isochrones Documentation

Release 2.1

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Contents

1 Install **3**
- 1.1 Conda environment and testing . . . 3
- 1.2 Installing MultiNest . . . 3

2 Quick Start **5**
- 2.1 Access stellar model grid data . . . 5
- 2.2 Interpolate stellar properites . . . 7
- 2.3 Generate synthetic properties of stars . . . 7
- 2.4 Fit physical parameters of a star to observed data . . . 9
- 2.5 Fit a binary star model . . . 13

3 Interpolation: the `DFInterpolator` **17**

4 Stellar model grids **21**
- 4.1 Background and EEPs . . . 21
- 4.2 Model Grid Objects and Interpolation . . . 21
- 4.3 Example visualization . . . 26

5 Bolometric correction grids **27**

6 ModelGridInterpolator **29**
- 6.1 Isochrones . . . 29
- 6.2 Evolution tracks . . . 30
- 6.3 Generating synthetic properties . . . 31
- 6.4 Demo: Visualize . . . 33

7 Fitting stellar parameters **35**
- 7.1 Defining a star model . . . 35
- 7.2 Priors . . . 37
- 7.3 Sampling the posterior . . . 40

8 Multiple star systems **47**
- 8.1 Unresolved multiple systems . . . 47
- 8.2 Resolved multiple system . . . 51
- 8.3 Unassociated companions . . . 54
- 8.4 More complex models . . . 54

9 Simulating stellar populations **57**

9.1 StarPopulation object 57
9.2 ModelGridInterpolator.generate_binary 60

Isochrones is a python package that provides a simple interface to grids of stellar evolution models, enabling the following common use cases:

- Interpolating stellar model values at desired locations.
- Generating properties of synthetic stellar populations.
- Determining stellar properties of either single- or multiple-star systems, based on arbitrary observables.

The central goal of **isochrones** is to standardize model-grid-based stellar parameter inference, and to enable such inference under different sets of stellar models. For now, only MIST models are included, but we hope to incorporate YAPSI and PARSEC models as well.

CHAPTER 1

Install

1.1 Conda environment and testing

Isochrones requires python 3. I also recommend using **isochrones** in its own conda environment, to help manage dependencies. For example:

```
conda create -n isochrones numpy numba nose pytables pandas
```

Then

```
conda activate isochrones
pip install isochrones
```

To make sure everything is working, run

```
nosetests isochrones
```

And if anything breaks, please raise an issue.

1.2 Installing MultiNest

It is highly recommended to install **MultiNest/PyMultiNest** for model fitting. First, install/build multinest with

```
git clone https://github.com/johannesBuchner/MultiNest
cd MultiNest/build
cmake -DCMAKE_INSTALL_PREFIX=~ ..  # or just "cmake .." if you have root permissions
make
make install
```

(Note that if you don't have `cmake` available on your system, that you can install it in your environment with `conda install -c conda-forge cmake`.)

If you do not have root permissions and thus installed the **MultiNest** libraries to your home directory, you will also need to make sure that `~/lib` is in your `LD_LIBRARY_PATH` environment variable; e.g., you can include the following line in your `~/.bash_profile` file:

```
export LD_LIBRARY_PATH=$HOME/lib
```

Then you can install pymultinest with

```
pip install pymultinest
```

(And run `nosetests isochrones` again, for good measure, to confirm that **MultiNest** works.)

CHAPTER 2

Quick Start

2.1 Access stellar model grid data

[1]:
```
from isochrones.mist import MISTIsochroneGrid

grid = MISTIsochroneGrid()
print(len(grid.df))
grid.df.head()  # Just the first few rows
```

```
1494453
```

[1]:
```
                                eep  age       feh      mass  initial_mass  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35   35  5.0 -3.978406  0.100000      0.100000
                            36   36  5.0 -3.978406  0.102885      0.102885
                            37   37  5.0 -3.978406  0.107147      0.107147
                            38   38  5.0 -3.978406  0.111379      0.111379
                            39   39  5.0 -3.978406  0.115581      0.115581

                                  radius   density   logTeff         Teff  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35  1.106082  0.104184  3.617011  4140.105252
                            36  1.122675  0.102507  3.618039  4149.909661
                            37  1.147702  0.099921  3.619556  4164.436984
                            38  1.173015  0.097287  3.621062  4178.903372
                            39  1.198615  0.094627  3.622555  4193.289262

                                    logg      logL      Mbol   delta_nu  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35  3.350571 -0.489734  5.964335  37.987066
                            36  3.347798 -0.472691  5.921728  37.739176
                            37  3.343658 -0.447471  5.858678  37.345115
                            38  3.339612 -0.422498  5.796244  36.923615
                            39  3.335660 -0.397776  5.734440  36.473151
```

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```
                                       nu_max  phase   dm_deep
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35    299.346079   -1.0  0.002885
                            36    298.570836   -1.0  0.003573
                            37    297.180748   -1.0  0.004247
                            38    295.526946   -1.0  0.004217
                            39    293.589960   -1.0  0.004189
```

[2]:
```python
from isochrones.mist import MISTEvolutionTrackGrid

grid_tracks = MISTEvolutionTrackGrid()
print(len(grid_tracks.df))
grid_tracks.df.head()
```

```
3619652
```

[2]:
```
                                    nu_max      logg  eep  initial_mass  \
initial_feh initial_mass EEP
-4.0        0.1          1     143.524548  3.033277  1.0           0.1
                         2     145.419039  3.038935  2.0           0.1
                         3     147.409881  3.044805  3.0           0.1
                         4     149.499346  3.050886  4.0           0.1
                         5     151.703570  3.057203  5.0           0.1

                                  radius   logTeff  mass   density      Mbol  \
initial_feh initial_mass EEP
-4.0        0.1          1     1.593804  3.620834   0.1  0.034823  5.132871
                         2     1.583455  3.620769   0.1  0.035510  5.147664
                         3     1.572790  3.620702   0.1  0.036237  5.163015
                         4     1.561817  3.620631   0.1  0.037006  5.178922
                         5     1.550499  3.620558   0.1  0.037823  5.195452

                               phase       feh         Teff      logL  \
initial_feh initial_mass EEP
-4.0        0.1          1      -1.0 -3.978406  4176.707371 -0.157148
                         2      -1.0 -3.978406  4176.085183 -0.163066
                         3      -1.0 -3.978406  4175.435381 -0.169206
                         4      -1.0 -3.978406  4174.757681 -0.175569
                         5      -1.0 -3.978406  4174.049081 -0.182181

                                delta_nu  interpolated      star_age       age  \
initial_feh initial_mass EEP
-4.0        0.1          1     21.776686         False  13343.289397  4.125263
                         2     21.993078         False  14171.978264  4.151430
                         3     22.219791         False  15048.910447  4.177505
                         4     22.457004         False  15975.827275  4.203463
                         5     22.706349         False  16962.744747  4.229496

                                dt_deep
initial_feh initial_mass EEP
-4.0        0.1          1     0.026168
                         2     0.026121
                         3     0.026016
                         4     0.025996
                         5     0.025996
```

2.2 Interpolate stellar properites

[3]:
```
from isochrones import get_ichrone
mist = get_ichrone('mist')
eep = mist.get_eep(1.01, 9.76, 0.03, accurate=True)
mist.interp_value([eep, 9.76, 0.03], ['Teff', 'logg', 'radius', 'density'])
```

[3]:
```
array([5.86016011e+03, 4.36634798e+00, 1.09151255e+00, 1.09589730e+00])
```

[4]:
```
mist.interp_mag([eep, 9.76, 0.03, 200, 0.1], bands=['G', 'BP', 'RP'])
```

[4]:
```
(5860.16011294621,
 4.366347981387894,
 -0.005536922088842331,
 array([10.99261956, 11.3150264 , 10.50313434]))
```

2.3 Generate synthetic properties of stars

[5]:
```
from isochrones import get_ichrone
tracks = get_ichrone('mist', tracks=True)

mass, age, feh = (1.03, 9.72, -0.11)

tracks.generate(mass, age, feh, return_dict=True)  # "accurate=True" makes more␣
↪accurate, but slower
```

[5]:
```
{'nu_max': 2275.6902092679834,
 'logg': 4.315208279229787,
 'eep': 394.24,
 'initial_mass': 1.03,
 'radius': 1.1692076259176427,
 'logTeff': 3.785191265391399,
 'mass': 1.0297274169057322,
 'density': 0.9097687776092286,
 'Mbol': 4.162373757546131,
 'phase': 0.0,
 'feh': -0.19095007384845408,
 'Teff': 6100.263434973235,
 'logL': 0.23105049698154745,
 'delta_nu': 114.32933695055772,
 'interpolated': 0.0,
 'star_age': 5302578707.515498,
 'age': 9.722480201790624,
 'dt_deep': 0.0036558739980003118,
 'J': 3.2044197352759696,
 'H': 2.91756110497181,
 'K': 2.890399473719951,
 'G': 4.085847599912897,
 'BP': 4.349405878788243,
 'RP': 3.6587316339856084,
 'W1': 2.8807983122840044,
 'W2': 2.885550073210391,
 'W3': 2.8685709557487264,
 'TESS': 3.653543903981804,
 'Kepler': 4.004222279916473}
```

[6]:
```
from isochrones.priors import ChabrierPrior
import numpy as np

# Simulate a 1000-star cluster at 8kpc

N = 1000
masses = ChabrierPrior().sample(N)
feh = -1.8
age = np.log10(6e9)  # 6 Gyr
distance = 8000.  # 8 kpc
AV = 0.15

# By default this will return a dataframe
%timeit tracks.generate(masses, age, feh, distance=distance, AV=AV)
df = tracks.generate(masses, age, feh, distance=distance, AV=AV)
```

