) are drawn at the head end. Arrows can be turned off with keyword arrows=False. Yes, it is ugly but drawing proper arrows with Matplotlib this way is tricky.

## Examples

```
>>> G=nx.dodecahedral_graph()
>>> edges=nx.draw_networkx_edges(G,pos=nx.spring_layout (G))
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html See also:
draw(), draw_networkx(), draw_networkx_nodes(), draw_networkx_labels(), draw_networkx_edge_labels()

### 10.1.6 draw_networkx_labels

draw_networkx_labels (G, pos, labels=None, font_size $=12$, font_color=' $k$ ', font_family='sans-serif', font_weight='normal', alpha=1.0, bbox=None, ax=None, **kwds)
Draw node labels on the graph G .

## Parameters

- G (graph) - A networkx graph
- pos (dictionary) - A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2.
- labels (dictionary, optional (default=None)) - Node labels in a dictionary keyed by node of text labels
- font_size (int) - Font size for text labels (default=12)
- font_color (string) - Font color string (default=' $k$ ' black)
- font_family (string) - Font family (default='sans-serif')
- font_weight (string) - Font weight (default='normal')
- alpha (float) - The text transparency (default=1.0)
- ax (Matplotlib Axes object, optional) - Draw the graph in the specified Matplotlib axes.

Returns dict of labels keyed on the nodes
Return type dict

## Examples

```
>>> G=nx.dodecahedral_graph()
>>> labels=nx.draw_networkx_labels(G,pos=nx.spring_layout (G))
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html

## See also:

draw(), draw_networkx(), draw_networkx_nodes(), draw_networkx_edges(), draw_networkx_edge_labels()

### 10.1.7 draw_networkx_edge_labels

draw_networkx_edge_labels $\left(G, \quad\right.$ pos, $\quad e d g e \_l a b e l s=N o n e, \quad$ label_pos=0.5, font_size $=10$, font_color=' $k$ ', font_family='sans-serif', font_weight='normal', alpha=1.0, bbox=None, ax=None, rotate=True, ${ }^{* *} k w d s$ )
Draw edge labels.

## Parameters

- G (graph) - A networkx graph
- pos (dictionary) - A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2 .
- ax (Matplotlib Axes object, optional) - Draw the graph in the specified Matplotlib axes.
- alpha $($ float $)$ - The text transparency (default=1.0)
- edge_labels (dictionary) - Edge labels in a dictionary keyed by edge two-tuple of text labels (default=None). Only labels for the keys in the dictionary are drawn.
- label_pos (float) - Position of edge label along edge ( $0=$ head, $0.5=$ center, $1=$ tail $)$
- font_size (int) - Font size for text labels (default=12)
- font_color (string) - Font color string (default=' $k$ ' black)
- font_weight (string) - Font weight (default='normal')
- font_family (string) - Font family (default='sans-serif')
- bbox (Matplotlib bbox) - Specify text box shape and colors.
- clip_on (bool) - Turn on clipping at axis boundaries (default=True)

Returns dict of labels keyed on the edges
Return type dict

## Examples

```
>>> G=nx.dodecahedral_graph()
>>> edge_labels=nx.draw_networkx_edge_labels(G,pos=nx.spring_layout(G))
```

Also see the NetworkX drawing examples at http://networkx.github.io/documentation/latest/gallery.html See also:

```
draw(), draw_networkx(), draw_networkx_nodes(), draw_networkx_edges(),
draw_networkx_labels()
```


### 10.1.8 draw_circular

draw_circular (G, **kwargs)
Draw the graph G with a circular layout.

## Parameters

- G (graph) - A networkx graph
- kwargs (optional keywords) - See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.


### 10.1.9 draw_random

draw_random ( $G$, **kwargs)
Draw the graph G with a random layout.

## Parameters

- G (graph) - A networkx graph
- kwargs (optional keywords) - See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.


### 10.1.10 draw_spectral

draw_spectral ( $G, * * k w a r g s$ )
Draw the graph $G$ with a spectral layout.

## Parameters

- G (graph) - A networkx graph
- kwargs (optional keywords) - See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.


### 10.1.11 draw_spring

draw_spring ( $G$, **kwargs)
Draw the graph $G$ with a spring layout.

## Parameters

- G (graph) - A networkx graph
- kwargs (optional keywords) - See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.


### 10.1.12 draw_shell

draw_shell ( $G$, **kwargs)
Draw networkx graph with shell layout.

## Parameters

- G (graph) - A networkx graph
- kwargs (optional keywords) - See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.


### 10.2 Graphviz AGraph (dot)

### 10.2.1 Graphviz AGraph

Interface to pygraphviz AGraph class.

## Examples

```
>>>G G nx.complete_graph (5)
>>> A = nx.nx_agraph.to_agraph(G)
>>> H = nx.nx_agraph.from_agraph(A)
```


## See also:

Pygraphviz http://pygraphviz.github.io/

| from_agraph(A[, create_using]) | Return a NetworkX Graph or DiGraph from a PyGraphviz <br> graph. |
| :--- | :--- |
| to_agraph(N) | Return a pygraphviz graph from a NetworkX graph N. |
| write_dot(G, path) | Write NetworkX graph G to Graphviz dot format on path. |
| read_dot(path) | Return a NetworkX graph from a dot file on path. |
| graphviz_layout(G[, prog, root, args]) | Create node positions for G using Graphviz. |
| pygraphviz_layout(G[, prog, root, args]) | Create node positions for G using Graphviz. |

### 10.2.2 from_agraph

from_agraph (A, create_using=None)
Return a NetworkX Graph or DiGraph from a PyGraphviz graph.

## Parameters

- A (PyGraphviz AGraph) - A graph created with PyGraphviz
- create_using (NetworkX graph class instance) - The output is created using the given graph class instance


## Examples

```
>>> K5 = nx.complete_graph(5)
```

$\ggg$ A $=$ nx.nx_agraph.to_agraph (K5)
$\ggg G=n x . n x \_a g r a p h . f r o m \_a g r a p h(A)$
>>> G $=$ nx.nx_agraph.from_agraph(A)

## Notes

The Graph G will have a dictionary G.graph_attr containing the default graphviz attributes for graphs, nodes and edges.

Default node attributes will be in the dictionary G.node_attr which is keyed by node.

Edge attributes will be returned as edge data in G. With edge_attr=False the edge data will be the Graphviz edge weight attribute or the value 1 if no edge weight attribute is found.

### 10.2.3 to_agraph

to_agraph ( $N$ )
Return a pygraphviz graph from a NetworkX graph N.
Parameters $\mathbf{N}$ (NetworkX graph) - A graph created with NetworkX

## Examples

```
>>> K5 = nx.complete_graph(5)
>>> A = nx.nx_agraph.to_agraph(K5)
```


## Notes

If N has an dict N. graph_attr an attempt will be made first to copy properties attached to the graph (see from_agraph) and then updated with the calling arguments if any.

### 10.2.4 write_dot

write_dot (G, path)
Write NetworkX graph G to Graphviz dot format on path.

## Parameters

- G (graph) - A networkx graph
- path (filename) - Filename or file handle to write


### 10.2.5 read_dot

read_dot (path)
Return a NetworkX graph from a dot file on path.
Parameters path (file or string) - File name or file handle to read.

### 10.2.6 graphviz_layout

graphviz_layout (G, prog='neato', root=None, args=' ')
Create node positions for G using Graphviz.

## Parameters

- G (NetworkX graph) - A graph created with NetworkX
- prog (string) - Name of Graphviz layout program
- root (string, optional) - Root node for twopi layout
- args (string, optional) - Extra arguments to Graphviz layout program
- Returns (dictionary) - Dictionary of $\mathrm{x}, \mathrm{y}$, positions keyed by node.


## Examples

```
>>> G = nx.petersen_graph()
>>> pos = nx.nx_agraph.graphviz_layout(G)
>>> pos = nx.nx_agraph.graphviz_layout(G, prog='dot')
```


## Notes

This is a wrapper for pygraphviz_layout.

### 10.2.7 pygraphviz_layout

pygraphviz_layout ( $G, \operatorname{prog=}=$ 'neato', root=None, $\operatorname{args=}=$ ' $)$
Create node positions for $G$ using Graphviz.

## Parameters

- G (NetworkX graph) - A graph created with NetworkX
- prog (string) - Name of Graphviz layout program
- root (string, optional) - Root node for twopi layout
- args (string, optional) - Extra arguments to Graphviz layout program
- Returns (dictionary) - Dictionary of $x, y$, positions keyed by node.


## Examples

```
>>> G = nx.petersen_graph()
>>> pos = nx.nx_agraph.graphviz_layout(G)
>>> pos = nx.nx_agraph.graphviz_layout(G, prog='dot')
```


### 10.3 Graphviz with pydot

### 10.3.1 Pydot

Import and export NetworkX graphs in Graphviz dot format using pydot.
Either this module or nx_agraph can be used to interface with graphviz.
See also:
pydot https://github.com/erocarrera/pydot
Graphviz http://www.research.att.com/sw/tools/graphviz/
DOT

| from_pydot $(\mathrm{P})$ | Return a NetworkX graph from a Pydot graph. |
| :--- | :--- |
| to_pydot $(\mathrm{N}[$, strict $])$ | Return a pydot graph from a NetworkX graph N. |
| write_dot $(\mathrm{G}$, path $)$ | Write NetworkX graph G to Graphviz dot format on path. |
|  | Continued on next page |

Table 10.3 - continued from previous page

| read_dot(path) | Return a NetworkX MultiGraph or MultiDiGraph <br> from the dot file with the passed path. |
| :--- | :--- |
| graphviz_layout(G[, prog, root]) | Create node positions using Pydot and Graphviz. |
| pydot_layout(G[, prog, root]) | Create node positions using pydot and Graphviz. |

### 10.3.2 from_pydot

from_pydot ( $P$ )
Return a NetworkX graph from a Pydot graph.
Parameters P (Pydot graph) - A graph created with Pydot
Returns G-A MultiGraph or MultiDiGraph.
Return type NetworkX multigraph

## Examples

```
>>> K5 = nx.complete_graph(5)
>>> A = nx.nx_pydot.to_pydot (K5)
>>> G = nx.nx_pydot.from_pydot(A) # return MultiGraph
```

\# make a Graph instead of MultiGraph >>> G = nx.Graph(nx.nx_pydot.from_pydot(A))

### 10.3.3 to_pydot

to_pydot ( $N$, strict=True)
Return a pydot graph from a NetworkX graph N .
Parameters $\mathbf{N}$ (NetworkX graph) - A graph created with NetworkX

## Examples

>>> K5 = nx.complete_graph (5)
$\ggg$ P $=$ nx.nx_pydot.to_pydot (K5)

## Notes

### 10.3.4 write_dot

write_dot (G, path)
Write NetworkX graph G to Graphviz dot format on path.
Path can be a string or a file handle.

### 10.3.5 read_dot

read_dot (path)
Return a NetworkX MultiGraph or MultiDiGraph from the dot file with the passed path.
If this file contains multiple graphs, only the first such graph is returned. All graphs _except_ the first are silently ignored.

Parameters path (str or file) - Filename or file handle.
Returns G-A MultiGraph or MultiDiGraph.
Return type MultiGraph or MultiDiGraph

## Notes

Use $G=n x . G r a p h\left(r e a d \_d o t(p a t h)\right)$ to return a Graph instead of a MultiGraph.

### 10.3.6 graphviz_layout

graphviz_layout (G, prog='neato', root=None, **kwds)
Create node positions using Pydot and Graphviz.
Returns a dictionary of positions keyed by node.

## Examples

```
>>> G = nx.complete_graph(4)
>>> pos = nx.nx_pydot.graphviz_layout(G)
>>> pos = nx.nx_pydot.graphviz_layout(G, prog='dot')
```


## Notes

This is a wrapper for pydot_layout.

### 10.3.7 pydot_layout

pydot_layout ( $G$, prog='neato', root=None, **kwds)
Create node positions using pydot and Graphviz.

## Parameters

- G (Graph) - NetworkX graph to be laid out.
- prog (optional[str]) - Basename of the GraphViz command with which to layout this graph. Defaults to neato, the default GraphViz command for undirected graphs.
Returns Dictionary of positions keyed by node.
Return type dict


## Examples

```
>>> G = nx.complete_graph(4)
>>> pos = nx.nx_pydot.pydot_layout(G)
>>> pos = nx.nx_pydot.pydot_layout(G, prog='dot')
```


### 10.4 Graph Layout

### 10.4.1 Layout

Node positioning algorithms for graph drawing.
For random_layout () the possible resulting shape is a square of side [0, scale] (default: [0, 1]) Changing center shifts the layout by that amount.

