
ORCA Documentation

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Installation and Configuration

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ORCA is a c++ whole-body reactive controller meant to compute the desired actuation torque of a robot given some tasks to perform and some constraints.

The problem is written as a **quadratic problem** :

$$\begin{aligned} \min_x \quad & \frac{1}{2}x^t H x + x^t g \\ \text{subject to} \quad & lb \leq x \leq ub \\ & lb_A \leq A x \leq ub_A \end{aligned}$$

- x the optimisation vector
- H the hessian matrix ($size(x) \times size(x)$)
- g the gradient vector ($size(x) \times 1$)
- A the constraint matrix ($size(x) \times size(x)$)
- lb and ub the lower and upper bounds of x ($size(x) \times 1$)
- lb_A and ub_A the lower and upper bounds of A ($size(x) \times 1$)

Tasks are written as **weighted euclidian distance function** :

$$w_{task} \|E x + f\|_{W_{norm}}^2$$

- x the optimisation vector, or **part** of the optimisation vector
- E the linear matrix of the affine function ($size(x) \times size(x)$)
- f the origin vector ($size(x) \times 1$)
- w_{task} the weight of the tasks in the overall quadratic cost (scalar $[0 : 1]$)
- W_{norm} the weight of the euclidean norm ($size(x) \times size(x)$)

Given n_t tasks, the **overall cost function** is such that:

$$\frac{1}{2}x^t H x + x^t g = \frac{1}{2} \sum_{i=1}^{n_t} w_{task,i} \|E_i x + f_i\|_{W_{norm,i}}^2$$

Constraints are written as **double bounded linear function** :

$$lb_C \leq C x \leq ub_C$$

- C the constraint matrix ($size(x) \times size(x)$)
- lb_C and ub_C the lower and upper bounds of A ($size(x) \times 1$)

The remainder of the documentation describes “classical” tasks and cosntraints which one may want to define

Optimisation Vector

The optimisation vector in the quadratic problem is written as follows :

$$X = \begin{pmatrix} \dot{\nu}^{fb} \\ \dot{\nu}^j \\ \tau^{fb} \\ \tau^j \\ {}^e w_0 \\ \vdots \\ {}^e w_n \end{pmatrix}$$

- $\dot{\nu}^{fb}$: Floating base joint acceleration (6×1)
- $\dot{\nu}^j$: Joint space acceleration ($n_{dof} \times 1$)
- τ^{fb} : Floating base joint torque (6×1)
- τ^j : Joint space joint torque ($n_{dof} \times 1$)
- ${}^e w_n$: External wrench (6×1)
- τ^{fb} : Floating base joint torque (6×1)
- τ^j : Joint space joint torque ($n_{dof} \times 1$)
- ${}^e w_n$: External wrench (6×1)

In ORCA those are called *Control variables* and should be used to define every task and constraint. In addition to those necessary variables, you can specify also a combination :

- $\dot{\nu}$: Generalised joint acceleration, concatenation of $\dot{\nu}^{fb}$ and $\dot{\nu}^j$ ($6 + n_{dof} \times 1$)
- τ : Generalised joint torque, concatenation of τ^{fb} and τ^j ($6 + n_{dof} \times 1$)
- X : The whole optimisation vector ($6 + n_{dof} + 6 + n_{dof} + n_{wrenches} \times 1$)
- ${}^e w$: External wrenches ($n_{wrenche} \times 6 \times 1$)
- X : The whole optimisation vector ($6 + n_{dof} + 6 + n_{dof} + n_{wrenches} \times 1$)
- ${}^e w$: External wrenches ($n_{wrenche} \times 6 \times 1$)

CHAPTER 2

Cartesian Acceleration

$$w_{task} \cdot \|\mathbf{E}x + \mathbf{f}\|_{W_{norm}}$$
$$\underset{n \times 1}{\mathbf{Y}} = \underset{n \times p}{X} \times \underset{p \times 1}{\boldsymbol{\theta}} + \underset{n \times 1}{\boldsymbol{\varepsilon}}$$

Dynamics Equation

- **Control variable** : X (whole optimisation vector)
- **Type** : Equality constraint
- **Size** : $ndof \times size(X)$

$$\begin{bmatrix} -M & S_\tau & J_{e_w} \end{bmatrix} X = C + G$$

```
orca::constraint::DynamicsEquationConstraint dyn_eq;  
dyn_eq.loadRobotModel( urdf );  
dyn_eq.setGravity( Eigen::Vector3d(0,0,-9.81) );  
dyn_eq.update(); // <-- Now initialized  
  
dyn_eq.activate(); // <-- Now activated  
dyn_eq.insertInProblem(); // <-- Now part of the optimisation problem
```