```
The slowest run took 158.58 times longer than the fastest. This could mean that an
↪intermediate result is being cached.
1 loop, best of 3: 9.04 ms per loop
```

[7]:
```
df = df.dropna()
print(len(df)) # about half of the original simulated stars are nans
df.head()
```

```
503
```

[7]:
```
         nu_max      logg         eep  initial_mass    radius   logTeff  \
0  10804.874097  4.914275  303.258462      0.418821  0.374324  3.631195
1  21841.644652  5.197122  252.271094      0.150592  0.161974  3.583987
7   2838.154305  4.435801  384.922283      0.849837  0.924219  3.833683
8    180.963558  3.194705  490.813513      0.968456  4.116643  3.742612
9   1931.725171  4.282014  416.535309      0.911882  1.142684  3.860309

       mass    density       Mbol  phase    ...              H          K  \
0  0.418811  11.354937   8.178493    0.0    ...      20.662206  20.501401
1  0.150591  50.030150  10.467041    0.0    ...      22.738821  22.531319
7  0.849572   1.517288   4.187702    0.0    ...      17.866850  17.848031
8  0.967435   0.019564   1.854663    2.0    ...      14.901613  14.851377
9  0.911438   0.861278   3.460707    0.0    ...      17.332797  17.316050

           G         BP         RP         W1         W2         W3  \
0  23.037457  23.718192  22.251047  20.363681  20.324516  20.219805
1  25.488818  26.383334  24.589471  22.380110  22.316618  22.177299
7  18.902509  19.108502  18.534769  17.834259  17.825916  17.803109
8  16.530374  16.884757  15.996079  14.816213  14.800818  14.770045
9  18.172883  18.341059  17.864912  17.304300  17.297109  17.275839

        TESS     Kepler
0  22.229761  22.950946
1  24.559047  25.416412
7  18.528245  18.837217
8  15.985842  16.451727
9  17.857626  18.111831

[5 rows x 29 columns]
```

[8]:
```
import holoviews as hv
hv.extension('bokeh')
```

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```
import hvplot.pandas

df['BP-RP'] = df.BP - df.RP
df.hvplot.scatter('BP-RP', 'G', hover_cols=['mass', 'radius', 'Teff', 'logg', 'eep']).
↪options(invert_yaxis=True, width=600)
```

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

[8]:
```
:Scatter   [BP-RP]   (G,mass,radius,Teff,logg,eep)
```

2.4 Fit physical parameters of a star to observed data

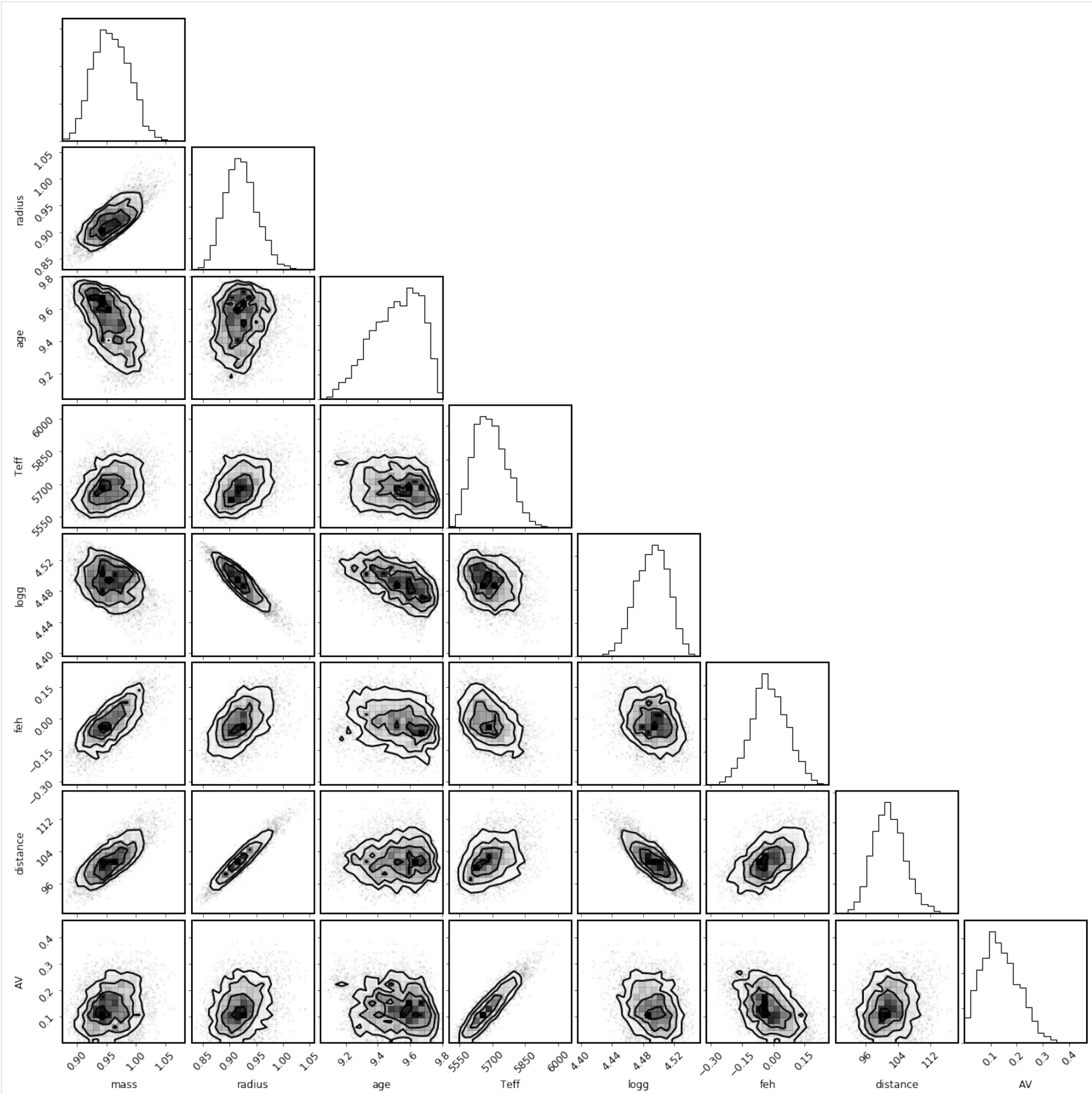
[9]:
```
from isochrones import get_ichrone, SingleStarModel

mist = get_ichrone('mist', bands=['BP', 'RP'])
params = {'Teff': (5700, 100), 'logg': (4.5, 0.1), 'feh': (0.0, 0.15),
          'BP': (10.42, 0.01), 'RP': (9.54, 0.01),
          'parallax': (10, 0.5)}  # mas
mod = SingleStarModel(mist, **params)
mod.fit()
```

```
INFO:root:MultiNest basename: ./chains/mist-single-
```

[10]:
```
%matplotlib inline

mod.corner_physical();
```



Check out the numerical sampling results:

[11]:
```
mod.samples.describe()
```

[11]:

	eep	age	feh	distance	AV	\
count	4643.000000	4643.000000	4643.000000	4643.000000	4643.000000	
mean	337.710149	9.509309	-0.020312	101.801691	0.136494	
std	9.624071	0.149170	0.078899	3.989609	0.069615	
min	304.868138	9.043279	-0.300996	88.634174	0.000291	
25%	330.856377	9.400976	-0.070581	99.008511	0.085589	
50%	339.214747	9.526285	-0.022509	101.582447	0.129668	
75%	345.473042	9.630834	0.033488	104.310959	0.183091	
max	364.435350	9.802944	0.243018	118.737714	0.466537	

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```
          lnprob
count  4643.000000
mean    -39.022183
std       1.311924
min     -48.789323
25%     -39.612822
50%     -38.744828
75%     -38.076162
max     -37.088602
```

And the derived parameters at those samples:

[12]:
```
mod.derived_samples.describe()
```

[12]:
```
              eep          age          feh         mass  initial_mass  \
count  4643.000000  4643.000000  4643.000000  4643.000000   4643.000000
mean    337.710149     9.509309    -0.018942     0.958078      0.958169
std       9.624071     0.149170     0.086400     0.030418      0.030410
min     304.868138     9.043279    -0.317669     0.876002      0.876104
25%     330.856377     9.400976    -0.076201     0.936277      0.936370
50%     339.214747     9.526285    -0.022622     0.956896      0.956970
75%     345.473042     9.630834     0.039230     0.979404      0.979464
max     364.435350     9.802944     0.267804     1.083840      1.083927

           radius      density      logTeff         Teff         logg  \
count  4643.000000  4643.000000  4643.000000  4643.000000  4643.000000
mean      0.921082     1.738516     3.755075  5690.649562     4.491137
std       0.031022     0.137653     0.005747    75.536605     0.020924
min       0.828304     1.209516     3.740466  5501.680997     4.396153
25%       0.899485     1.643165     3.750771  5634.112958     4.476522
50%       0.919331     1.740170     3.754596  5683.909988     4.492272
75%       0.940155     1.832595     3.758868  5740.000379     4.506173
max       1.058840     2.217147     3.782650  6062.846454     4.553723

          ...         Mbol     delta_nu       nu_max   phase  \
count     ...  4643.000000  4643.000000  4643.000000  4643.0
mean      ...     4.982881   157.087407  3538.322417     0.0
std       ...     0.107383     6.095095   177.717410     0.0
min       ...     4.568552   131.705546  2801.309165     0.0
25%       ...     4.913786   152.933682  3412.744754     0.0
50%       ...     4.985717   157.283081  3545.198779     0.0
75%       ...     5.061043   161.294116  3664.317241     0.0
max       ...     5.303972   177.056425  4130.359858     0.0

          dm_deep       BP_mag       RP_mag     parallax     distance  \
count  4643.000000  4643.000000  4643.000000  4643.000000  4643.000000
mean      0.008306    10.420067     9.539072     9.837970   101.801691
std       0.000654     0.009287     0.009087     0.382167     3.989609
min       0.002593    10.386946     9.505070     8.421924    88.634174
25%       0.007966    10.413741     9.533035     9.586720    99.008511
50%       0.008247    10.420135     9.539162     9.844220   101.582447
75%       0.008619    10.426205     9.545132    10.100142   104.310959
max       0.011076    10.453965     9.574724    11.282330   118.737714