For the other layout routines, the extent is [center - scale, center + scale] (default: $[-1,1]$ ).
Warning: Most layout routines have only been tested in 2-dimensions.

| circular_layout(G[, scale, center, dim]) | Position nodes on a circle. |
| :--- | :--- |
| random_layout $(\mathrm{G}[$, center, dim]) | Position nodes uniformly at random in the unit square. |
| rescale_layout(pos[, scale]) | Return scaled position array to (-scale, scale) in all axes. |
| shell_layout(G[, nlist, scale, center, dim]) | Position nodes in concentric circles. <br> spring_layout(G[, k, pos, fixed, ...]) <br> algorithm. |
| spectral_layout(G[, weight, scale, center, dim]) | Position nodes using the eigenvectors of the graph Lapla- <br> cian. |

### 10.4.2 circular_layout

circular_layout ( $G$, scale $=1$, center $=$ None, dim=2)
Position nodes on a circle.

## Parameters

- G (NetworkX graph or list of nodes)
- scale (float) - Scale factor for positions
- center (array-like or None) - Coordinate pair around which to center the layout.
- $\operatorname{dim}(i n t)$ - Dimension of layout, currently only dim=2 is supported

Returns pos - A dictionary of positions keyed by node
Return type dict

## Examples

```
>>> G = nx.path_graph(4)
>>> pos = nx.circular_layout(G)
```


## Notes

This algorithm currently only works in two dimensions and does not try to minimize edge crossings.

### 10.4.3 random_layout

random_layout ( $G$, center $=$ None, dim=2)
Position nodes uniformly at random in the unit square.
For every node, a position is generated by choosing each of dim coordinates uniformly at random on the interval [0.0, 1.0).

NumPy (http://scipy.org) is required for this function.

## Parameters

- G (NetworkX graph or list of nodes) - A position will be assigned to every node in G .
- center (array-like or None) - Coordinate pair around which to center the layout.
- $\operatorname{dim}(i n t)$ - Dimension of layout.

Returns pos - A dictionary of positions keyed by node
Return type dict

Examples
>>> G = nx.lollipop_graph(4, 3)
$\ggg$ pos $=n x . r a n d o m \_l a y o u t(G)$

### 10.4.4 rescale_layout

rescale_layout (pos, scale=1)
Return scaled position array to (-scale, scale) in all axes.
The function acts on NumPy arrays which hold position information. Each position is one row of the array. The dimension of the space equals the number of columns. Each coordinate in one column.

To rescale, the mean (center) is subtracted from each axis separately. Then all values are scaled so that the largest magnitude value from all axes equals scale (thus, the aspect ratio is preserved). The resulting NumPy Array is returned (order of rows unchanged).

## Parameters

- pos (numpy array) - positions to be scaled. Each row is a position.
- scale (number (default: 1)) - The size of the resulting extent in all directions.

Returns pos - scaled positions. Each row is a position.
Return type numpy array

### 10.4.5 shell_layout

shell_layout ( $G$, nlist $=$ None, , scale $=1$, center=None, dim=2)
Position nodes in concentric circles.

## Parameters

- G (NetworkX graph or list of nodes)
- nlist (list of lists) - List of node lists for each shell.
- scale (float) - Scale factor for positions
- center (array-like or None) - Coordinate pair around which to center the layout.
- $\operatorname{dim}(i n t)$ - Dimension of layout, currently only dim=2 is supported

Returns pos - A dictionary of positions keyed by node
Return type dict

## Examples

```
>>> G = nx.path_graph(4)
>>> shells = [[0], [1, 2, 3]]
>>> pos = nx.shell_layout(G, shells)
```


## Notes

This algorithm currently only works in two dimensions and does not try to minimize edge crossings.

### 10.4.6 spring_layout

spring_layout ( $G, k=$ None, pos=None, fixed=None, iterations $=50$, weight='weight', scale $=1.0$, center $=$ None, dim=2)
Position nodes using Fruchterman-Reingold force-directed algorithm.

## Parameters

- G (NetworkX graph or list of nodes)
- $\mathbf{k}($ float $($ default=None $)$ ) - Optimal distance between nodes. If None the distance is set to $1 /$ sqrt( $n$ ) where $n$ is the number of nodes. Increase this value to move nodes farther apart.
- pos (dict or None optional (default=None)) - Initial positions for nodes as a dictionary with node as keys and values as a coordinate list or tuple. If None, then use random initial positions.
- fixed (list or None optional (default=None)) - Nodes to keep fixed at initial position.
- iterations (int optional (default=50) - Number of iterations of spring-force relaxation
- weight (string or None optional (default='weight')) - The edge attribute that holds the numerical value used for the edge weight. If None, then all edge weights are 1.
- scale (float (default=1.0)) - Scale factor for positions. The nodes are positioned in a box of size [0, scale] x [0, scale].
- center (array-like or None) - Coordinate pair around which to center the layout.
- $\operatorname{dim}(i n t)$ - Dimension of layout

Returns pos - A dictionary of positions keyed by node
Return type dict

## Examples

```
>>> G = nx.path_graph(4)
>>> pos = nx.spring_layout(G)
```

\# The same using longer but equivalent function name >>> pos = nx.fruchterman_reingold_layout(G)

### 10.4.7 spectral_layout

spectral_layout ( $G$, weight='weight', scale $=1$, center=None, dim=2)
Position nodes using the eigenvectors of the graph Laplacian.

## Parameters

- G (NetworkX graph or list of nodes)
- weight (string or None optional (default='weight')) - The edge attribute that holds the numerical value used for the edge weight. If None, then all edge weights are 1.
- scale (float) - Scale factor for positions
- center (array-like or None) - Coordinate pair around which to center the layout.
- $\operatorname{dim}(i n t)$ - Dimension of layout

Returns pos - A dictionary of positions keyed by node
Return type dict

## Examples

$\ggg G=n x \cdot p a t h \_g r a p h(4)$
$\ggg$ pos $=$ nx.spectral_layout (G)

## Notes

Directed graphs will be considered as undirected graphs when positioning the nodes.
For larger graphs ( $>500$ nodes) this will use the SciPy sparse eigenvalue solver (ARPACK).

## Exceptions

### 11.1 Exceptions

Base exceptions and errors for NetworkX.

## class NetworkXException

Base class for exceptions in NetworkX.

## class NetworkXError

Exception for a serious error in NetworkX
class NetworkXPointlessConcept
Harary, F. and Read, R. "Is the Null Graph a Pointless Concept?" In Graphs and Combinatorics Conference, George Washington University. New York: Springer-Verlag, 1973.

## class NetworkXAlgorithmError

Exception for unexpected termination of algorithms.

## class NetworkXUnfeasible

Exception raised by algorithms trying to solve a problem instance that has no feasible solution.

## class NetworkXNoPath

Exception for algorithms that should return a path when running on graphs where such a path does not exist.

## class NodeNotFound

Exception raised if requested node is not present in the graph

## class NetworkXUnbounded

Exception raised by algorithms trying to solve a maximization or a minimization problem instance that is unbounded.

## Utilities

### 12.1 Helper Functions

Miscellaneous Helpers for NetworkX.
These are not imported into the base networkx namespace but can be accessed, for example, as

```
>>> import networkx
>>> networkx.utils.is_string_like('spam')
True
```

| is_string_like(obj) | Check if obj is string. |
| :--- | :--- |
| flatten(obj[, result]) | Return flattened version of (possibly nested) iterable ob- <br> ject. |
| iterable(obj) | Return True if obj is iterable with a well-defined len(). |
| is_list_of_ints(intlist) | Return True if list is a list of ints. |
| make_str(x) | Return the string representation of t. |
| generate_unique_node() | Generate a unique node label. |
| default_opener(filename) | Opens filename using system's default program. |
| pairwise(iterable[, cyclic]) | s -> (s0, s1), (s1, s2), (s2, s3), .. |
| groups(many_to_one) | Converts a many-to-one mapping into a one-to-many map- <br>  <br> ping. |

### 12.1.1 is_string_like

is_string_like (obj)
Check if obj is string.

### 12.1.2 flatten

## flatten (obj, result=None)

Return flattened version of (possibly nested) iterable object.

### 12.1.3 iterable

## iterable (obj)

Return True if obj is iterable with a well-defined len().

### 12.1.4 is_list_of_ints

is_list_of_ints (intlist)
Return True if list is a list of ints.

### 12.1.5 make_str

make_str ( $x$ )
Return the string representation of $t$.

### 12.1.6 generate_unique_node

generate_unique_node ()
Generate a unique node label.

### 12.1.7 default_opener

## default_opener (filename)

Opens filename using system's default program.
Parameters filename (str) - The path of the file to be opened.

### 12.1.8 pairwise

pairwise (iterable, cyclic=False)
$\mathrm{s}->(\mathrm{s} 0, \mathrm{~s} 1),(\mathrm{s} 1, \mathrm{~s} 2),(\mathrm{s} 2, \mathrm{~s} 3), \ldots$

### 12.1.9 groups

groups (many_to_one)
Converts a many-to-one mapping into a one-to-many mapping.
many_to_one must be a dictionary whose keys and values are all hashable.
The return value is a dictionary mapping values from many_to_one to sets of keys from many_to_one that have that value.

For example:

```
>>> from networkx.utils import groups
>>> many_to_one = {'a': 1, 'b': 1, 'c': 2, 'd': 3, 'e': 3}
>>> groups(many_to_one)
{1: {'a', 'b'}, 2: {'c'}, 3: {'d', 'e'}}
```


### 12.2 Data Structures and Algorithms

Union-find data structure.

### 12.2.1 union

UnionFind. union (*objects)
Find the sets containing the objects and merge them all.

### 12.3 Random Sequence Generators

Utilities for generating random numbers, random sequences, and random selections.

| create_degree_sequence(n[, <br> max_tries]) | sfunction, |
| :--- | :--- |
| pareto_sequence(n[, exponent]) | Return sample sequence of length n from a Pareto distribu- <br> tion. |
| powerlaw_sequence(n[, exponent]) | Return sample sequence of length n from a power law dis- <br> tribution. |
| uniform_sequence(n) | Return sample sequence of length n from a uniform distri- <br> bution. |
| cumulative_distribution(distribution) | Return normalized cumulative distribution from discrete <br> distribution. |
| discrete_sequence(n[, distribution, ...]) | Return sample sequence of length n from a given discrete <br> distribution or discrete cumulative distribution. |
| zipf_sequence(n[, alpha, xmin]) | Return a sample sequence of length n from a Zipf distri- <br> bution with exponent parameter alpha and minimum value <br> xmin. |
| zipf_rv(alpha[, xmin, seed]) | Return a random value chosen from the Zipf distribution. |
| random_weighted_sample(mapping, k$)$ | Return k items without replacement from a weighted sam- <br> ple. |
| weighted_choice(mapping) | Return a single element from a weighted sample. |

### 12.3.1 create_degree_sequence

create_degree_sequence ( $n$, sfunction=None, max_tries $=50, * * k w d s$ )

### 12.3.2 pareto_sequence

pareto_sequence ( $n$, exponent $=1.0$ )
Return sample sequence of length n from a Pareto distribution.

### 12.3.3 powerlaw_sequence

powerlaw_sequence ( $n$, exponent $=2.0$ )
Return sample sequence of length n from a power law distribution.

### 12.3.4 uniform_sequence

## uniform_sequence ( $n$ )

Return sample sequence of length n from a uniform distribution.

### 12.3.5 cumulative_distribution

## cumulative_distribution (distribution)

Return normalized cumulative distribution from discrete distribution.

### 12.3.6 discrete_sequence

discrete_sequence ( $n$, distribution=None, cdistribution=None)
Return sample sequence of length n from a given discrete distribution or discrete cumulative distribution.
One of the following must be specified.
distribution $=$ histogram of values, will be normalized
cdistribution $=$ normalized discrete cumulative distribution

### 12.3.7 zipf_sequence

zipf_sequence ( $n$, alpha $=2.0, x \min =1$ )
Return a sample sequence of length $n$ from a Zipf distribution with exponent parameter alpha and minimum value xmin.

## See also:

zipf_rv()

### 12.3.8 zipf_rv

zipf_rv (alpha, $x$ min $=1$, seed $=$ None $)$
Return a random value chosen from the Zipf distribution.
The return value is an integer drawn from the probability distribution ::math:

$$
p(x)=\backslash \operatorname{frac}\left\{x^{\wedge}\{-\backslash a l p h a\}\right\}\left\{\text { zeta }\left(\backslash a l p h a, x_{-}\{\min \}\right)\right\},
$$

where zeta (alpha, $\left.x_{-}\{\min \}\right)$ is the Hurwitz zeta function.

## Parameters

- alpha (float) - Exponent value of the distribution
- xmin (int) - Minimum value
- seed (int) - Seed value for random number generator

Returns $\mathbf{x}$ - Random value from Zipf distribution
Return type int
Raises ValueError: - If xmin $<1$ or If alpha $<=1$

## Notes

The rejection algorithm generates random values for a the power-law distribution in uniformly bounded expected time dependent on parameters. See [1] for details on its operation.

## Examples

```
>>> nx.zipf_rv(alpha=2, xmin=3, seed=42)
```


## References

..[1] Luc Devroye, Non-Uniform Random Variate Generation, Springer-Verlag, New York, 1986.

### 12.3.9 random_weighted_sample

random_weighted_sample (mapping, $k$ )
Return k items without replacement from a weighted sample.
The input is a dictionary of items with weights as values.

### 12.3.10 weighted_choice

weighted_choice (mapping)
Return a single element from a weighted sample.
The input is a dictionary of items with weights as values.

### 12.4 Decorators

open_file(path_arg[, mode]) Decorator to ensure clean opening and closing of files.

### 12.4.1 open_file

open_file (path_arg, mode='r')
Decorator to ensure clean opening and closing of files.

## Parameters

- path_arg (int) - Location of the path argument in args. Even if the argument is a named positional argument (with a default value), you must specify its index as a positional argument.
- mode (str) - String for opening mode.

Returns _open_file - Function which cleanly executes the io.
Return type function

## Examples

Decorate functions like this:

```
@open_file(0,'r')
def read_function(pathname):
    pass
```

```
@open_file(1,'w')
def write_function(G,pathname):
    pass
@open_file(1,'w')
def write_function(G, pathname='graph.dot')
    pass
@open_file('path', 'w+')
def another_function(arg, **kwargs):
    path = kwargs['path']
    pass
```


### 12.5 Cuthill-Mckee Ordering

Cuthill-McKee ordering of graph nodes to produce sparse matrices

| cuthill_mckee_ordering(G[, heuristic $])$ | Generate an ordering (permutation) of the graph nodes to <br> make a sparse matrix. |
| :--- | :--- |
| reverse_cuthill_mckee_ordering(G[, heuris- Generate an ordering (permutation) of the graph nodes to <br> tic])  |  |

### 12.5.1 cuthill_mckee_ordering

```
cuthill_mckee_ordering(G,heuristic=None)
```

Generate an ordering (permutation) of the graph nodes to make a sparse matrix.
Uses the Cuthill-McKee heuristic (based on breadth-first search) ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- heuristic (function, optional) - Function to choose starting node for RCM algorithm. If None a node from a pseudo-peripheral pair is used. A user-defined function can be supplied that takes a graph object and returns a single node.

Returns nodes - Generator of nodes in Cuthill-McKee ordering.
Return type generator

## Examples

```
>>> from networkx.utils import cuthill_mckee_ordering
>>> G = nx.path_graph(4)
>>> rcm = list(cuthill_mckee_ordering(G))
>>> A = nx.adjacency_matrix(G, nodelist=rcm)
```

Smallest degree node as heuristic function:

[^113]```
>>> def smallest_degree(G):
... return min(G, key=G.degree)
>>> rcm = list(cuthill_mckee_ordering(G, heuristic=smallest_degree))
```


## See also:

```
reverse_cuthill_mckee_ordering()
```


## Notes

The optimal solution the the bandwidth reduction is NP-complete ${ }^{2}$.

## References

### 12.5.2 reverse_cuthill_mckee_ordering

reverse_cuthill_mckee_ordering ( $G$, heuristic=None)
Generate an ordering (permutation) of the graph nodes to make a sparse matrix.
Uses the reverse Cuthill-McKee heuristic (based on breadth-first search) ${ }^{1}$.

## Parameters

- G (graph) - A NetworkX graph
- heuristic (function, optional) - Function to choose starting node for RCM algorithm. If None a node from a pseudo-peripheral pair is used. A user-defined function can be supplied that takes a graph object and returns a single node.
Returns nodes - Generator of nodes in reverse Cuthill-McKee ordering.
Return type generator

Examples

```
>>> from networkx.utils import reverse_cuthill_mckee_ordering
>>> G = nx.path_graph(4)
>>> rcm = list(reverse_cuthill_mckee_ordering(G))
>>> A = nx.adjacency_matrix(G, nodelist=rcm)
```

Smallest degree node as heuristic function:

```
>>> def smallest_degree(G):
... return min(G, key=G.degree)
>>> rcm = list(reverse_cuthill_mckee_ordering(G, heuristic=smallest_degree))
```


## See also:

```
cuthill_mckee_ordering()
```

[^114]
## Notes

The optimal solution the the bandwidth reduction is NP-complete ${ }^{2}$.

References

### 12.6 Context Managers

```
reversed(\*args, \*\*kwds)
```

A context manager for temporarily reversing a directed graph in place.

### 12.6.1 reversed

reversed (*args, **kwds)
A context manager for temporarily reversing a directed graph in place.
This is a no-op for undirected graphs.
Parameters G (graph) - A NetworkX graph.

[^115]
## License

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```
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## CHAPTER 14

## Citing

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### 15.1 Contributions

This section aims to provide a list of people and projects that have contributed to networkx. It is intended to be an inclusive list, and anyone who has contributed and wishes to make that contribution known is welcome to add an entry into this file. Generally, no name should be added to this list without the approval of the person associated with that name.

Creating a comprehensive list of contributors can be difficult, and the list within this file is almost certainly incomplete. Contributors include testers, bug reporters, contributors who wish to remain anonymous, funding sources, academic advisors, end users, and even build/integration systems (such as TravisCI, coveralls, and readthedocs).

Do you want to make your contribution known? If you have commit access, edit this file and add your name. If you do not have commit access, feel free to open an issue, submit a pull request, or get in contact with one of the official team members.

A supplementary (but still incomplete) list of contributors is given by the list of names that have commits in networkx's git repository. This can be obtained via:

```
git log --raw | grep "^Author: " | sort | uniq
```

A historical, partial listing of contributors and their contributions to some of the earlier versions of NetworkX can be found here.

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Dan Schult
Pieter Swart

### 15.1.2 Contributors

Optionally, add your desired name and include a few relevant links. The order is partially historical, and now, mostly arbitrary.

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## Glossary

dictionary A Python dictionary maps keys to values. Also known as "hashes", or "associative arrays". See http: //docs.python.org/tutorial/datastructures.html\#dictionaries
ebunch An iteratable container of edge tuples like a list, iterator, or file.
edge Edges are either two-tuples of nodes ( $u, v$ ) or three tuples of nodes with an edge attribute dictionary ( $u, v, d i c t$ ).
edge attribute Edges can have arbitrary Python objects assigned as attributes by using keyword/value pairs when adding an edge assigning to the G.edge $[\mathrm{u}][\mathrm{v}]$ attribute dictionary for the specified edge $\mathrm{u}-\mathrm{v}$.
hashable An object is hashable if it has a hash value which never changes during its lifetime (it needs a __hash__() method), and can be compared to other objects (it needs an __eq_() or __cmp_() method). Hashable objects which compare equal must have the same hash value.

Hashability makes an object usable as a dictionary key and a set member, because these data structures use the hash value internally.
All of Python's immutable built-in objects are hashable, while no mutable containers (such as lists or dictionaries) are. Objects which are instances of user-defined classes are hashable by default; they all compare unequal, and their hash value is their id().
Definition from http://docs.python.org/glossary.html
nbunch An nbunch is any iterable container of nodes that is not itself a node in the graph. It can be an iterable or an iterator, e.g. a list, set, graph, file, etc..
node A node can be any hashable Python object except None.
node attribute Nodes can have arbitrary Python objects assigned as attributes by using keyword/value pairs when adding a node or assigning to the G.node[ n$]$ attribute dictionary for the specified node n .
[atlas] Ronald C. Read and Robin J. Wilson, An Atlas of Graphs. Oxford University Press, 1998.
[atlas] Ronald C. Read and Robin J. Wilson, An Atlas of Graphs. Oxford University Press, 1998.

## a

networkx．algorithms．bipartite．spectral， 151
networkx．algorithms．approximation， 119
networkx．algorithms．approximation．clique，networkx．algorithms．boundary， 165 123
networkx．algorithms．centrality， 166

124 networkx．algorithms．chordal， 192

119 networkx．algorithms．cluster， 199
networkx．algorithms．approximation．dominađetyofkx，algorithms．coloring， 203 124 networkx．algorithms．communicability＿alg，
networkx．algorithms．approximation．independent＿set， 126 networkx．algorithms．community， 208
networkx．algorithms．approximation．kcompo月êter，rx．algorithms．community．asyn＿lpa， 122210
networkx．algorithms．approximation．matchi月ध̧，workx．algorithms．community．centrality， 126 212
networkx．algorithms．approximation．ramsey，networkx．algorithms．community．kclique， 127

## 209

networkx．algorithms．approximation．vertex＿甜民ま，k．algorithms．community．kernighan＿lin， 127

208
networkx．algorithms．assortativity， 128
networkx．algorithms．bipartite， 137
networkx．algorithms．bipartite．basic， 138 networkx．algorithms．components， 214
networkx．algorithms．bipartite．centrality，networkx．algorithms．connectivity， 230

157
networkx．algorithms．bipartite．cluster， 152
networkx．algorithms．bipartite．covering， 164
networkx．algorithms．bipartite．generators， 160
networkx．algorithms．bipartite．matching， 142
networkx．algorithms．bipartite．matrix， 144
networkx．algorithms．bipartite．projection， 145
networkx．algorithms．bipartite．redundancy，networkx．algorithms．covering， 252 156
networkx．algorithms．connectivity．connectivity， 233
networkx．algorithms．connectivity．cuts， 241
networkx．algorithms．connectivity．kcomponents， 230
networkx．algorithms．connectivity．kcutsets， 232
networkx．algorithms．connectivity．stoerwagner，
networkx．algorithms．connectivity．utils，
248
networkx．algorithms．core， 249
networkx．algorithms．cuts， 257
networkx．algorithms．cycles， 254
networkx．algorithms．dag， 261
networkx．algorithms．distance＿measures， 266
networkx．algorithms．distance＿regular， 268
networkx．algorithms．dominance， 270
networkx．algorithms．dominating， 272
networkx．algorithms．efficiency， 273
networkx．algorithms．euler， 275
networkx．algorithms．flow， 276
networkx．algorithms．graphical， 301
networkx．algorithms．hierarchy， 305
networkx．algorithms．hybrid， 305
networkx．algorithms．isolate， 307
networkx．algorithms．isomorphism， 308
networkx．algorithms．isomorphism．isomorph伊民政orkx．drawing．layout， 556 311
networkx．algorithms．link＿analysis．hits＿an＠tworkx．drawing．nx＿pydot， 553 326
networkx．algorithms．link＿analysis．pagerank＿alg， 322
networkx．algorithms．link＿prediction， 328
networkx．algorithms．matching， 334
networkx．algorithms．minors， 336
networkx．algorithms．mis， 342
networkx．algorithms．operators．all， 346
networkx．algorithms．operators．binary， 343
networkx．algorithms．operators．product， 348
networkx．algorithms．operators．unary， 342
networkx．algorithms．reciprocity， 352
networkx．algorithms．richclub， 353
networkx．algorithms．shortest＿paths．astar 380
networkx．algorithms．shortest＿paths．dense， 378
networkx．algorithms．shortest＿paths．generígtworkx．generators．nonisomorphic＿trees， 354
networkx．algorithms．shortest＿paths．unweiqhetarkx．generators．random＿clustered， 358
networkx．algorithms．shortest＿paths．weighfêtworkx．generators．random＿graphs， 436 360
networkx．algorithms．simple＿paths， 381
networkx．algorithms．swap， 384
networkx．algorithms．tournament， 386
networkx．algorithms．traversal．beamsearch， 396
networkx．algorithms．traversal．breadth＿firstwserkchinalg．algebraicconnectivity， 393
networkx．algorithms．traversal．depth＿firstesearch，linalg．attrmatrix， 483 389
networkx．algorithms．traversal．edgedfs， 397
networkx．algorithms．tree．branchings， 401
networkx．drawing．nx＿agraph， 551
networkx．drawing．nx＿pylab， 543
e
networkx．exception， 561

## g

networkx．generators．atlas， 421
networkx．generators．classic， 422
networkx．generators．community， 466
networkx．generators．community， 466
networkx．generators．degree＿seq， 446
networkx．generators．directed， 453
networkx．generators．duplication， 445
networkx．generators．ego， 462
networkx．generators．expanders， 430
networkx．generators．geometric， 456
networkx．generators．intersection， 463
networkx．generators．joint＿degree＿seq， 472
networkx．generators．line， 461 471 452
networkx．generators．small， 432
networkx．generators．social， 465
networkx．generators．stochastic， 463
networkx．generators．triads， 471
networkx．algorithms．tree．mst， 403
networkx．algorithms．tree．recognition， 398
networkx．algorithms．triads， 407
networkx．algorithms．vitality， 408
networkx．algorithms．voronoi， 408
networkx．algorithms．wiener， 409

## C

networkx．classes．function， 411
networkx．convert， 487
networkx．convert＿matrix， 490

## d

etworkx．generators．nonisomorphic＿trees，
i 480
networkx．linalg．graphmatrix， 475
networkx．linalg．laplacianmatrix， 477
networkx．linalg．spectrum， 479