               AV
count  4643.000000
mean      0.136494
```

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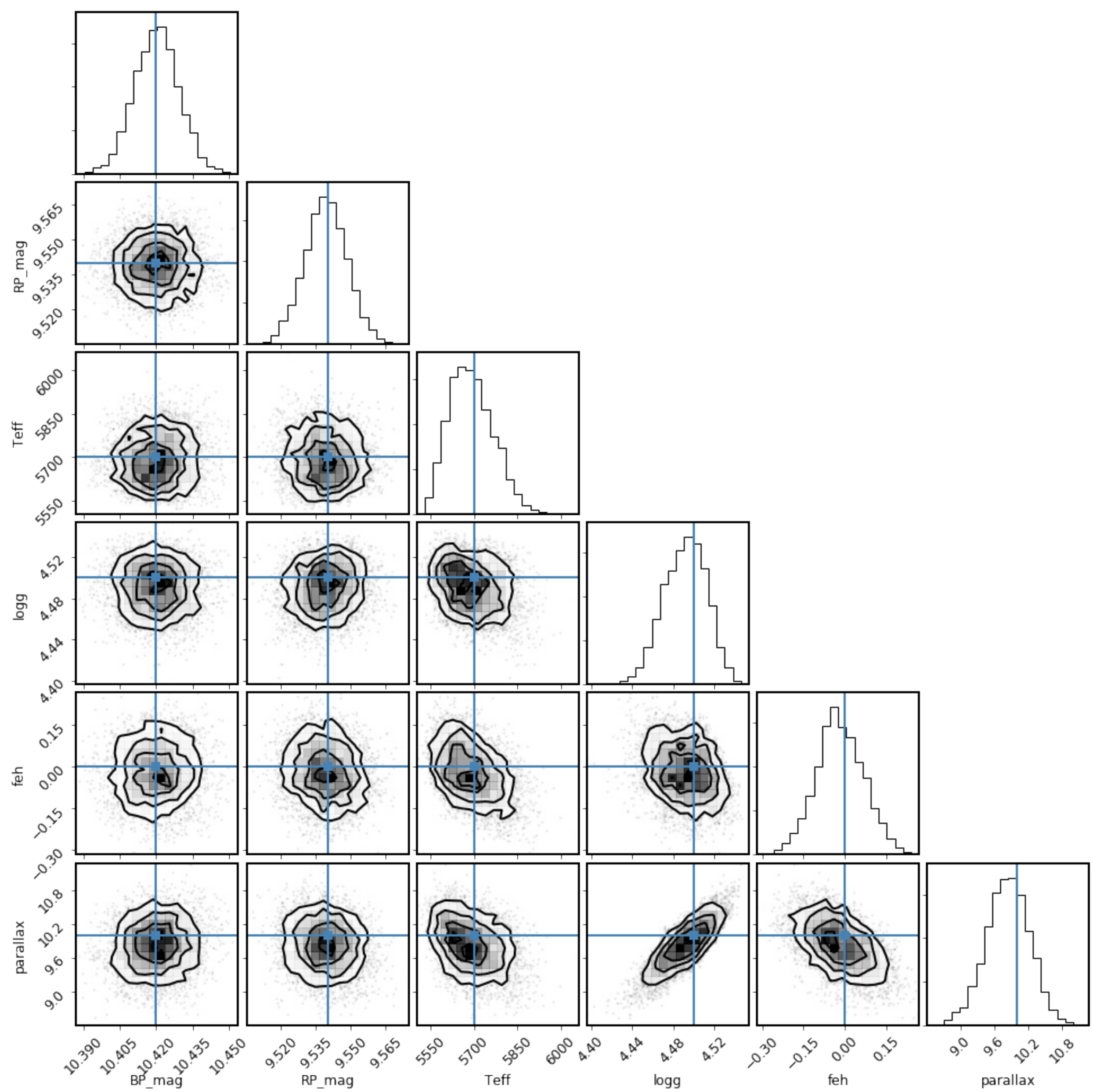
```
std       0.069615
min       0.000291
25%       0.085589
50%       0.129668
75%       0.183091
max       0.466537

[8 rows x 21 columns]
```

Eyeball your posterior predictive with:

[13]:
```
mod.corner_observed();
```

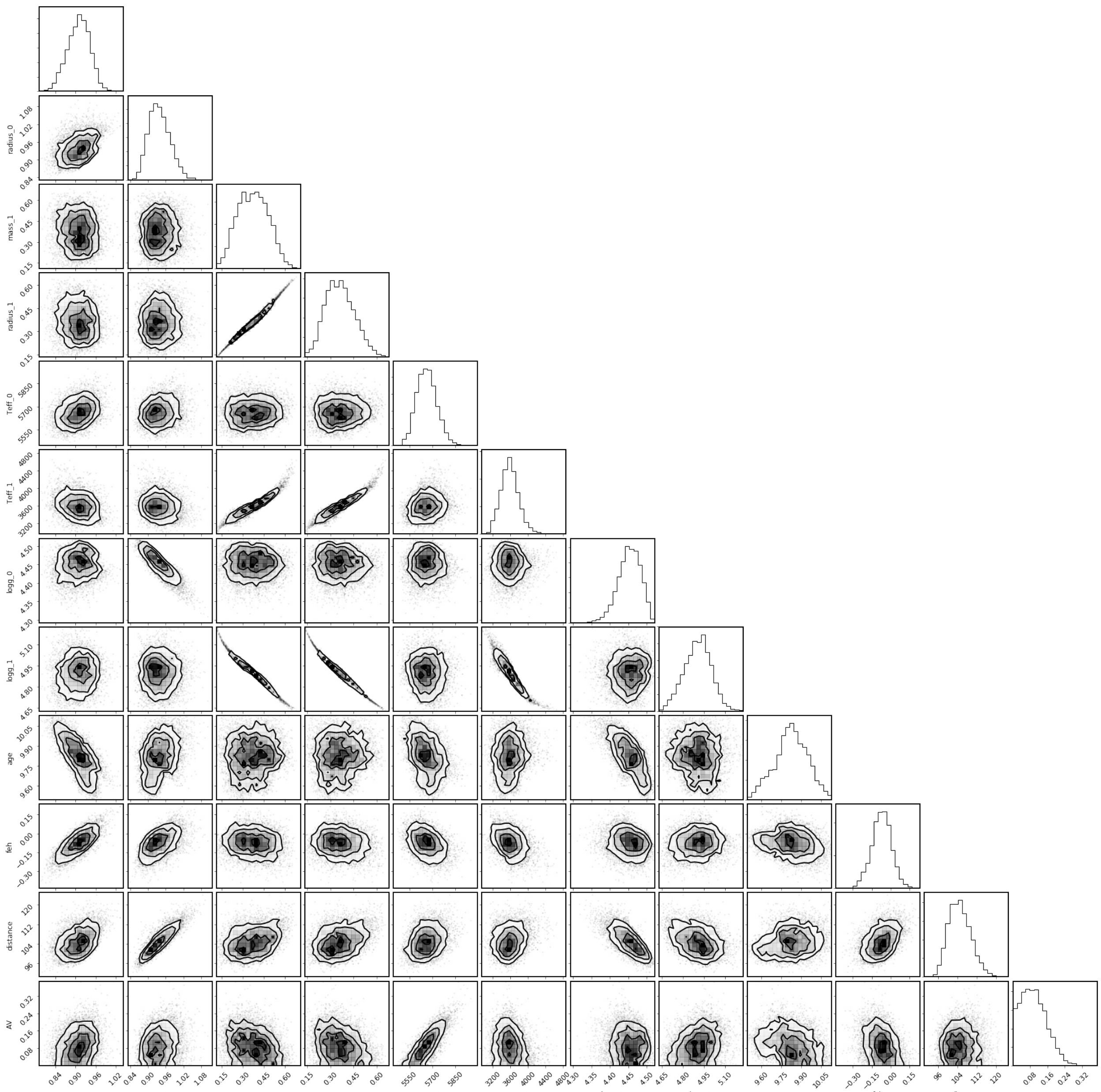


2.5 Fit a binary star model

[14]:
```python
from isochrones import BinaryStarModel
mod2 = BinaryStarModel(mist, **params)
```

[15]:
```python
mod2.fit()
mod2.corner_physical();
```