```
r
networkx.readwrite.adjlist,501
networkx.readwrite.edgelist,508
networkx.readwrite.gexf,514
networkx.readwrite.gml,516
networkx.readwrite.gpickle,520
networkx.readwrite.graph6,532
networkx.readwrite.graphml,521
networkx.readwrite.json_graph,523
networkx.readwrite.leda,528
networkx.readwrite.multiline_adjlist,
        504
networkx.readwrite.nx_shp,538
networkx.readwrite.nx_yaml,529
networkx.readwrite.pajek, 537
networkx.readwrite.sparse6,534
U
networkx.utils,563
networkx.utils.contextmanagers, 570
networkx.utils.decorators,567
networkx.utils.misc,563
networkx.utils.random_sequence, 565
networkx.utils.rcm,568
networkx.utils.union_find,564
```


## Index

## Symbols

__contains_ () (DiGraph method), 52
__contains__() (Graph method), 26
__contains__() (MultiDiGraph method), 108
__contains__() (MultiGraph method), 80
__getitem__() (DiGraph method), 49
__getitem__() (Graph method), 24
__getitem__() (MultiDiGraph method), 105
__getitem__() (MultiGraph method), 77
__init__() (DiGraph method), 38
__init__() (DiGraphMatcher method), 315
__init__() (Edmonds method), 403
__init__() (Graph method), 13
__init__() (GraphMatcher method), 313
__init__() (MultiDiGraph method), 93
__init__() (MultiGraph method), 66
__iter__() (DiGraph method), 46
__iter__() (Graph method), 21
__iter__() (MultiDiGraph method), 101
__iter__() (MultiGraph method), 75
__len__() (DiGraph method), 53
__len__() (Graph method), 27
__len_() (MultiDiGraph method), 109
__len_() (MultiGraph method), 81

## A

adamic_adar_index() (in module net-
workx.algorithms.link_prediction), 330
add_cycle() (in module networkx.classes.function), 413
add_edge() (DiGraph method), 41
add_edge() (Graph method), 16
add_edge() (MultiDiGraph method), 95
add_edge() (MultiGraph method), 69
add_edges_from() (DiGraph method), 42
add_edges_from() (Graph method), 17
add_edges_from() (MultiDiGraph method), 96
add_edges_from() (MultiGraph method), 70
add_node() (DiGraph method), 39
add_node() (Graph method), 14
add_node() (MultiDiGraph method), 93
add_node() (MultiGraph method), 66
add_nodes_from() (DiGraph method), 40
add_nodes_from() (Graph method), 15
add_nodes_from() (MultiDiGraph method), 94
add_nodes_from() (MultiGraph method), 67
add_path() (in module networkx.classes.function), 413
add_star() (in module networkx.classes.function), 413
add_weighted_edges_from() (DiGraph method), 43
add_weighted_edges_from() (Graph method), 18
add_weighted_edges_from() (MultiDiGraph method), 97
add_weighted_edges_from() (MultiGraph method), 71
adjacency() (DiGraph method), 50
adjacency() (Graph method), 24
adjacency() (MultiDiGraph method), 106
adjacency() (MultiGraph method), 78
adjacency_data() (in module networkx.readwrite.json_graph), 525
adjacency_graph() (in module networkx.readwrite.json_graph), 526
adjacency_matrix() (in module networkx.linalg.graphmatrix), 475
adjacency_spectrum() (in module networkx.linalg.spectrum), 479
algebraic_connectivity() (in module networkx.linalg.algebraicconnectivity), 480
all_neighbors() (in module networkx.classes.function), 414
all_node_cuts() (in module networkx.algorithms.connectivity.kcutsets), 232
all_pairs_bellman_ford_path() (in module networkx.algorithms.shortest_paths.weighted), 374
all_pairs_bellman_ford_path_length() (in module networkx.algorithms.shortest_paths.weighted), 374
all_pairs_dijkstra_path() (in module networkx.algorithms.shortest_paths.weighted), 368
all_pairs_dijkstra_path_length() (in module networkx.algorithms.shortest_paths.weighted),

369
all_pairs_node_connectivity() (in module networkx.algorithms.approximation.connectivity), 119
all_pairs_node_connectivity() (in module networkx.algorithms.connectivity.connectivity), 234
all_pairs_shortest_path() (in module networkx.algorithms.shortest_paths.unweighted), 359
all_pairs_shortest_path_length() (in module networkx.algorithms.shortest_paths.unweighted), 359
all_shortest_paths() (in module networkx.algorithms.shortest_paths.generic), 355
all_simple_paths() (in module networkx.algorithms.simple_paths), 381
alternating_havel_hakimi_graph() (in module networkx.algorithms.bipartite.generators), 162
ancestors() (in module networkx.algorithms.dag), 261
antichains() (in module networkx.algorithms.dag), 265
approximate_current_flow_betweenness_centrality() (in module networkx.algorithms.centrality), 181
articulation_points() (in module networkx.algorithms.components), 229
astar_path() (in module networkx.algorithms.shortest_paths.astar), 380
astar_path_length() (in module networkx.algorithms.shortest_paths.astar), 381
asyn_lpa_communities() (in module networkx.algorithms.community.asyn_lpa), 210
attr_matrix() (in module networkx.linalg.attrmatrix), 483
attr_sparse_matrix() (in module networkx.linalg.attrmatrix), 484
attracting_component_subgraphs() (in module networkx.algorithms.components), 224
attracting_components() (in module workx.algorithms.components), 224
attribute_assortativity_coefficient() (in module net workx.algorithms.assortativity), 129
attribute_mixing_dict() (in module networkx.algorithms.assortativity), 137
attribute_mixing_matrix() (in module workx.algorithms.assortativity), 135
authority_matrix() (in module networkx.algorithms.link_analysis.hits_alg), 328
average_clustering() (in module networkx.algorithms.approximation.clustering_coeffibiddjacency_matrix() (in module net124 (in workx.algorithms.bipartite.matrix), 144
average_clustering() (in module net- biconnected_component_edges() (in module networkx.algorithms.bipartite.cluster), 153
average_clustering() (in module networkx.algorithms.cluster), 201
average_degree_connectivity() (in module networkx.algorithms.assortativity), 133
average_neighbor_degree() (in module networkx.algorithms.assortativity), 132
average_node_connectivity() (in module networkx.algorithms.connectivity.connectivity), 233
average_shortest_path_length() (in module networkx.algorithms.shortest_paths.generic), 357
B
balanced_tree() (in module networkx.generators.classic), 423
barabasi_albert_graph() (in module networkx.generators.random_graphs), 441
barbell_graph() (in module networkx.generators.classic), 423
bellman_ford_path() (in module networkx.algorithms.shortest_paths.weighted), 371
bellman_ford_path_length() (in module networkx.algorithms.shortest_paths.weighted), 372
bellman_ford_predecessor_and_distance() (in module networkx.algorithms.shortest_paths.weighted), 376
betweenness_centrality() (in module networkx.algorithms.bipartite.centrality), 159
betweenness_centrality () (in module networkx.algorithms.centrality), 175
betweenness_centrality_subset() (in module networkx.algorithms.centrality), 177
bfs_beam_edges() (in module networkx.algorithms.traversal.beamsearch), 396
bfs_edges() (in module networkx.algorithms.traversal.breadth_first_search), 394
bfs_predecessors() (in module networkx.algorithms.traversal.breadth_first_search), 395
net- bfs_successors() (in module networkx.algorithms.traversal.breadth_first_search), 395
bfs_tree() (in module networkx.algorithms.traversal.breadth_first_search), 394 workx.algorithms.components), 227
biconnected_component_subgraphs() (in module networkx.algorithms.components), 228
biconnected_components() (in module workx.algorithms.components), 226
bidirectional_dijkstra() (in module networkx.algorithms.shortest_paths.weighted), 370
binomial_graph() (in module networkx.generators.random_graphs), 439
blockmodel() (in module networkx.algorithms.minors), 341
boundary_expansion() (in module networkx.algorithms.cuts), 257
boykov_kolmogorov() (in module workx.algorithms.flow), 290
branching_weight() (in module workx.algorithms.tree.branchings), 401
build_auxiliary_edge_connectivity() (in module workx.algorithms.connectivity.utils), 248
build_auxiliary_node_connectivity() (in module workx.algorithms.connectivity.utils), 249
build_residual_network() (in module networkx.algorithms.flow), 292
bull_graph() (in module networkx.generators.small), 433

## C

candidate_pairs_iter() (DiGraphMatcher method), 316
candidate_pairs_iter() (GraphMatcher method), 314
capacity_scaling() (in module networkx.algorithms.flow), 299
cartesian_product() (in module networkx.algorithms.operators.product), 348
categorical_edge_match() (in module networkx.algorithms.isomorphism), 317
categorical_multiedge_match() (in module networkx.algorithms.isomorphism), 318
categorical_node_match() (in module workx.algorithms.isomorphism), 317
caveman_graph() (in module workx.generators.community), 466
center() (in module workx.algorithms.distance_measures), 266
chain_decomposition() (in module networkx.algorithms.chains), 191
chordal_cycle_graph() (in module networkx.generators.expanders), 431
chordal_graph_cliques() (in module workx.algorithms.chordal), 193
chordal_graph_treewidth() (in module workx.algorithms.chordal), 193
chvatal_graph() (in module networkx.generators.small), 433
circular_ladder_graph() (in module networkx.generators.classic), 425
circular_layout() (in module networkx.drawing.layout), 556
clear() (DiGraph method), 44
clear() (Graph method), 19
clear() (MultiDiGraph method), 100
clear() (MultiGraph method), 73
clique_removal() (in module networkx.algorithms.approximation.clique), 124
cliques_containing_node() (in module networkx.algorithms.clique), 198
closeness_centrality() (in module networkx.algorithms.bipartite.centrality), 157
net- closeness_centrality() (in module networkx.algorithms.centrality), 173
closeness_vitality() (in module networkx.algorithms.vitality), 408
clustering() (in module networkx.algorithms.bipartite.cluster), 152
clustering() (in module networkx.algorithms.cluster), 200
cn_soundarajan_hopcroft() (in module networkx.algorithms.link_prediction), 331
collaboration_weighted_projected_graph() (in module networkx.algorithms.bipartite.projection), 147
color() (in module networkx.algorithms.bipartite.basic), 140
common_neighbors() (in module networkx.classes.function), 415
communicability() (in module networkx.algorithms.communicability_alg), 207
communicability_betweenness_centrality() (in module networkx.algorithms.centrality), 184
communicability_exp() (in module networkx.algorithms.communicability_alg), 208
net- complement() (in module networkx.algorithms.operators.unary), 343
net- complete_bipartite_graph() (in module networkx.algorithms.bipartite.generators), 160
complete_graph() (in module networkx.generators.classic), 424
complete_multipartite_graph() (in module networkx.generators.classic), 424
compose() (in module networkx.algorithms.operators.binary), 344
net- compose_all() (in module networkx.algorithms.operators.all), 346
condensation() (in module networkx.algorithms.components), 220
conductance() (in module networkx.algorithms.cuts), 257
configuration_model() (in module networkx.algorithms.bipartite.generators), 161
configuration_model() (in module workx.generators.degree_seq), 446
connected_caveman_graph() (in module workx.generators.community), 467
connected_component_subgraphs() (in module workx.algorithms.components), 215
connected_components() (in module workx.algorithms.components), 215
connected_double_edge_swap() (in module workx.algorithms.swap), 385
connected_watts_strogatz_graph() (in module workx.generators.random_graphs), 440
contracted_edge() (in module workx.algorithms.minors), 336
contracted_nodes() (in module workx.algorithms.minors), 337
copy() (DiGraph method), 59
copy() (Graph method), 31
copy() (MultiDiGraph method), 115
copy() (MultiGraph method), 85
core_number() (in module networkx.algorithms.core), degree_mixing_matrix() (in module net249
cost_of_flow() (in module networkx.algorithms.flow) 297
could_be_isomorphic() (in module workx.algorithms.isomorphism), 310
coverage() (in module
workx.algorithms.community.quality), 211
create_degree_sequence() (in module workx.utils.random_sequence), 565
create_empty_copy() (in module workx.classes.function), 412
cubical_graph() (in module networkx.generators.small), 433
cumulative_distribution() (in module networkx.utils.random_sequence), 566
current_flow_betweenness_centrality() (in module networkx.algorithms.centrality), 179
current_flow_betweenness_centrality_subset() (in module networkx.algorithms.centrality), 182
current_flow_closeness_centrality() (in module networkx.algorithms.centrality), 174
cut_size() (in module networkx.algorithms.cuts), 258
cuthill_mckee_ordering() (in module networkx.utils.rem), 568
cycle_basis() (in module networkx.algorithms.cycles), 254
cycle_graph() (in module networkx.generators.classic), 425

## D

dag_longest_path() (in module networkx.algorithms.dag), 265
net- dag_longest_path_length() (in module networkx.algorithms.dag), 266
net- davis_southern_women_graph() (in module networkx.generators.social), 465
net- default_opener() (in module networkx.utils.misc), 564
degree() (DiGraph method), 54
net- degree() (Graph method), 27
degree() (in module networkx.classes.function), 411
net- degree() (MultiDiGraph method), 110
degree() (MultiGraph method), 81
net- degree_assortativity_coefficient() (in module networkx.algorithms.assortativity), 129
net- degree_centrality() (in module networkx.algorithms.bipartite.centrality), 158
net- degree_centrality() (in module networkx.algorithms.centrality), 167
degree_histogram() (in module networkx.classes.function), 411
degree_mixing_dict() (in module networkx.algorithms.assortativity), 136
ixing_matrix() (in module networkx.algorithms.assortativity), 136
degree_pearson_correlation_coefficient() (in module networkx.algorithms.assortativity), 131
net- degree_sequence_tree() (in module networkx.generators.degree_seq), 451
net- degrees() (in module networkx.algorithms.bipartite.basic), 141
dense_gnm_random_graph() (in module networkx.generators.random_graphs), 437
net- density() (in module networkx.algorithms.bipartite.basic), 141
density() (in module networkx.classes.function), 412
desargues_graph() (in module networkx.generators.small), 433
descendants() (in module networkx.algorithms.dag), 261 dfs_edges() (in module networkx.algorithms.traversal.depth_first_search), 389
dfs_labeled_edges() (in module networkx.algorithms.traversal.depth_first_search), 392
dfs_postorder_nodes() (in module networkx.algorithms.traversal.depth_first_search), 392
dfs_predecessors() (in module networkx.algorithms.traversal.depth_first_search), 390
dfs_preorder_nodes() (in module networkx.algorithms.traversal.depth_first_search), 391
dfs_successors() (in module networkx.algorithms.traversal.depth_first_search), 391
dfs_tree() (in module networkx.algorithms.traversal.depth_first_search), 390
diameter() (in module networkx.algorithms.distance_measures), 267
diamond_graph() (in module networkx.generators.small), 434
dictionary, 579
difference() (in module
workx.algorithms.operators.binary), 345
DiGraph() (in module networkx), 34
dijkstra_path() (in module networkx.algorithms.shortest_paths.weighted), 362
dijkstra_path_length() (in module networkx.algorithms.shortest_paths.weighted), 363
dijkstra_predecessor_and_distance() (in module networkx.algorithms.shortest_paths.weighted), 361
dinitz() (in module networkx.algorithms.flow), 288
directed_configuration_model() (in module workx.generators.degree_seq), 447
directed_havel_hakimi_graph() (in module networkx.generators.degree_seq), 450
directed_laplacian_matrix() (in module workx.linalg.laplacianmatrix), 478
discrete_sequence() (in module workx.utils.random_sequence), 566
disjoint_union() (in module workx.algorithms.operators.binary), 345
disjoint_union_all() (in module networkx.algorithms.operators.all), 347
dodecahedral_graph() (in module networkx.generators.small), 434
dominance_frontiers() (in module networkx.algorithms.dominance), 271
dominating_set() (in module workx.algorithms.dominating), 272
dorogovtsev_goltsev_mendes_graph() (in module networkx.generators.classic), 426
double_edge_swap() (in module networkx.algorithms.swap), 385
draw() (in module networkx.drawing.nx_pylab), 543
draw_circular() (in module networkx.drawing.nx_pylab), 549
draw_networkx() (in module networkx.drawing.nx_pylab), 544
draw_networkx_edge_labels() (in module workx.drawing.nx_pylab), 549
draw_networkx_edges() (in module workx.drawing.nx_pylab), 547
draw_networkx_labels() (in module networkx.drawing.nx_pylab), 548
net-
draw_networkx_nodes() (in module networkx.drawing.nx_pylab), 546
draw_random() (in module networkx.drawing.nx_pylab), 550
draw_shell() (in module networkx.drawing.nx_pylab), 550
draw_spectral() (in module networkx.drawing.nx_pylab), 550
draw_spring() (in module networkx.drawing.nx_pylab), 550
duplication_divergence_graph() (in module networkx.generators.duplication), 445
$E$
ebunch, 579
eccentricity() (in module networkx.algorithms.distance_measures), 267
edge, 579
edge attribute, 579
edge_betweenness_centrality() (in module networkx.algorithms.centrality), 176
edge_betweenness_centrality_subset() (in module networkx.algorithms.centrality), 178
edge_boundary() (in module networkx.algorithms.boundary), 165
edge_connectivity() (in module networkx.algorithms.connectivity.connectivity), 234
edge_current_flow_betweenness_centrality() (in module networkx.algorithms.centrality), 180
edge_current_flow_betweenness_centrality_subset() (in module networkx.algorithms.centrality), 183
edge_dfs() (in module networkx.algorithms.traversal.edgedfs), 397
edge_expansion() (in module networkx.algorithms.cuts), 259
edge_load_centrality() (in module networkx.algorithms.centrality), 185
edge_subgraph() (DiGraph method), 61
edge_subgraph() (Graph method), 34
edge_subgraph () (MultiDiGraph method), 117
edge_subgraph() (MultiGraph method), 88
edges() (DiGraph method), 46
edges() (Graph method), 21
edges() (in module networkx.classes.function), 415
edges() (MultiDiGraph method), 102
edges() (MultiGraph method), 75
Edmonds (class in networkx.algorithms.tree.branchings), 403
edmonds_karp() (in module networkx.algorithms.flow), 283
efficiency() (in module networkx.algorithms.efficiency), 273
ego_graph() (in module networkx.generators.ego), 462
eigenvector_centrality() (in module workx.algorithms.centrality), 168
eigenvector_centrality_numpy() (in module networkx.algorithms.centrality), 169
empty_graph() (in module networkx.generators.classic), 426
enumerate_all_cliques() (in module networkx.algorithms.clique), 195
eppstein_matching() (in module networkx.algorithms.bipartite.matching), 142
erdos_renyi_graph() (in module networkx.generators.random_graphs), 438
estrada_index() (in module networkx.algorithms.centrality), 188
eulerian_circuit() (in module networkx.algorithms.euler), 275
expected_degree_graph() (in module networkx.generators.degree_seq), 449

## F

fast_could_be_isomorphic() (in module networkx.algorithms.isomorphism), 310
fast_gnp_random_graph() (in module workx.generators.random_graphs), 436
faster_could_be_isomorphic() (in module workx.algorithms.isomorphism), 310
fiedler_vector() (in module workx.linalg.algebraicconnectivity), 481
find_cliques() (in module networkx.algorithms.clique), 195
find_cycle() (in module networkx.algorithms.cycles), 256
find_induced_nodes() (in module networkx.algorithms.chordal), 194
flatten() (in module networkx.utils.misc), 563
florentine_families_graph() (in module workx.generators.social), 466
flow_hierarchy() (in module workx.algorithms.hierarchy), 305
floyd_warshall() (in module workx.algorithms.shortest_paths.dense), 379
floyd_warshall_numpy() (in module networkx.algorithms.shortest_paths.dense), 380
floyd_warshall_predecessor_and_distance() (in module networkx.algorithms.shortest_paths.dense), 379
freeze() (in module networkx.classes.function), 419
from_agraph() (in module networkx.drawing.nx_agraph), 551
from_biadjacency_matrix() (in module net- girvan_newman() (in module networkx.algorithms.bipartite.matrix), 145
from_dict_of_dicts() (in module networkx.convert), 488
from_dict_of_lists() (in module networkx.convert), 489
from_edgelist() (in module networkx.convert), 490
from_numpy_matrix() (in module networkx.convert_matrix), 493
from_pandas_dataframe() (in module networkx.convert_matrix), 498
from_pydot() (in module networkx.drawing.nx_pydot), 554
from_scipy_sparse_matrix() (in module networkx.convert_matrix), 495
frucht_graph() (in module networkx.generators.small), 434
$G$
gaussian_random_partition_graph() (in module networkx.generators.community), 469
general_random_intersection_graph() (in module networkx.generators.intersection), 464
generalized_degree() (in module networkx.algorithms.cluster), 202
generate_adjlist() (in module networkx.readwrite.adjlist), 504
generate_edgelist() (in module networkx.readwrite.edgelist), 512
generate_gml() (in module networkx.readwrite.gml), 519 net- generate_graph6() (in module networkx.readwrite.graph6), 533
net- generate_multiline_adjlist() (in module networkx.readwrite.multiline_adjlist), 507
generate_sparse6() (in module networkx.readwrite.sparse6), 536
generate_unique_node() (in module networkx.utils.misc), 564
generic_edge_match() (in module networkx.algorithms.isomorphism), 320
net- generic_multiedge_match() (in module networkx.algorithms.isomorphism), 321
net- generic_node_match() (in module networkx.algorithms.isomorphism), 320
net- generic_weighted_projected_graph() (in module networkx.algorithms.bipartite.projection), 150
geographical_threshold_graph() (in module networkx.generators.geometric), 458
get_edge_attributes() (in module networkx.classes.function), 418
get_edge_data() (DiGraph method), 49
get_edge_data() (Graph method), 22
get_edge_data() (MultiDiGraph method), 104
get_edge_data() (MultiGraph method), 76
get_node_attributes() (in module networkx.classes.function), 417
 212
global_efficiency() (in module workx.algorithms.efficiency), 274
global_parameters() (in module workx.algorithms.distance_regular), 270
global_reaching_centrality() (in module networkx.algorithms.centrality), 190
gn_graph() (in module networkx.generators.directed), 453
gnc_graph() (in module networkx.generators.directed), havel_hakimi_graph() (in module net454
gnm_random_graph() (in module net- heawood_graph() (in module networkx.generators.small), workx.generators.random_graphs), 438
gnmk_random_graph() (in module net- hits() (in module networkx.algorithms.bipartite.generators), 164
gnp_random_graph() (in module networkx.generators.random_graphs), 437
gnr_graph() (in module networkx.generators.directed), 454
google_matrix() (in module networkx.algorithms.link_analysis.pagerank_alg), 325
Graph() (in module networkx), 9
graph_atlas() (in module networkx.generators.atlas), 421
graph_atlas_g() (in module networkx.generators.atlas), 421
graph_clique_number() (in module workx.algorithms.clique), 197
graph_number_of_cliques() (in module workx.algorithms.clique), 198
graphviz_layout() (in module networkx.drawing.nx_agraph), 552
graphviz_layout() (in module networkx.drawing.nx_pydot), 555
greedy_branching() (in module networkx.algorithms.tree.branchings), 401
greedy_color() (in module workx.algorithms.coloring), 203
grid_2d_graph() (in module networkx.generators.classic), 427
grid_graph() (in module networkx.generators.classic), 427
groups() (in module networkx.utils.misc), 564

## H

hamiltonian_path() (in module workx.algorithms.tournament), 387
harmonic_centrality() (in module workx.algorithms.centrality), 188
has_edge() (DiGraph method), 52
has_edge() (Graph method), 26
has_edge() (MultiDiGraph method), 108
has_edge() (MultiGraph method), 80
has_node() (DiGraph method), 52
has_node() (Graph method), 25
net- has_node() (MultiDiGraph method), 107
has_node() (MultiGraph method), 79
has_path() (in module networkx.algorithms.shortest_paths.generic), 357
hashable, 579
havel_hakimi_graph() (in module networkx.algorithms.bipartite.generators), 161 workx.generators.degree_seq), 450 434