```
INFO:root:MultiNest basename: ./chains/mist-binary-
```



[16]:
```python
mod2.derived_samples.head()
```

[16]:
```
        eep_0       eep_1       age       feh   distance        AV  \
0  359.086356  249.801089  9.821476 -0.073397  98.727349  0.178347
```

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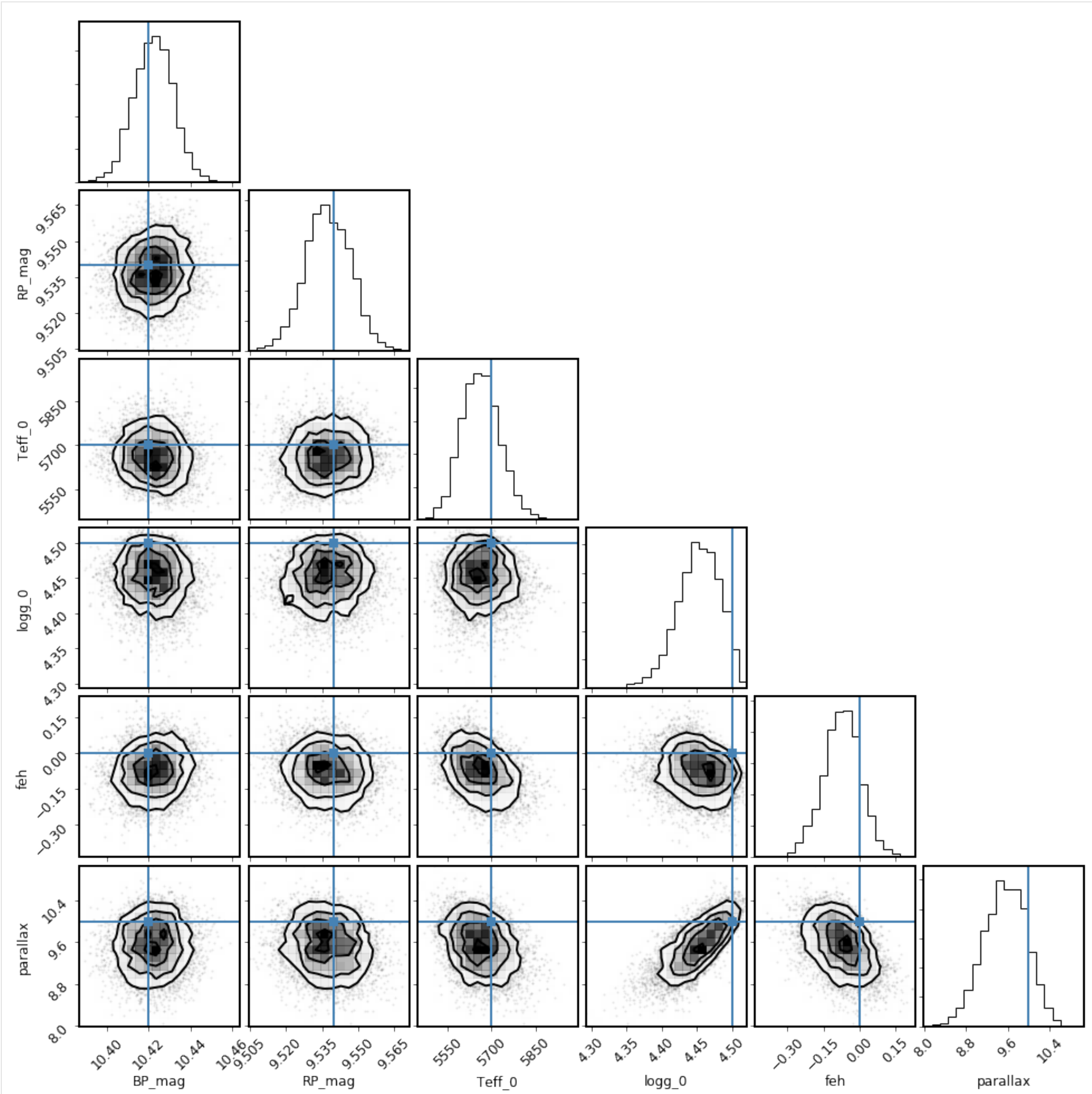
```
1  398.209362  283.317080  10.067148 -0.280222  107.778948  0.088950
2  389.035743  301.852188   9.971050 -0.207856  106.526665  0.166244
3  397.668498  266.911796   9.915806 -0.109190  120.644797  0.249099
4  370.982797  256.436383   9.816281 -0.205884  103.041704  0.385523

      lnprob       eep_0     feh_0    mass_0    ...        Mbol_1  \
0 -54.808864  359.086356 -0.104045  0.902009    ...     10.367668
1 -54.414634  398.209362 -0.372728  0.826538    ...      9.308760
2 -54.189376  389.035743 -0.278190  0.869004    ...      9.014070
3 -54.100849  397.668498 -0.176003  0.936127    ...      9.592630
4 -53.934142  370.982797 -0.260589  0.914513    ...      9.905946

   delta_nu_1      nu_max_1  phase_1  dm_deep_1   BP_mag_1   RP_mag_1  \
0  615.092182  16669.820345      0.0   0.006549  17.795078  15.260853
1  463.937102  12708.865085      0.0   0.001280  16.173813  14.156854
2  414.876808  11597.777825      0.0   0.002549  15.919824  13.882693
3  486.986772  13422.770521      0.0   0.006254  17.143644  14.860496
4  562.841780  15244.528561      0.0   0.007484  17.169701  14.890180

      BP_mag    RP_mag   parallax
0  10.463678  9.547117  10.128906
1  10.414967  9.566663   9.278250
2  10.391647  9.518239   9.387321
3  10.432474  9.553840   8.288795
4  10.439209  9.511726   9.704808

[5 rows x 44 columns]
```

[17]:
```
mod2.corner_observed();
```



RP_mag
Teff_0
logg_0
feh
parallax
BP_mag
RP_mag
Teff_0
logg_0
feh
parallax

CHAPTER 3

Interpolation: the `DFInterpolator`

Linear interpolation between gridded datapoints lies at the heart of much of what **isochrones** does. The custom `DFInterpolator` object manages this interpolation, implemented to optimize speed and convenience for large grids. A `DFInterpolator` is built on top of a pandas multi-indexed dataframe, and while designed with stellar model grids in mind, it can be used with any similarly structured data.

Let's demonstrate with a small example of data on a 2-dimensional grid.

[1]:
```
import itertools
import numpy as np
import pandas as pd

x = np.arange(1, 4)
y = np.arange(1, 6)

index = pd.MultiIndex.from_product((x, y), names=['x', 'y'])
df = pd.DataFrame(index=index)

df['sum'] = [x + y for x, y in itertools.product(x, y)]
df['product'] = [x * y for x, y in itertools.product(x, y)]
df['power'] = [x**y for x, y in itertools.product(x, y)]

df
```

[1]:
```
     sum  product  power
x y
1 1    2        1      1
  2    3        2      1
  3    4        3      1
  4    5        4      1
  5    6        5      1
2 1    3        2      2
  2    4        4      4
  3    5        6      8
  4    6        8     16
  5    7       10     32
```

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```
3 1    4         3      3
  2    5         6      9
  3    6         9     27
  4    7        12     81
  5    8        15    243
```

The DFInterpolator is initialized with this dataframe and then can interpolate the values of the columns at any location within the grid defined by the multiindex.

[2]:
```
from isochrones.interp import DFInterpolator

interp = DFInterpolator(df)
interp([1.4, 2.1])
```

[2]:
```
array([3.5 , 2.94, 2.36])
```

Individual columns may also be accessed by name:

[3]:
```
interp([2.2, 4.6], ['product'])
```

[3]:
```
array([10.12])
```

This object is very similar to the linear interpolation objects available in scipy, but it is significantly faster for single interpolation evaluations:

[4]:
```
from scipy.interpolate import RegularGridInterpolator

nx, ny = len(x), len(y)
grid = np.reshape(df['sum'].values, (nx, ny))
scipy_interp = RegularGridInterpolator([x, y], grid)

# Values are the same
assert(scipy_interp([1.3, 2.2])==interp([1.3, 2.2], ['sum']))

# Timings are different
%timeit scipy_interp([1.3, 2.2])
%timeit interp([1.3, 2.2])
```

```
10000 loops, best of 3: 176 µs per loop
The slowest run took 7.10 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
100000 loops, best of 3: 7.71 µs per loop
```

The DFInterpolator is about 30x faster than the scipy regular grid interpolation, for a single point. However, for vectorized calculations, scipy is indeed faster:

[5]:
```
N = 10000
pts = [1.3 * np.ones(N), 2.2 * np.ones(N)]
%timeit scipy_interp(np.array(pts).T)
%timeit interp(pts, ['sum'])
```

```
The slowest run took 7.51 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
100 loops, best of 3: 1.52 ms per loop
The slowest run took 30.75 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
1 loop, best of 3: 15.1 ms per loop
```

However, the `DFInterpolator` has an additional advantage of being able to manage missing data—that is, the grid doesn't have to be completely filled to construct the interpolator, as it does with scipy:

[6]:
```
df_missing = df.drop([(3, 3), (3, 4)])
df_missing
```

[6]:
```
     sum  product  power
x y
1 1    2        1      1
  2    3        2      1
  3    4        3      1
  4    5        4      1
  5    6        5      1
2 1    3        2      2
  2    4        4      4
  3    5        6      8
  4    6        8     16
  5    7       10     32
3 1    4        3      3
  2    5        6      9
  5    8       15    243
```

[7]:
```
interp_missing = DFInterpolator(df_missing)
interp_missing([1.3, 2.2])
```

[7]:
```
array([3.5 , 2.86, 2.14])
```

However, if the grid cell that the requested point is in is adjacent to one of these missing points, the interpolation will return nans:

[8]:
```
interp_missing([2.3, 3])
```

[8]:
```
array([nan, nan, nan])
```

In other words, the interpolator can be constructed with an incomplete grid, but it does not fill values for the missing points.

CHAPTER 4

Stellar model grids

4.1 Background and EEPs

Stellar model grids are typically constructed as a set of evolutionary tracks, where models of stellar evolution are run on grids of initial mass and metallicity, often with some other physical parameter varied as well (e.g., rotation, helium fraction, α-abundance, etc.). Each of these evolutionary tracks predicts various physical properties (temperature, luminosity, etc.) of a star with given initial mass and metallicity, as a function of age.

It is also often of interest to re-organize these evolution track grids into "isochrones"—sets of stars at a range of masses, all with the same age. As described in this reference, in order to construct these isochrones, the time axis of each evolution track gets mapped into a new coordinate, called "equivalent evolutionary phase," or EEP. The principle of the EEPs is to first identify physically significant stages in stellar evolution, and then subdivide each of these stages into a number of equal steps. This adaptive sampling enables accurate interpolation between evolution tracks even at ages when stars are evolving quickly, in the post-main sequence phases.