workx.algorithms.link_analysis.hits_alg), 326
hits_numpy() (in module networkx.algorithms.link_analysis.hits_alg), 327
hits_scipy() (in module networkx.algorithms.link_analysis.hits_alg), 327
hopcroft_karp_matching() (in module networkx.algorithms.bipartite.matching), 143
house_graph() (in module networkx.generators.small), 434
house_x_graph() (in module networkx.generators.small), 434
net- hub_matrix() (in module networkx.algorithms.link_analysis.hits_alg), 328
hypercube_graph() (in module net-
net- workx.generators.classic), 427
net-
icosahedral_graph() (in module net-
net-
identified_nodes() (in module networkx.algorithms.minors), 338
immediate_dominators() (in module networkx.algorithms.dominance), 271
in_degree() (DiGraph method), 54
in_degree() (MultiDiGraph method), 110
in_degree_centrality() (in module networkx.algorithms.centrality), 167
net- in_edges() (DiGraph method), 48
in_edges() (MultiDiGraph method), 104
net- incidence_matrix () (in module networkx.linalg.graphmatrix), 476
info() (in module networkx.classes.function), 412
initialize() (DiGraphMatcher method), 315
initialize() (GraphMatcher method), 313
intersection() (in module networkx.algorithms.operators.binary), 345
intersection_all() (in module workx.algorithms.operators.all), 348
intersection_array() (in module workx.algorithms.distance_regular), 269
is_aperiodic() (in module networkx.algorithms.dag), 264
is_arborescence() (in module networkx.algorithms.tree.recognition), 400
is_attracting_component() (in module workx.algorithms.components), 223
is_biconnected() (in module workx.algorithms.components), 225
is_bipartite() (in module workx.algorithms.bipartite.basic), 139
is_bipartite_node_set() (in module workx.algorithms.bipartite.basic), 139
is_branching() (in module workx.algorithms.tree.recognition), 400
is_chordal() (in module networkx.algorithms.chordal) 192
is_connected () (in module workx.algorithms.components), 214
is_digraphical() (in module workx.algorithms.graphical), 302
is_directed() (in module networkx.classes.function), 41
is_directed_acyclic_graph() (in module networkx.algorithms.dag), 263
is_distance_regular() (in module workx.algorithms.distance_regular), 268
is_dominating_set() (in module workx.algorithms.dominating), 273
is_edge_cover() (in module workx.algorithms.covering), 253
is_eulerian() (in module networkx.algorithms.euler), 275
is_forest() (in module networkx.algorithms.tree.recognition), 400
is_frozen() (in module networkx.classes.function), 419
is_graphical() (in module networkx.algorithms.graphical), 302
is_isolate() (in module networkx.algorithms.isolate), 307
is_isomorphic() (DiGraphMatcher method), 315
is_isomorphic() (GraphMatcher method), 313
is_isomorphic() (in module workx.algorithms.isomorphism), 308
is_kl_connected() (in module workx.algorithms.hybrid), 306
is_list_of_ints() (in module networkx.utils.misc), 564
is_matching() (in module networkx.algorithms.matching), 334
is_maximal_matching() (in module workx.algorithms.matching), 335
is_multigraphical() (in module workx.algorithms.graphical), 303
is_pseudographical() (in module workx.algorithms.graphical), 303
net- is_reachable() (in module networkx.algorithms.tournament), 387 net- is_semiconnected() (in module networkx.algorithms.components), 230
is_simple_path() (in module networkx.algorithms.simple_paths), 383
is_string_like() (in module networkx.utils.misc), 563
net- is_strongly_connected() (in module networkx.algorithms.components), 217
net- is_strongly_connected() (in module networkx.algorithms.tournament), 387
net- is_strongly_regular() (in module networkx.algorithms.distance_regular), 269
net- is_tournament() (in module networkx.algorithms.tournament), 388
net- is_tree() (in module networkx.algorithms.tree.recognition), 399
is_valid_degree_sequence_erdos_gallai() (in module networkx.algorithms.graphical), 304
net- is_valid_degree_sequence_havel_hakimi() (in module networkx.algorithms.graphical), 303
net- is_valid_joint_degree() (in module networkx.generators.joint_degree_seq), 472
is_weakly_connected() (in module networkx.algorithms.components), 221
isolates() (in module networkx.algorithms.isolate), 307
isomorphisms_iter() (DiGraphMatcher method), 316
isomorphisms_iter() (GraphMatcher method), 314
net- iterable() (in module networkx.utils.misc), 563
net- J
jaccard_coefficient() (in module networkx.algorithms.link_prediction), 329
jit_data() (in module networkx.readwrite.json_graph), 528
jit_graph() (in module networkx.readwrite.json_graph), 528
johnson() (in module networkx.algorithms.shortest_paths.weighted), 377
joint_degree_graph() (in module networkx.generators.joint_degree_seq), 473
net K
k_clique_communities() (in module networkx.algorithms.community.kclique), 209
k_components() (in module networkx.algorithms.approximation.kcomponents), 122
k_components() (in module networkx.algorithms.connectivity.kcomponents), 231
k_core() (in module networkx.algorithms.core), 250
k_corona() (in module networkx.algorithms.core), 252
k_crust() (in module networkx.algorithms.core), 251
k_nearest_neighbors() (in module networkx.algorithms.assortativity), 134
k_random_intersection_graph() (in module networkx.generators.intersection), 464
k_shell() (in module networkx.algorithms.core), 251
karate_club_graph() (in module networkx.generators.social), 465
katz_centrality() (in module workx.algorithms.centrality), 170
katz_centrality_numpy() (in module workx.algorithms.centrality), 172
kernighan_lin_bisection() (in module networkx.algorithms.community.kernighan_lin), 209
kl_connected_subgraph() (in module networkx.algorithms.hybrid), 306
kosaraju_strongly_connected_components() (in module networkx.algorithms.components), 220
krackhardt_kite_graph() (in module networkx.generators.small), 434

## L

ladder_graph() (in module networkx.generators.classic), 427
laplacian_matrix() (in module networkx.linalg.laplacianmatrix), 477
laplacian_spectrum() (in module networkx.linalg.spectrum), 479
latapy_clustering() (in module networkx.algorithms.bipartite.cluster), 154
LCF_graph() (in module networkx.generators.small), 433
lexicographic_product() (in module networkx.algorithms.operators.product), 349
lexicographical_topological_sort() (in module networkx.algorithms.dag), 263
line_graph() (in module networkx.generators.line), 461
literal_destringizer() (in module networkx.readwrite.gml), 519
literal_stringizer() (in module networkx.readwrite.gml), 519
load_centrality() (in module networkx.algorithms.centrality), 185
local_edge_connectivity() (in module networkx.algorithms.connectivity.connectivity), 235
local_efficiency() (in module networkx.algorithms.efficiency), 274
local_node_connectivity() (in module networkx.algorithms.approximation.connectivity), 120
local_node_connectivity() (in module networkx.algorithms.connectivity.connectivity), 237
local_reaching_centrality() (in module networkx.algorithms.centrality), 189
lollipop_graph() (in module networkx.generators.classic), 428
M
et- make_clique_bipartite() (in module networkx.algorithms.clique), 197
net- make_max_clique_graph() (in module networkx.algorithms.clique), 196
make_small_graph() (in module networkx.generators.small), 432
make_str() (in module networkx.utils.misc), 564
margulis_gabber_galil_graph() (in module networkx.generators.expanders), 431
match() (DiGraphMatcher method), 316
match() (GraphMatcher method), 314
max_clique() (in module networkx.algorithms.approximation.clique), 123
max_flow_min_cost() (in module networkx.algorithms.flow), 298
max_weight_matching() (in module networkx.algorithms.matching), 335
maximal_independent_set() (in module networkx.algorithms.mis), 342
maximal_matching() (in module networkx.algorithms.matching), 335
maximum_branching() (in module networkx.algorithms.tree.branchings), 402
maximum_flow() (in module networkx.algorithms.flow), 276
maximum_flow_value() (in module networkx.algorithms.flow), 278
maximum_independent_set() (in module networkx.algorithms.approximation.independent_set), 126
maximum_spanning_arborescence() (in module networkx.algorithms.tree.branchings), 402
maximum_spanning_edges() (in module networkx.algorithms.tree.mst), 406
maximum_spanning_tree() (in module networkx.algorithms.tree.mst), 404
min_cost_flow() (in module networkx.algorithms.flow), 296
min_cost_flow_cost() (in module networkx.algorithms.flow), 295
min_edge_cover() (in module networkx.algorithms.bipartite.covering), 164
min_edge_cover() (in module networkx.algorithms.covering), 253
min_edge_dominating_set() (in module networkx.algorithms.approximation.dominating_set), 125
min_maximal_matching() (in module networkx.algorithms.approximation.matching), 127
min_weighted_dominating_set() (in module networkx.algorithms.approximation.dominating_set), 125
min_weighted_vertex_cover() (in module networkx.algorithms.approximation.vertex_cover), 128
minimum_branching() (in module networkx.algorithms.tree.branchings), 402
minimum_cut() (in module networkx.algorithms.flow), 280
minimum_cut_value() (in module workx.algorithms.flow), 282
minimum_edge_cut() (in module workx.algorithms.connectivity.cuts), 241
minimum_node_cut() (in module workx.algorithms.connectivity.cuts), 242
minimum_spanning_arborescence() (in module workx.algorithms.tree.branchings), 403
minimum_spanning_edges() (in module workx.algorithms.tree.mst), 405
minimum_spanning_tree() (in module workx.algorithms.tree.mst), 403
minimum_st_edge_cut() (in module workx.algorithms.connectivity.cuts), 244
minimum_st_node_cut() (in module workx.algorithms.connectivity.cuts), 245
mixing_expansion() (in module workx.algorithms.cuts), 259
moebius_kantor_graph() (in module workx.generators.small), 435
multi_source_dijkstra_path() (in module networkx.algorithms.shortest_paths.weighted), 366
multi_source_dijkstra_path_length() (in module networkx.algorithms.shortest_paths.weighted), 367
MultiDiGraph() (in module networkx), 89
MultiGraph() (in module networkx), 62

## N

navigable_small_world_graph() (in module networkx.generators.geometric), 460
nbunch, 579
nbunch_iter() (DiGraph method), 51
nbunch_iter() (Graph method), 24
nbunch_iter() (MultiDiGraph method), 106
nbunch_iter() (MultiGraph method), 78
negative_edge_cycle() (in module networkx.algorithms.shortest_paths.weighted), 377
neighbors() (DiGraph method), 49
neighbors() (Graph method), 23
neighbors() (MultiDiGraph method), 105
neighbors() (MultiGraph method), 77
network_simplex() (in module networkx.algorithms.flow), 293
networkx.algorithms.approximation (module), 119
networkx.algorithms.approximation.clique (module), 123
networkx.algorithms.approximation.clustering_coefficient (module), 124
networkx.algorithms.approximation.connectivity (module), 119
networkx.algorithms.approximation.dominating_set (module), 124
networkx.algorithms.approximation.independent_set (module), 126
net- networkx.algorithms.approximation.kcomponents (module), 122
net- networkx.algorithms.approximation.matching (module), 126
net- networkx.algorithms.approximation.ramsey (module), 127
net- networkx.algorithms.approximation.vertex_cover (module), 127
net- networkx.algorithms.assortativity (module), 128 networkx.algorithms.bipartite (module), 137
net- networkx.algorithms.bipartite.basic (module), 138 networkx.algorithms.bipartite.centrality (module), 157
net- networkx.algorithms.bipartite.cluster (module), 152 networkx.algorithms.bipartite.covering (module), 164 net- networkx.algorithms.bipartite.generators (module), 160 networkx.algorithms.bipartite.matching (module), 142
net- networkx.algorithms.bipartite.matrix (module), 144 networkx.algorithms.bipartite.projection (module), 145 networkx.algorithms.bipartite.redundancy (module), 156 networkx.algorithms.bipartite.spectral (module), 151 networkx.algorithms.boundary (module), 165 networkx.algorithms.centrality (module), 166 networkx.algorithms.chains (module), 191 networkx.algorithms.chordal (module), 192 networkx.algorithms.clique (module), 194 networkx.algorithms.cluster (module), 199 networkx.algorithms.coloring (module), 203 networkx.algorithms.communicability_alg (module), 207 networkx.algorithms.community (module), 208 networkx.algorithms.community.asyn_lpa (module), 210 networkx.algorithms.community.centrality (module), 212 networkx.algorithms.community.kclique (module), 209 networkx.algorithms.community.kernighan_lin (module), 208 networkx.algorithms.community.quality (module), 211 networkx.algorithms.components (module), 214 networkx.algorithms.connectivity (module), 230 networkx.algorithms.connectivity.connectivity (module), 233
networkx.algorithms.connectivity.cuts (module), 241
networkx.algorithms.connectivity.kcomponents (module), 230
networkx.algorithms.connectivity.kcutsets (module), 232
networkx.algorithms.connectivity.stoerwagner (module), 247
networkx.algorithms.connectivity.utils (module), 248
networkx.algorithms.core (module), 249
networkx.algorithms.covering (module), 252
networkx.algorithms.cuts (module), 257
networkx.algorithms.cycles (module), 254
networkx.algorithms.dag (module), 261
networkx.algorithms.distance_measures (module), 266
networkx.algorithms.distance_regular (module), 268
networkx.algorithms.dominance (module), 270
networkx.algorithms.dominating (module), 272
networkx.algorithms.efficiency (module), 273
networkx.algorithms.euler (module), 275
networkx.algorithms.flow (module), 276
networkx.algorithms.graphical (module), 301
networkx.algorithms.hierarchy (module), 305
networkx.algorithms.hybrid (module), 305
networkx.algorithms.isolate (module), 307
networkx.algorithms.isomorphism (module), 308
networkx.algorithms.isomorphism.isomorphvf2 (module), 311
networkx.algorithms.link_analysis.hits_alg (module), 326
networkx.algorithms.link_analysis.pagerank_alg (module), 322
networkx.algorithms.link_prediction (module), 328
networkx.algorithms.matching (module), 334
networkx.algorithms.minors (module), 336
networkx.algorithms.mis (module), 342
networkx.algorithms.operators.all (module), 346
networkx.algorithms.operators.binary (module), 343
networkx.algorithms.operators.product (module), 348
networkx.algorithms.operators.unary (module), 342
networkx.algorithms.reciprocity (module), 352
networkx.algorithms.richclub (module), 353
networkx.algorithms.shortest_paths.astar (module), 380
networkx.algorithms.shortest_paths.dense (module), 378
networkx.algorithms.shortest_paths.generic (module), 354
networkx.algorithms.shortest_paths.unweighted (module), 358
networkx.algorithms.shortest_paths.weighted (module), 360
networkx.algorithms.simple_paths (module), 381
networkx.algorithms.swap (module), 384
networkx.algorithms.tournament (module), 386
networkx.algorithms.traversal.beamsearch (module), 396
networkx.algorithms.traversal.breadth_first_search (module), 393
networkx.algorithms.traversal.depth_first_search (module), 389
networkx.algorithms.traversal.edgedfs (module), 397
networkx.algorithms.tree.branchings (module), 401
networkx.algorithms.tree.mst (module), 403
networkx.algorithms.tree.recognition (module), 398
networkx.algorithms.triads (module), 407
networkx.algorithms.vitality (module), 408
networkx.algorithms.voronoi (module), 408
networkx.algorithms.wiener (module), 409
networkx.classes.function (module), 411
networkx.convert (module), 487
networkx.convert_matrix (module), 490
networkx.drawing.layout (module), 556
networkx.drawing.nx_agraph (module), 551
networkx.drawing.nx_pydot (module), 553
networkx.drawing.nx_pylab (module), 543
networkx.exception (module), 561
networkx.generators.atlas (module), 421
networkx.generators.classic (module), 422
networkx.generators.community (module), 466
networkx.generators.degree_seq (module), 446
networkx.generators.directed (module), 453
networkx.generators.duplication (module), 445
networkx.generators.ego (module), 462
networkx.generators.expanders (module), 430
networkx.generators.geometric (module), 456
networkx.generators.intersection (module), 463
networkx.generators.joint_degree_seq (module), 472
networkx.generators.line (module), 461
networkx.generators.nonisomorphic_trees (module), 471
networkx.generators.random_clustered (module), 452
networkx.generators.random_graphs (module), 436
networkx.generators.small (module), 432
networkx.generators.social (module), 465
networkx.generators.stochastic (module), 463
networkx.generators.triads (module), 471
networkx.linalg.algebraicconnectivity (module), 480
networkx.linalg.attrmatrix (module), 483
networkx.linalg.graphmatrix (module), 475
networkx.linalg.laplacianmatrix (module), 477
networkx.linalg.spectrum (module), 479
networkx.readwrite.adjlist (module), 501
networkx.readwrite.edgelist (module), 508
networkx.readwrite.gexf (module), 514
networkx.readwrite.gml (module), 516
networkx.readwrite.gpickle (module), 520
networkx.readwrite.graph6 (module), 532
networkx.readwrite.graphml (module), 521
networkx.readwrite.json_graph (module), 523
networkx.readwrite.leda (module), 528
networkx.readwrite.multiline_adjlist (module), 504
networkx.readwrite.nx_shp (module), 538
networkx.readwrite.nx_yaml (module), 529
networkx.readwrite.pajek (module), 537
networkx.readwrite.sparse6 (module), 534
networkx.utils (module), 563
networkx.utils.contextmanagers (module), 570
networkx.utils.decorators (module), 567
networkx.utils.misc (module), 563
networkx.utils.random_sequence (module), 565
networkx.utils.rcm (module), 568
networkx.utils.union_find (module), 564
NetworkXAlgorithmError (class in networkx), 561
NetworkXError (class in networkx), 561
NetworkXException (class in networkx), 561
NetworkXNoPath (class in networkx), 561
NetworkXPointlessConcept (class in networkx), 561
NetworkXUnbounded (class in networkx), 561
NetworkXUnfeasible (class in networkx), 561
new_edge_key() (MultiDiGraph method), 98
new_edge_key() (MultiGraph method), 71
newman_watts_strogatz_graph() (in module workx.generators.random_graphs), 439
node, 579
node attribute, 579
node_boundary() (in module workx.algorithms.boundary), 166
node_clique_number() (in module workx.algorithms.clique), 198
node_connected_component() (in module workx.algorithms.components), 216
node_connectivity() (in module networkx.algorithms.approximation.connectivity), 121
node_connectivity() (in module networkx.algorithms.connectivity.connectivity), 239
node_expansion() (in module networkx.algorithms.cuts), 259
node_link_data() (in module networkx.readwrite.json_graph), 523
node_link_graph() (in module networkx.readwrite.json_graph), 524
node_redundancy() (in module networkx.algorithms.bipartite.redundancy), 156
NodeNotFound (class in networkx), 561
nodes() (DiGraph method), 45
nodes() (Graph method), 20
nodes() (in module networkx.classes.function), 414
nodes() (MultiDiGraph method), 100
nodes() (MultiGraph method), 74
nodes_with_selfloops() (DiGraph method), 57
nodes_with_selfloops() (Graph method), 29
nodes_with_selfloops() (MultiDiGraph method), 113
nodes_with_selfloops() (MultiGraph method), 83
non_edges() (in module networkx.classes.function), 416
non_neighbors() (in module networkx.classes.function), 415
nonisomorphic_trees() (in module networkx.generators.nonisomorphic_trees), 471
normalized_cut_size() (in module networkx.algorithms.cuts), 260
normalized_laplacian_matrix() (in module networkx.linalg.laplacianmatrix), 477
null_graph() (in module networkx.generators.classic), 428
number_attracting_components() (in module networkx.algorithms.components), 224
number_connected_components() (in module networkx.algorithms.components), 214
number_of_cliques() (in module networkx.algorithms.clique), 198
number_of_edges() (DiGraph method), 56
net- number_of_edges() (Graph method), 29
number_of_edges() (in module networkx.classes.function), 416
number_of_edges() (MultiDiGraph method), 112
number_of_edges() (MultiGraph method), 83
number_of_nodes() (DiGraph method), 53
number_of_nodes() (Graph method), 27
number_of_nodes() (in module networkx.classes.function), 414
number_of_nodes() (MultiDiGraph method), 109
number_of_nodes() (MultiGraph method), 81
number_of_nonisomorphic_trees() (in module networkx.generators.nonisomorphic_trees), 471
number_of_selfloops() (DiGraph method), 58
number_of_selfloops() (Graph method), 30
number_of_selfloops() (MultiDiGraph method), 114
number_of_selfloops() (MultiGraph method), 84
number_strongly_connected_components() (in module networkx.algorithms.components), 217
number_weakly_connected_components() (in module networkx.algorithms.components), 221
numeric_assortativity_coefficient() (in module networkx.algorithms.assortativity), 130
numerical_edge_match() (in module networkx.algorithms.isomorphism), 319
numerical_multiedge_match() (in module networkx.algorithms.isomorphism), 319
numerical_node_match() (in module networkx.algorithms.isomorphism), 318
0
octahedral_graph() (in module networkx.generators.small), 435
open_file() (in module networkx.utils.decorators), 567
order() (DiGraph method), 53
order() (Graph method), 26
order() (MultiDiGraph method), 109
order() (MultiGraph method), 81
out_degree() (DiGraph method), 55
out_degree() (MultiDiGraph method), 111
out_degree_centrality() (in module workx.algorithms.centrality), 167
out_edges() (DiGraph method), 47
out_edges() (MultiDiGraph method), 103
overall_reciprocity() (in module workx.algorithms.reciprocity), 352
overlap_weighted_projected_graph() (in module networkx.algorithms.bipartite.projection), 148

## P

pagerank() (in module networkx.algorithms.link_analysis.pagerank_alg), 322
pagerank_numpy () (in module networkx.algorithms.link_analysis.pagerank_alg), 323
pagerank_scipy() (in module networkx.algorithms.link_analysis.pagerank_alg), 324
pairwise() (in module networkx.utils.misc), 564
pappus_graph() (in module networkx.generators.small), 435
pareto_sequence() (in module networkx.utils.random_sequence), 565
parse_adjlist() (in module networkx.readwrite.adjlist), 503
parse_edgelist() (in module networkx.readwrite.edgelist), 513
parse_gml() (in module networkx.readwrite.gml), 518
parse_graph6() (in module networkx.readwrite.graph6), 532
parse_leda() (in module networkx.readwrite.leda), 529
parse_multiline_adjlist() (in module networkx.readwrite.multiline_adjlist), 507
parse_pajek() (in module networkx.readwrite.pajek), 538
parse_sparse6() (in module networkx.readwrite.sparse6), 535
partial_duplication_graph() (in module networkx.generators.duplication), 445
path_graph() (in module networkx.generators.classic), 428
performance() (in module networkx.algorithms.community.quality), 211
periphery() (in module networkx.algorithms.distance_measures), 267
petersen_graph() (in module networkx.generators.small), 435
planted_partition_graph() (in module networkx.generators.community), 469
power() (in module networkx.algorithms.operators.product), 351
powerlaw_cluster_graph() (in module networkx.generators.random_graphs), 441
net- powerlaw_sequence() (in module networkx.utils.random_sequence), 565
predecessor() (in module networkx.algorithms.shortest_paths.unweighted), 360
predecessors() (DiGraph method), 50
predecessors() (MultiDiGraph method), 106
preferential_attachment() (in module networkx.algorithms.link_prediction), 331
preferential_attachment_graph() (in module networkx.algorithms.bipartite.generators), 163
preflow_push() (in module networkx.algorithms.flow), 287
projected_graph() (in module networkx.algorithms.bipartite.projection), 145
pydot_layout() (in module networkx.drawing.nx_pydot), 555
pygraphviz_layout() (in module networkx.drawing.nx_agraph), 553
$Q$
quotient_graph() (in module networkx.algorithms.minors), 339
$R$
ra_index_soundarajan_hopcroft() (in module networkx.algorithms.link_prediction), 332
radius() (in module networkx.algorithms.distance_measures), 268
ramsey_R2() (in module networkx.algorithms.approximation.ramsey), 127