Previous versions of **isochrones** relied directly on these precomputed isochrone grids and interpolated between grid points in `(mass, age, feh)` space. This returned inaccurate results for post-MS stages of stellar evolution, and thus was not reliable for modeling evolved stars. However, beginning with v2.0, **isochrones** now implements all interpolation using EEPs. In addition, it provides direct access to the evolution track grids, in addition to precomputed isochrone grids. Note that version 2.0 includes only the MIST models; future updates will include more (e.g. PARSEC, YAPSI).

4.2 Model Grid Objects and Interpolation

Isochrones provides a simple and direct interface to full grids of stellar models. Upon first access, the grids are downloaded in original form, reorganized, and written to disk in binary format in order to load quickly with subsequent access. The grids are loaded as pandas dataframes with multi-level indexing that reflects the structure of the grids: evolution track grids are indexed by metallicity, initial mass, and EEP; and isochone grids by metallicity, age, and EEP.

[1]:
```
from isochrones.mist import MISTEvolutionTrackGrid, MISTIsochroneGrid

track_grid = MISTEvolutionTrackGrid()
track_grid.df.head()   # just show first few rows
```

[1]:
```
                                  nu_max      logg  eep  initial_mass  \
initial_feh initial_mass EEP
-4.0        0.1          1    143.524548  3.033277  1.0           0.1
                         2    145.419039  3.038935  2.0           0.1
                         3    147.409881  3.044805  3.0           0.1
                         4    149.499346  3.050886  4.0           0.1
                         5    151.703570  3.057203  5.0           0.1

                                radius   logTeff  mass   density      Mbol  \
initial_feh initial_mass EEP
-4.0        0.1          1    1.593804  3.620834   0.1  0.034823  5.132871
                         2    1.583455  3.620769   0.1  0.035510  5.147664
                         3    1.572790  3.620702   0.1  0.036237  5.163015
                         4    1.561817  3.620631   0.1  0.037006  5.178922
                         5    1.550499  3.620558   0.1  0.037823  5.195452

                              phase        feh         Teff      logL  \
initial_feh initial_mass EEP
-4.0        0.1          1     -1.0 -3.978406  4176.707371 -0.157148
                         2     -1.0 -3.978406  4176.085183 -0.163066
                         3     -1.0 -3.978406  4175.435381 -0.169206
                         4     -1.0 -3.978406  4174.757681 -0.175569
                         5     -1.0 -3.978406  4174.049081 -0.182181

                               delta_nu  interpolated      star_age       age  \
initial_feh initial_mass EEP
-4.0        0.1          1    21.776686         False  13343.289397  4.125263
                         2    21.993078         False  14171.978264  4.151430
                         3    22.219791         False  15048.910447  4.177505
                         4    22.457004         False  15975.827275  4.203463
                         5    22.706349         False  16962.744747  4.229496

                               dt_deep
initial_feh initial_mass EEP
-4.0        0.1          1    0.026168
                         2    0.026121
                         3    0.026016
                         4    0.025996
                         5    0.025996
```

[2]:
```
iso_grid = MISTIsochroneGrid()
iso_grid.df.head()   # just show first few rows
```

[2]:
```
                                   eep  age       feh      mass  initial_mass  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35     35  5.0 -3.978406  0.100000      0.100000
                            36     36  5.0 -3.978406  0.102885      0.102885
                            37     37  5.0 -3.978406  0.107147      0.107147
                            38     38  5.0 -3.978406  0.111379      0.111379
                            39     39  5.0 -3.978406  0.115581      0.115581

                                     radius   density   logTeff         Teff  \
log10_isochrone_age_yr feh  EEP
```

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```
5.0                  -4.0 35   1.106082  0.104184  3.617011  4140.105252
                          36   1.122675  0.102507  3.618039  4149.909661
                          37   1.147702  0.099921  3.619556  4164.436984
                          38   1.173015  0.097287  3.621062  4178.903372
                          39   1.198615  0.094627  3.622555  4193.289262

                                   logg      logL      Mbol   delta_nu  \
log10_isochrone_age_yr feh  EEP
5.0                  -4.0 35   3.350571 -0.489734  5.964335  37.987066
                          36   3.347798 -0.472691  5.921728  37.739176
                          37   3.343658 -0.447471  5.858678  37.345115
                          38   3.339612 -0.422498  5.796244  36.923615
                          39   3.335660 -0.397776  5.734440  36.473151

                                   nu_max  phase   dm_deep
log10_isochrone_age_yr feh  EEP
5.0                  -4.0 35   299.346079   -1.0  0.002885
                          36   298.570836   -1.0  0.003573
                          37   297.180748   -1.0  0.004247
                          38   295.526946   -1.0  0.004217
                          39   293.589960   -1.0  0.004189
```

This generally contains only a subset of the original columns provided by the underlying grid, with standardized names. There are also additional computed columns, such as stellar radius and density. The full, original grids, can be found with the `.df_orig` attribute if desired:

[3]:
```
iso_grid.df_orig.head()  # just show first few rows
```

[3]:
```
                               EEP  log10_isochrone_age_yr  initial_mass  \
log10_isochrone_age_yr feh  EEP
5.0                  -4.0 35    35                     5.0      0.100000
                          36    36                     5.0      0.102885
                          37    37                     5.0      0.107147
                          38    38                     5.0      0.111379
                          39    39                     5.0      0.115581

                               star_mass     star_mdot  he_core_mass  \
log10_isochrone_age_yr feh  EEP
5.0                  -4.0 35    0.100000 -1.455094e-13           0.0
                          36    0.102885 -1.562027e-13           0.0
                          37    0.107147 -1.707298e-13           0.0
                          38    0.111379 -1.836256e-13           0.0
                          39    0.115581 -1.949639e-13           0.0

                               c_core_mass  o_core_mass     log_L  \
log10_isochrone_age_yr feh  EEP
5.0                  -4.0 35           0.0          0.0 -0.489734
                          36           0.0          0.0 -0.472691
                          37           0.0          0.0 -0.447471
                          38           0.0          0.0 -0.422498
                          39           0.0          0.0 -0.397776

                               log_L_div_Ledd ...       nu_max  \
log10_isochrone_age_yr feh  EEP                ...
5.0                  -4.0 35       -4.167035 ...   299.346079
                          36       -4.129623 ...   298.570836
                          37       -4.073591 ...   297.180748
```

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```
                             38        -4.017245 ...   295.526946
                             39        -3.960633 ...   293.589960

                                  acoustic_cutoff  max_conv_vel_div_csound  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35       2233.536029                 0.127243
                            36       2228.014832                 0.128938
                            37       2218.440338                 0.130528
                            38       2207.403678                 0.132657
                            39       2194.776391                 0.134294

                                  max_gradT_div_grada  gradT_excess_alpha  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35              1.095544                 0.0
                            36              1.101114                 0.0
                            37              1.109114                 0.0
                            38              1.116760                 0.0
                            39              1.124050                 0.0

                                  min_Pgas_div_P  max_L_rad_div_Ledd  \
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35         0.999989            0.000016
                            36         0.999989            0.000017
                            37         0.999988            0.000019
                            38         0.999987            0.000021
                            39         0.999986            0.000022

                                     e_thermal  phase  feh
log10_isochrone_age_yr feh  EEP
5.0                    -4.0 35   3.002314e+46   -1.0 -4.0
                            36   3.106838e+46   -1.0 -4.0
                            37   3.264230e+46   -1.0 -4.0
                            38   3.424460e+46   -1.0 -4.0
                            39   3.587613e+46   -1.0 -4.0

[5 rows x 80 columns]
```

[4]:
```
iso_grid.df_orig.columns
```

[4]:
```
Index(['EEP', 'log10_isochrone_age_yr', 'initial_mass', 'star_mass',
       'star_mdot', 'he_core_mass', 'c_core_mass', 'o_core_mass', 'log_L',
       'log_L_div_Ledd', 'log_LH', 'log_LHe', 'log_LZ', 'log_Teff',
       'log_abs_Lgrav', 'log_R', 'log_g', 'log_surf_z', 'surf_avg_omega',
       'surf_avg_v_rot', 'surf_num_c12_div_num_o16', 'v_wind_Km_per_s',
       'surf_avg_omega_crit', 'surf_avg_omega_div_omega_crit',
       'surf_avg_v_crit', 'surf_avg_v_div_v_crit', 'surf_avg_Lrad_div_Ledd',
       'v_div_csound_surf', 'surface_h1', 'surface_he3', 'surface_he4',
       'surface_li7', 'surface_be9', 'surface_b11', 'surface_c12',
       'surface_c13', 'surface_n14', 'surface_o16', 'surface_f19',
       'surface_ne20', 'surface_na23', 'surface_mg24', 'surface_si28',
       'surface_s32', 'surface_ca40', 'surface_ti48', 'surface_fe56',
       'log_center_T', 'log_center_Rho', 'center_degeneracy', 'center_omega',
       'center_gamma', 'mass_conv_core', 'center_h1', 'center_he4',
       'center_c12', 'center_n14', 'center_o16', 'center_ne20', 'center_mg24',
       'center_si28', 'pp', 'cno', 'tri_alfa', 'burn_c', 'burn_n', 'burn_o',
       'c12_c12', 'delta_nu', 'delta_Pg', 'nu_max', 'acoustic_cutoff',
       'max_conv_vel_div_csound', 'max_gradT_div_grada', 'gradT_excess_alpha',
```

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```
       'min_Pgas_div_P', 'max_L_rad_div_Ledd', 'e_thermal', 'phase', 'feh'],
      dtype='object')
```

Any property (or properties) of these grids can be interpolated to any value of the index parameters via the `.interp` method:

```
[5]: track_grid.interp([-0.12, 1.01, 353.1], ['mass', 'radius', 'logg', 'Teff'])
```

```
[5]: array([1.00983180e+00, 1.04691913e+00, 4.40266419e+00, 6.03383320e+03])
```

Similarly, the `.interp_orig` method interpolates any of the original columns by name:

```
[6]: track_grid.interp_orig([-0.12, 1.01, 353.1], ['v_wind_Km_per_s'])
```

```
[6]: array([2.87408918e-05])
```

Note that these interpolations are fast—30-40x faster than the equivalent interpolation in scipy, for evaluating at a single point:

```
[7]: from scipy.interpolate import RegularGridInterpolator

grid = track_grid.interp.grid[:, :, :, 4]  # subgrid corresponding to radius
interp = RegularGridInterpolator(track_grid.interp.index_columns, grid)
assert track_grid.interp([-0.12, 1.01, 353.1], ['radius']) == interp([-0.12, 1.01,␣
↪353.1])
```

```
[8]: %timeit interp([-0.12, 1.01, 353.1])
%timeit track_grid.interp([-0.12, 1.01, 353.1], ['radius'])
```

```
The slowest run took 14.68 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
100 loops, best of 3: 1.13 ms per loop
The slowest run took 5.04 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
10000 loops, best of 3: 12.5 µs per loop
```

In order to select a subset of these grids, you can use pandas multi-index magic:

```
[9]: iso_grid.df.xs((9.0, 0.0), level=(0, 1)).head()  # just show first few rows
```

```
[9]:     eep  age       feh      mass  initial_mass    radius    density  \
EEP
193  193  9.0  0.042799  0.100000      0.100000  0.126216  70.115891
194  194  9.0  0.042805  0.103449      0.103449  0.129201  67.622476
195  195  9.0  0.042814  0.108103      0.108103  0.133343  64.282466
196  196  9.0  0.042824  0.112932      0.112932  0.137791  60.858291
197  197  9.0  0.042834  0.117543      0.117544  0.142177  57.660112

      logTeff         Teff      logg      logL       Mbol     delta_nu  \
EEP
193  3.460248  2885.680821  5.235913 -3.002129  12.245322  1045.120425
194  3.462649  2901.679870  5.227759 -2.972222  12.170556  1025.261668
195  3.465890  2923.411541  5.216743 -2.931855  12.069637   998.429682
196  3.469256  2946.160133  5.205249 -2.889887  11.964717   970.463953
197  3.472471  2968.048014  5.194272 -2.849811  11.864529   943.753111

          nu_max  phase   dm_deep
EEP
```

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```
193  27533.135930   -1.0  0.003449
194  27025.521973   -1.0  0.004051
195  26339.315728   -1.0  0.004742
196  25622.963627   -1.0  0.004720
197  24938.691941   -1.0  0.004735
```

4.3 Example visualization

Just for fun, let's plot a few isochrones:

[10]:
```
import hvplot.pandas

# Select two isochrones from the grid
iso_df1 = iso_grid.df.xs((9.0, 0.0), level=(0, 1))
iso_df2 = iso_grid.df.xs((9.5, 0.0), level=(0, 1))

options = dict(invert_xaxis=True, legend_position='bottom_left')

# Isn't hvplot/holoviews great?
plot1 = iso_df1.hvplot.line('logTeff', 'logL', label='Log(age) = 9.0')
plot2 = iso_df2.hvplot.line('logTeff', 'logL', label='Log(age) = 9.5')
(plot1 * plot2).options(**options)
```

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

[10]:
```
:Overlay
   .Curve.Log_left_parenthesis_age_right_parenthesis_equals_9_full_stop_0 :Curve   ␣
↪[logTeff]   (logL)
   .Curve.Log_left_parenthesis_age_right_parenthesis_equals_9_full_stop_5 :Curve   ␣
↪[logTeff]   (logL)
```

CHAPTER 5

Bolometric correction grids

Bolometric correction is defined as the difference between the apparent bolometric magnitude of a star and its apparent magnitude in a particular bandpass:

$$BC_x = m_{bol} - m_x$$

The MIST project provide grids of bolometric corrections in many photometric systems as a function of stellar temperature, surface gravity, metallicity, and A_V extinction. This allows for accurate conversion of bolometric magnitude of a star (available from the theoretical grids) to magnitude in any band, at any extinction (and distance), without the need for any "effective wavelength" approximation (used in **isochrones** prior to v2.0), which breaks down for broad bandpasses and large extinctions. These grids are downloaded, organized, stored, and interpolated in much the same manner as the model grids.

[1]:
```
from isochrones.mist.bc import MISTBolometricCorrectionGrid

bc_grid = MISTBolometricCorrectionGrid(['J', 'H', 'K', 'G', 'BP', 'RP', 'g', 'r', 'i
↪'])
```

[2]:
```
bc_grid.df.head()
```

[2]:
```
                                 g         r         i         J         H  \
Teff   logg [Fe/H] Av
2500.0 -4.0 -4.0   0.00 -6.534742 -3.332877 -1.617626  1.845781  2.927064
                   0.05 -6.590469 -3.375570 -1.650338  1.831466  2.917990
                   0.10 -6.646182 -3.418258 -1.683043  1.817153  2.908916
                   0.15 -6.701881 -3.460939 -1.715740  1.802841  2.899842
                   0.20 -6.757566 -3.503615 -1.748429  1.788530  2.890769

                                 K         G        BP        RP
Teff   logg [Fe/H] Av
2500.0 -4.0 -4.0   0.00  3.436304 -2.181986 -4.652544 -0.881255
                   0.05  3.430463 -2.211637 -4.697700 -0.909057
                   0.10  3.424623 -2.241240 -4.742838 -0.936829
                   0.15  3.418782 -2.270797 -4.787959 -0.964571
                   0.20  3.412942 -2.300306 -4.833062 -0.992285
```

```
[3]: bc_grid.interp.index_names
```

```
[3]: FrozenList(['Teff', 'logg', '[Fe/H]', 'Av'])
```

```
[4]: bc_grid.interp([5770, 4.44, 0.0, 0.], ['G', 'K'])
```

```
[4]: array([0.0819599 , 1.45398088])
```

The bandpasses provided to initialize the grid object are parsed according to the `.get_band` method, which returns the photometric system and the name of the band in the system:

```
[5]: bc_grid.get_band('G'), bc_grid.get_band('g')
```

```
[5]: (('UBVRIplus', 'Gaia_G_DR2Rev'), ('SDSSugriz', 'SDSS_g'))
```

Not all bands have cute nicknames to them, so you can also be explicit, e.g.:

```
[6]: bc_grid.get_band('DECam_g')
```

```
[6]: ('DECam', 'DECam_g')
```

See the implementation of `.get_band` for details.

CHAPTER 6

ModelGridInterpolator

In practice, interaction with the model grid and bolometric correction objects is easiest through a `ModelGridInterpolator` object, which brings the two together. This object is the replacement of the `Isochrone` object from previous generations of this package, though it has a slightly different API. It is mostly backward compatible, except for the removal of the `.mag` function dictionary for interpolating apparent magnitudes, this being replaced by the `.interp_mag` method.

6.1 Isochrones

An `IsochroneInterpolator` object takes `[EEP, log(age), feh]` as parameters.

[1]:
```
from isochrones.mist import MIST_Isochrone

mist = MIST_Isochrone()

pars = [353, 9.78, -1.24]  # eep, log(age), feh
mist.interp_value(pars, ['mass', 'radius', 'Teff'])
```

[1]:
```
array([7.93829519e-01, 7.91444054e-01, 6.30305932e+03])
```

To interpolate apparent magnitudes, add distance [pc] and A_V extinction as parameters.

[2]:
```
mist.interp_mag(pars + [200, 0.11], ['K', 'BP', 'RP'])  # Returns Teff, logg, feh,␣
↪mags
```

[2]:
```
(6303.059322477636,
 4.540738764316164,
 -1.377262817643937,
 array([10.25117074, 11.73997159, 11.06529993]))
```

6.2 Evolution tracks

Note that you can do the same using an `EvolutionTrackInterpolator` rather than an isochrone grid, using `[mass, EEP, feh]` as parameters:

```
[3]: from isochrones.mist import MIST_EvolutionTrack

     mist_track = MIST_EvolutionTrack()

     pars = [0.794, 353, -1.24]  # mass, eep, feh [matching above]
     mist_track.interp_value(pars, ['mass', 'radius', 'Teff', 'age'])
```

```
[3]: array([7.93843749e-01, 7.91818696e-01, 6.31006708e+03, 9.77929505e+00])
```

```
[4]: mist_track.interp_mag(pars + [200, 0.11], ['K', 'BP', 'RP'])
```

```
[4]: (6310.067080800683,
      4.54076772643659,
      -1.372925841944066,
      array([10.24893319, 11.73358578, 11.06056746]))
```

There are also convenience methods (for both isochrones and tracks) if you prefer (and for backward compatibility—note that the parameters must be unpacked, unlike the calls to `.interp_value` and `.interp_mag`), though it is slower to call multiple of these than to call `.interp_value` once with several desired outputs:

```
[5]: mist_track.mass(*pars)
```

```
[5]: array(0.79384375)
```