random_clustered_graph() (in module networkx.generators.random_clustered), 452
random_degree_sequence_graph() (in module networkx.generators.degree_seq), 451
random_geometric_graph() (in module networkx.generators.geometric), 457
random_graph() (in module networkx.algorithms.bipartite.generators), 163
random_k_out_graph() (in module networkx.generators.directed), 455
random_kernel_graph() (in module networkx.generators.random_graphs), 442
random_layout() (in module networkx.drawing.layout), 557
random_lobster() (in module networkx.generators.random_graphs), 443
random_partition_graph() (in module networkx.generators.community), 468
random_powerlaw_tree() (in module networkx.generators.random_graphs), 444
random_powerlaw_tree_sequence() (in module workx.generators.random_graphs), 444
random_regular_graph() (in module workx.generators.random_graphs), 441
random_shell_graph() (in module workx.generators.random_graphs), 443
random_tournament() (in module workx.algorithms.tournament), 388
random_weighted_sample() (in module networkx.utils.random_sequence), 567
read_adjlist() (in module networkx.readwrite.adjlist), 501 read_dot() (in module networkx.drawing.nx_agraph), 552
read_dot() (in module networkx.drawing.nx_pydot), 555
read_edgelist() (in module networkx.readwrite.edgelist), 509
read_gexf() (in module networkx.readwrite.gexf), 515
read_gml() (in module networkx.readwrite.gml), 517
read_gpickle() (in module networkx.readwrite.gpickle), 520
read_graph6() (in module networkx.readwrite.graph6), 533
read_graphml() (in module networkx.readwrite.graphml), 522
read_leda() (in module networkx.readwrite.leda), 529
read_multiline_adjlist() (in module networkx.readwrite.multiline_adjlist), 505
read_pajek() (in module networkx.readwrite.pajek), 537
read_shp() (in module networkx.readwrite.nx_shp), 540
read_sparse6() (in module networkx.readwrite.sparse6), 535
read_weighted_edgelist() (in module networkx.readwrite.edgelist), 511
read_yaml() (in module networkx.readwrite.nx_yaml), 531
reciprocity () (in module networkx.algorithms.reciprocity), 352
relabel_gexf_graph() (in module workx.readwrite.gexf), 516
relaxed_caveman_graph() (in module workx.generators.community), 467
remove_edge() (DiGraph method), 43
remove_edge() (Graph method), 18
remove_edge() (MultiDiGraph method), 98
remove_edge() (MultiGraph method), 72
remove_edges_from() (DiGraph method), 44
remove_edges_from() (Graph method), 19
remove_edges_from() (MultiDiGraph method), 99
remove_edges_from() (MultiGraph method), 72
remove_node() (DiGraph method), 40
remove_node() (Graph method), 15
remove_node() (MultiDiGraph method), 94
remove_node() (MultiGraph method), 68
remove_nodes_from() (DiGraph method), 41
remove_nodes_from() (Graph method), 16
remove_nodes_from() (MultiDiGraph method), 95
remove_nodes_from() (MultiGraph method), 68
rescale_layout() (in module networkx.drawing.layout), 557
net- resource_allocation_index() (in module net-
workx.algorithms.link_prediction), 329
reverse() (DiGraph method), 62
reverse() (in module networkx.algorithms.operators.unary), 343
reverse() (MultiDiGraph method), 118
reverse_cuthill_mckee_ordering() (in module networkx.utils.rcm), 569
reverse_havel_hakimi_graph() (in module networkx.algorithms.bipartite.generators), 162
reversed() (in module networkx.utils.contextmanagers), 570
rich_club_coefficient() (in module networkx.algorithms.richclub), 353
ring_of_cliques() (in module networkx.generators.community), 470
robins_alexander_clustering() (in module networkx.algorithms.bipartite.cluster), 155

S
scale_free_graph() (in module networkx.generators.directed), 456
score_sequence() (in module networkx.algorithms.tournament), 389
sedgewick_maze_graph() (in module networkx.generators.small), 435
selfloop_edges() (DiGraph method), 57
selfloop_edges() (Graph method), 30
selfloop_edges() (MultiDiGraph method), 113
selfloop_edges() (MultiGraph method), 84
semantic_feasibility() (DiGraphMatcher method), 316
semantic_feasibility() (GraphMatcher method), 314
set_edge_attributes() (in module networkx.classes.function), 417
net- set_node_attributes() (in module networkx.classes.function), 416
sets() (in module networkx.algorithms.bipartite.basic), 140
shell_layout() (in module networkx.drawing.layout), 558 shortest_augmenting_path() (in module networkx.algorithms.flow), 285
shortest_path() (in module networkx.algorithms.shortest_paths.generic), 354
shortest_path_length() (in module networkx.algorithms.shortest_paths.generic), 356
shortest_simple_paths() (in module workx.algorithms.simple_paths), 383
simple_cycles() (in module networkx.algorithms.cycles), 255
single_source_bellman_ford() (in module networkx.algorithms.shortest_paths.weighted), 375
single_source_bellman_ford_path() (in module networkx.algorithms.shortest_paths.weighted), 372
single_source_bellman_ford_path_length() (in module networkx.algorithms.shortest_paths.weighted), 373
single_source_dijkstra() (in module networkx.algorithms.shortest_paths.weighted), 364
single_source_dijkstra_path() (in module networkx.algorithms.shortest_paths.weighted), 365
single_source_dijkstra_path_length() (in module networkx.algorithms.shortest_paths.weighted), 366
single_source_shortest_path() (in module networkx.algorithms.shortest_paths.unweighted), 358
single_source_shortest_path_length() (in module networkx.algorithms.shortest_paths.unweighted), 358
size() (DiGraph method), 56
size() (Graph method), 28
size() (MultiDiGraph method), 112
size() (MultiGraph method), 82
spectral_bipartivity() (in module networkx.algorithms.bipartite.spectral), 151
spectral_layout() (in module networkx.drawing.layout), 559
spectral_ordering() (in module networkx.linalg.algebraicconnectivity), 482
spring_layout() (in module networkx.drawing.layout), 558
square_clustering() (in module networkx.algorithms.cluster), 202
star_graph() (in module networkx.generators.classic), 428
stochastic_graph() (in module networkx.generators.stochastic), 463
stoer_wagner() (in module networkx.algorithms.connectivity.stoerwagner), 247
strategy_connected_sequential() (in module networkx.algorithms.coloring), 205
strategy_connected_sequential_bfs() (in module networkx.algorithms.coloring), 205
strategy_connected_sequential_dfs() (in module networkx.algorithms.coloring), 205
strategy_independent_set() (in module networkx.algorithms.coloring), 206
strategy_largest_first() (in module networkx.algorithms.coloring), 206
strategy_random_sequential() (in module networkx.algorithms.coloring), 206
strategy_saturation_largest_first() (in module networkx.algorithms.coloring), 206
strategy_smallest_last() (in module networkx.algorithms.coloring), 206
strong_product() (in module networkx.algorithms.operators.product), 349
strongly_connected_component_subgraphs() (in module networkx.algorithms.components), 218
strongly_connected_components() (in module networkx.algorithms.components), 218
strongly_connected_components_recursive() (in module networkx.algorithms.components), 219
subgraph() (DiGraph method), 61
subgraph() (Graph method), 33
subgraph() (MultiDiGraph method), 118
subgraph() (MultiGraph method), 87
subgraph_centrality() (in module networkx.algorithms.centrality), 186
subgraph_centrality_exp() (in module networkx.algorithms.centrality), 187
subgraph_is_isomorphic() (DiGraphMatcher method), 315
subgraph_is_isomorphic() (GraphMatcher method), 313
subgraph_isomorphisms_iter() (DiGraphMatcher method), 316
subgraph_isomorphisms_iter() (GraphMatcher method), 314
successors() (DiGraph method), 50
successors() (MultiDiGraph method), 106
symmetric_difference() (in module networkx.algorithms.operators.binary), 346
syntactic_feasibility() (DiGraphMatcher method), 316
syntactic_feasibility() (GraphMatcher method), 314
T
tensor_product() (in module networkx.algorithms.operators. product), 350
tetrahedral_graph() (in module networkx.generators.small), 435
to_agraph() (in module networkx.drawing.nx_agraph), 552
to_dict_of_dicts() (in module networkx.convert), 488
to_dict_of_lists() (in module networkx.convert), 489
to_directed() (DiGraph method), 60
to_directed() (Graph method), 32
to_directed() (MultiDiGraph method), 116
to_directed() (MultiGraph method), 87
to_edgelist() (in module networkx.convert), 489
to_networkx_graph() (in module networkx.convert), 487
to_numpy_matrix() (in module networkx.convert_matrix), 490
to_numpy_recarray() (in module networkx.convert_matrix), 492
to_pandas_dataframe() (in module networkx.convert_matrix), 497
to_pydot() (in module networkx.drawing.nx_pydot), 554
to_scipy_sparse_matrix() (in module networkx.convert_matrix), 494
to_undirected() (DiGraph method), 59
to_undirected() (Graph method), 32
to_undirected() (MultiDiGraph method), 115
to_undirected() (MultiGraph method), 86
to_vertex_cover() (in module networkx.algorithms.bipartite.matching), 143
topological_sort() (in module networkx.algorithms.dag), 262
transitive_closure() (in module networkx.algorithms.dag), 264
transitive_reduction() (in module networkx.algorithms.dag), 264
transitivity() (in module networkx.algorithms.cluster), 199
tree_data() (in module networkx.readwrite.json_graph), 526
tree_graph() (in module networkx.readwrite.json_graph), 527
triad_graph() (in module networkx.generators.triads), 472
triadic_census() (in module networkx.algorithms.triads), 407
triangles() (in module networkx.algorithms.cluster), 199
trivial_graph() (in module networkx.generators.classic), 429
truncated_cube_graph() (in module networkx.generators.small), 435
truncated_tetrahedron_graph() (in module networkx.generators.small), 435
turan_graph() (in module networkx.generators.classic), 429
tutte_graph() (in module networkx.generators.small), 435

## U

uniform_random_intersection_graph() (in module networkx.generators.intersection), 464
uniform_sequence() (in module
workx.utils.random_sequence), 565
union() (in module
workx.algorithms.operators.binary), 344
union() (UnionFind method), 565
union_all() (in module workx.algorithms.operators.all), 347

## V

volume() (in module networkx.algorithms.cuts), 260
voronoi_cells() (in module networkx.algorithms.voronoi), 409
W
watts_strogatz_graph() (in module networkx.generators.random_graphs), 440
waxman_graph() (in module networkx.generators.geometric), 459
weakly_connected_component_subgraphs() (in module networkx.algorithms.components), 222
weakly_connected_components() (in module networkx.algorithms.components), 222
weighted_choice() (in module networkx.utils.random_sequence), 567
weighted_projected_graph() (in module networkx.algorithms.bipartite.projection), 146
wheel_graph() (in module networkx.generators.classic), 429
wiener_index() (in module networkx.algorithms.wiener), 410
within_inter_cluster() (in module networkx.algorithms.link_prediction), 333
write_adjlist() (in module networkx.readwrite.adjlist), 502
write_dot() (in module networkx.drawing.nx_agraph), 552
write_dot() (in module networkx.drawing.nx_pydot), 554
write_edgelist() (in module networkx.readwrite.edgelist), 510
write_gexf() (in module networkx.readwrite.gexf), 515
write_gml() (in module networkx.readwrite.gml), 517
write_gpickle() (in module networkx.readwrite.gpickle), 521
write_graph6() (in module networkx.readwrite.graph6), 534
write_graphml() (in module networkx.readwrite.graphml), 522
write_multiline_adjlist() (in module networkx.readwrite.multiline_adjlist), 506
write_pajek() (in module networkx.readwrite.pajek), 538
write_shp() (in module networkx.readwrite.nx_shp), 540
write_sparse6() (in module networkx.readwrite.sparse6), 536
net- write_weighted_edgelist() (in module networkx.readwrite.edgelist), 511
net- write_yaml() (in module networkx.readwrite.nx_yaml), 531
zipf_rv() (in module networkx.utils.random_sequence), 566
zipf_sequence() (in module net-
workx.utils.random_sequence), 566


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