You can also get the dataframe of a single isochrone (interpolated to any age or metallicity) as follows:

```
[6]: mist.isochrone(9.53, 0.1).head()  # just show first few rows
```

```
[6]:      eep   age       feh      mass  initial_mass    radius    density  \
223  223.0  9.53  0.150280  0.143050      0.143050  0.174516  42.182044
224  224.0  9.53  0.150322  0.147584      0.147584  0.178799  40.088758
225  225.0  9.53  0.150371  0.152520      0.152521  0.183594  37.948464
226  226.0  9.53  0.150419  0.157318      0.157319  0.184463  37.208965
227  227.0  9.53  0.150468  0.161795      0.161796  0.189168  35.381629

      logTeff         Teff      logg    ...       H_mag     K_mag  \
223  3.477544  3003.536405  5.121475    ...    8.785652  8.559155
224  3.479902  3019.769652  5.112821    ...    8.713187  8.487450
225  3.482375  3036.910262  5.103613    ...    8.635963  8.411037
226  3.480519  3024.116433  5.101786    ...    8.629300  8.403586
227  3.482801  3040.176145  5.093340    ...    8.558717  8.333774

        G_mag     BP_mag     RP_mag    W1_mag    W2_mag    W3_mag   TESS_mag  \
223  12.766111  14.751368  11.522764  8.398324  8.200245  8.032482  11.381237
224  12.662468  14.612205  11.426131  8.327414  8.129809  7.964879  11.287794
225  12.552453  14.464800  11.323512  8.251886  8.054820  7.892865  11.188540
226  12.569050  14.507862  11.334820  8.243224  8.045057  7.881000  11.197325
227  12.467864  14.371759  11.240553  8.174286  7.976668  7.815386  11.106209

     Kepler_mag
223   12.864034
224   12.755405
225   12.640135
```

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```
226    12.660600
227    12.554499

[5 rows x 27 columns]
```

6.3 Generating synthetic properties

Often one wants to use stellar model grids to generate synthetic properties of stars. This can be done in a couple different ways, depending on what information you are able to provide. If you happen to have EEP values, you can use the fact that a `ModelGridInterpolator` is callable. Note that it takes the same parameters as all the other interpolation calls, with `distance` and `AV` as optional keyword parameters.

[7]:
```
from isochrones.mist import MIST_EvolutionTrack

mist_track = MIST_EvolutionTrack()
mist_track([0.8, 0.9, 1.0], 350, 0.0, distance=100, AV=0.1)
```

[7]:
```
        nu_max      logg    eep  initial_mass    radius   logTeff      mass  \
0  4254.629601  4.548780  350.0           0.8  0.787407  3.707984  0.799894
1  3622.320906  4.495440  350.0           0.9  0.888064  3.741043  0.899876
2  3041.107996  4.432089  350.0           1.0  1.006928  3.766249  0.999860

    density      Mbol  phase     ...          H_mag     K_mag      G_mag  \
0  2.309938  5.792554    0.0     ...       9.040105  8.972502  10.872154
1  1.811405  5.200732    0.0     ...       8.667003  8.614974  10.224076
2  1.380733  4.675907    0.0     ...       8.312159  8.270380   9.679997

      BP_mag     RP_mag    W1_mag    W2_mag    W3_mag   TESS_mag  Kepler_mag
0  11.328425  10.258543  8.945414  8.989254  8.921756  10.247984   10.773706
1  10.602874   9.678976  8.593946  8.622577  8.575349   9.671007   10.129692
2  10.005662   9.186910  8.253638  8.269467  8.238306   9.180275    9.590731

[3 rows x 29 columns]
```

Often, however, you will not know the EEP values at which you wish to simulate your synthetic population. In this case, you can use the `.generate()` method.

[8]:
```
mist_track.generate([0.81, 0.91, 1.01], 9.51, 0.01)
```

[8]:
```
        nu_max      logg      eep  initial_mass    radius   logTeff      mass  \
0  4787.598310  4.595858  320.808          0.81  0.750611  3.699978  0.809963
1  3986.671794  4.535170  332.280          0.91  0.853120  3.737424  0.909935
2  3154.677953  4.447853  343.800          1.01  0.993830  3.766201  1.009887

    density      Mbol  phase    ...            H         K         G  \
0  2.703461  5.977047    0.0    ...     4.154396  4.088644  5.988091
1  2.066995  5.324246    0.0    ...     3.747329  3.699594  5.264620
2  1.451510  4.705019    0.0    ...     3.322241  3.286761  4.620132

         BP        RP        W1        W2        W3      TESS    Kepler
0  6.444688  5.375415  4.066499  4.117992  4.047535  5.365712  5.887722
1  5.632088  4.731978  3.684034  3.718112  3.670736  4.725020  5.169229
2  4.925805  4.148936  3.276062  3.295002  3.266166  4.143362  4.531319

[3 rows x 29 columns]
```

Under the hood, `.generate()` uses an interpolation step to approximate the eep value(s) corresponding to the requested value(s) of mass, age, and metallicity:

[9]:
```
mist_track.get_eep(1.01, 9.51, 0.01)
```

[9]:
```
343.8
```

Because this is fast, it is pretty inexpensive to generate a population of stars with given properties:

[10]:
```
import numpy as np

N = 10000
mass = np.ones(N) * 1.01
age = np.ones(N) * 9.82
feh = np.ones(N) * 0.02
%timeit mist_track.generate(mass, age, feh)
```

```
10 loops, best of 3: 112 ms per loop
```

Note though, that this interpolation doesn't do great for evolved stars (this is the fundamental reason why **isochrones** always fits with EEP as one of the parameters). However, if you do want to compute more precise EEP values for given physical properties, you can set the `accurate` keyword parameter, which performs a function minimization:

[11]:
```
mist_track.get_eep(1.01, 9.51, 0.01, accurate=True)
```

[11]:
```
343.1963539123535
```

This is more accurate, but slow because it is actually performing a function minimization:

[12]:
```
%timeit mist_track.get_eep(1.01, 9.51, 0.01, accurate=True)
%timeit mist_track.get_eep(1.01, 9.51, 0.01)
```

```
100 loops, best of 3: 4.56 ms per loop
The slowest run took 4.98 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
100000 loops, best of 3: 4.26 µs per loop
```

Here we can see the effect of accuracy by plugging back in the estimated EEP into the interpolation:

[13]:
```
[mist_track.interp_value([1.01, e, 0.01], ['age']) for e in [343.8, 343.
↪1963539123535]]
```

[13]:
```
[array([9.51806019]), array([9.50999994])]
```

So if accuracy is required, definitely use `accurate=True`, but for most purposes, the default should be fine. You can request that `.generate()` run in "accurate" mode, which uses this more expensive EEP computation (it will be correspondingly slower).

[14]:
```
mist_track.generate([0.81, 0.91, 1.01], 9.51, 0.01, accurate=True)
```

[14]:
```
        nu_max      logg         eep  initial_mass    radius   logTeff  \
0  4794.035436  4.596385  320.219650          0.81  0.750156  3.699863
1  3995.692509  4.536089  331.721363          0.91  0.852218  3.737300
2  3168.148566  4.449647  343.196354          1.01  0.991781  3.766083

       mass   density      Mbol  phase    ...            H         K  \
0  0.809963  2.708365  5.979507    0.0    ...     4.156117  4.090301
1  0.909936  2.073560  5.327785    0.0    ...     3.750018  3.702214
2  1.009890  1.460523  4.710671    0.0    ...     3.327067  3.291533
```

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```
          G         BP         RP         W1         W2         W3       TESS  \
0  5.990784  6.447681  5.377849  4.068141  4.119700  4.049167  5.368138
1  5.268320  5.636100  4.735394  3.686635  3.720795  3.673334  4.728428
2  4.625783  4.931724  4.154311  3.280826  3.299859  3.270940  4.148735

     Kepler
0  5.890400
1  5.172899
2  4.536929

[3 rows x 29 columns]
```

Just for curiosity, let's look at the difference in the predictions:

[15]:
```
df0 = mist_track.generate([0.81, 0.91, 1.01], 9.51, 0.01, accurate=True)
df1 = mist_track.generate([0.81, 0.91, 1.01], 9.51, 0.01)
((df1 - df0) / df0).mean()
```

[15]:
```
nu_max          -0.002617
logg            -0.000240
eep              0.001760
initial_mass     0.000000
radius           0.001243
logTeff          0.000032
mass            -0.000002
density         -0.003716
Mbol            -0.000759
phase                 NaN
feh             -0.057173
Teff             0.000273
logL             0.061576
delta_nu        -0.001803
interpolated          NaN
star_age         0.018487
age              0.000837
dt_deep         -0.007171
J               -0.000848
H               -0.000861
K               -0.000854
G               -0.000791
BP              -0.000792
RP              -0.000823
W1              -0.000854
W2              -0.000869
W3              -0.000857
TESS            -0.000823
Kepler          -0.000800
dtype: float64
```

Not too bad, for this example!

6.4 Demo: Visualize

Now let's make sure that interpolated isochrones fall nicely between ones that are actually part of the grid. In order to execute this code, you will need to

```
conda install -c pyviz pyviz
```

and to execute in JupyterLab, you will need to

```
jupyter labextension install @pyviz/jupyterlab_pyviz
```

[16]:
```
import hvplot.pandas

iso1 = mist.model_grid.df.xs((9.5, 0.0), level=(0, 1))    # extract subgrid at log_
↪age=9.5, feh=0.0
iso2 = mist.model_grid.df.xs((9.5, 0.25), level=(0, 1))  # extract subgrid at log_
↪age=9.5, feh=0.25
iso3 = mist.isochrone(9.5, 0.12)  # should be between the other two

plot1 = iso1.hvplot.line('logTeff', 'logL', label='[Fe/H] = 0.0')
plot2 = iso2.hvplot.line('logTeff', 'logL', label='[Fe/H] = 0.25')
plot3 = iso3.hvplot.line('logTeff', 'logL', label='[Fe/H] = 0.12')

(plot1 * plot2 * plot3).options(invert_xaxis=True, legend_position='bottom_left',␣
↪width=600)
```

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

[16]:
```
:Overlay
   .Curve.Left_square_bracket_Fe_over_H_right_square_bracket_equals_0_full_stop_0  :
↪Curve   [logTeff]   (logL)
   .Curve.Left_square_bracket_Fe_over_H_right_square_bracket_equals_0_full_stop_25 :
↪Curve   [logTeff]   (logL)
   .Curve.Left_square_bracket_Fe_over_H_right_square_bracket_equals_0_full_stop_12 :
↪Curve   [logTeff]   (logL)
```

CHAPTER 7

Fitting stellar parameters

The central purpose of **isochrones** is to infer the physical properties of stars given arbitrary observations. This is accomplished via the `StarModel` object. For simplest usage, a `StarModel` is initialized with a `ModelGridInterpolator` and observed properties, provided as `(value, uncertainty)` pairs. Also, while the vanilla `StarModel` object (which is mostly the same as the **isochrones** v1 `StarModel` object) can still be used to fit a single star, **isochrones** v2 has a new `SingleStarModel` available, that has a more optimized likelihood implementation, for significantly faster inference.

7.1 Defining a star model

First, let's generate some "observed" properties according to the model grids themselves. Remember that `.generate()` only works with the evolution track interpolator.

[1]:
```
from isochrones.mist import MIST_EvolutionTrack, MIST_Isochrone

track = MIST_EvolutionTrack()

mass, age, feh, distance, AV = 1.0, 9.74, -0.05, 100, 0.02

# Using return_dict here rather than return_df, because we just want scalar values
true_props = track.generate(mass, age, feh, distance=distance, AV=AV, return_
↪dict=True)
true_props
```

[1]:
```
{'nu_max': 2617.5691700617886,
 'logg': 4.370219109480715,
 'eep': 380.0,
 'initial_mass': 1.0,
 'radius': 1.0813017873811603,
 'logTeff': 3.773295968705084,
 'mass': 0.9997797219140423,
 'density': 1.115827651504971,
 'Mbol': 4.4508474939623826,
```

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```
 'phase': 0.0,
 'feh': -0.09685557997282962,
 'Teff': 5934.703385987951,
 'logL': 0.11566100241504726,
 'delta_nu': 126.60871562200438,
 'interpolated': 0.0,
 'star_age': 5522019067.711771,
 'age': 9.74119762492735,
 'dt_deep': 0.0036991465241712263,
 'J': 8.435233804866742,
 'H': 8.124109062114325,
 'K': 8.09085566863133,
 'G': 9.387465543790636,
 'BP': 9.680097761608252,
 'RP': 8.928888526297722,
 'W1': 8.079124865544092,
 'W2': 8.090757185192754,
 'W3': 8.06683507215844,
 'TESS': 8.923262483762786,
 'Kepler': 9.301490687837552}
```

Now, we can define a starmodel with these "observations", this time using the isochrone grid interpolator. We use the optimized `SingleStarModel` object.

[2]:
```
from isochrones import SingleStarModel, get_ichrone

mist = get_ichrone('mist')

uncs = dict(Teff=80, logg=0.1, feh=0.1, phot=0.02)
props = {p: (true_props[p], uncs[p]) for p in ['Teff', 'logg', 'feh']}
props.update({b: (true_props[b], uncs['phot']) for b in 'JHK'})

# Let's also give an appropriate parallax, in mas
props.update({'parallax': (1000./distance, 0.1)})

mod = SingleStarModel(mist, name='demo', **props)
```

And we can see the prior, likelihood, and posterior at the true parameters:

[3]:
```
eep = mist.get_eep(mass, age, feh, accurate=True)
pars = [eep, age, feh, distance, AV]

mod.lnprior(pars), mod.lnlike(pars), mod.lnpost(pars)
```

[3]:
```
(-23.05503287088296, -20.716150242083508, -43.77118311296647)
```

If we stray from these parameters, we can see the likelihood decrease:

[4]:
```
pars2 = [eep + 3, age - 0.05, feh + 0.02, distance, AV]
mod.lnprior(pars2), mod.lnlike(pars2), mod.lnpost(pars2)
```

[4]:
```
(-23.251706955307853, -85.08590699022739, -108.33761394553524)
```

How long does a posterior evaluation take?

[5]:
```
%timeit mod.lnpost(pars)
```

```
1000 loops, best of 3: 369 µs per loop
```

[6]:
```
from isochrones import BinaryStarModel

mod2 = BinaryStarModel(mist, **props)
```

[7]:
```
pars2 = [eep, eep - 20, age, feh, distance, AV]
%timeit mod2.lnpost(pars2)
```

```
The slowest run took 373.39 times longer than the fastest. This could mean that an␣
↪intermediate result is being cached.
1000 loops, best of 3: 429 µs per loop
```

[8]:
```
from isochrones import TripleStarModel
mod3 = TripleStarModel(mist, **props)
pars3 = [eep, eep-20, eep-40, age, feh, distance, AV]
%timeit mod3.lnpost(pars3)
```

```
1000 loops, best of 3: 541 µs per loop
```

7.2 Priors

As you may have noticed, we have not explictly defined any priors on our parameters. They were defined for you, but you may wish to know what they are, and/or to change them.

[9]:
```
mod._priors
```

[9]:
```
{'mass': <isochrones.priors.ChabrierPrior at 0x1c47e270f0>,
 'feh': <isochrones.priors.FehPrior at 0x1c47e27358>,
 'age': <isochrones.priors.AgePrior at 0x1c47e27748>,
 'distance': <isochrones.priors.DistancePrior at 0x1c47e27390>,
 'AV': <isochrones.priors.AVPrior at 0x1c47e27400>,
 'eep': <isochrones.priors.EEP_prior at 0x1c47e274e0>}
```

You can sample from these priors:

[10]:
```
samples = mod.sample_from_prior(1000)
samples
```

[10]:
```
          age       feh     distance        AV   eep
0    9.775384  0.004928  9585.312354  0.058294   415
1    9.690678  0.318313  5460.742158  0.212007   295
2    9.317426 -0.008935  5381.226921  0.144259   265
3    9.721345 -0.131058  7867.502875  0.228851   295
4    9.374286 -0.325079  9590.728624  0.954659   350
5    9.293293  0.229220  9574.055273  0.713866   293
6    9.941975  0.178338  7788.336554  0.100102   272
7    9.436477  0.231631  9148.585364  0.204715   314
8    9.743647 -0.396267  6767.456426  0.383272   460
9    9.607588 -0.236938  4317.243131  0.795216   327
10  10.057273 -0.236801  9031.515061  0.488995   314
11   9.645887 -0.338786  9055.303060  0.408045  1702
12   9.997611 -0.214726  8452.833760  0.398581   327
13   9.926111 -0.063765  9726.761618  0.544188   321
14   9.845896 -0.106017  9148.167681  0.455272   292
```

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```
15     9.761015 -0.051480  8247.211954  0.379722   253
16     9.286601 -0.186755  7113.433648  0.504276   451
17     8.124025 -0.138695  9335.360257  0.445926   380
18     9.357514  0.067101  7737.470105  0.752912   300
19     9.595645  0.107345  7115.589991  0.231624   420
20     9.933118 -0.048234  8424.490753  0.139511  1692
21    10.105196  0.292670  9485.836532  0.366055   331
22    10.040531  0.014999  9296.367644  0.185203   314
23     9.967137  0.000672  8552.101985  0.791577   254
24    10.128790 -0.176654  6727.495813  0.494750   345
25     9.854273 -0.183929  5380.753272  0.945978   407
26     9.808855  0.074433  8664.219066  0.692098   455
27     9.917164  0.202846  7496.956324  0.363808   389
28     9.630302 -0.373880  8555.632299  0.287120   322
29     9.179908 -0.199734  9856.026771  0.303459   300
..          ...       ...          ...       ...   ...
970   10.086391  0.106938  7592.165249  0.180709   314
971   10.085083 -0.036016  7027.634747  0.314156   299
972    9.287464  0.265288  6042.848103  0.144724   402
973    9.955503 -0.080510  6262.492050  0.611200   327
974    9.616142 -0.342509  5069.839242  0.567513   349
975   10.075706 -0.130103  5829.432844  0.892020   299
976   10.028535  0.193184  4257.798238  0.293645   329
977    9.574676  0.085395  9752.502653  0.703944   357
978    9.556853 -0.140753  8530.526505  0.871235   334
979    9.821810 -0.118098  8972.965633  0.026728   397
980   10.105103  0.104010  9992.769437  0.343932   292
981    9.853596 -0.118408  6035.299187  0.686813   254
982    9.312975 -0.038113  8689.991781  0.047170   324
983    8.971418  0.238024  5797.572151  0.773175   268
984    9.664546 -0.028603  9719.254429  0.707218   347
985    9.924745 -0.216946  9422.814918  0.292175   292
986    9.624291 -0.034933  4359.825182  0.661057   317
987    9.947794 -0.264508  8355.572420  0.301372   292
988    9.766796  0.070148  9155.900363  0.597846   292
989    9.960194  0.026427  7655.336536  0.002166   265
990    9.488527 -0.094431  9896.426901  0.662185   271
991   10.105075  0.145627  3359.853867  0.416843   489
992    9.994699 -0.246844  6033.657596  0.198885   271
993    9.369871  0.052412  2669.191340  0.294969   317
994    9.903893  0.176871  8832.480953  0.128152   295
995    9.864971  0.142656  9366.389176  0.782361   347
996    9.968736  0.027372  8748.808096  0.300267   351
997    9.525780  0.005426  6651.084393  0.508013   247
998    9.290521 -0.064370  8489.972371  0.615413   296
999   10.131466  0.077783  8694.197822  0.833533   271

[1000 rows x 5 columns]
```

Remember, these are the fit parameters:

[11]:
```
mod.param_names
```

[11]:
```
('eep', 'age', 'feh', 'distance', 'AV')
```

Let's turn this into a dataframe, and visualize it.

[12]:
```
import pandas as pd
import holoviews as hv
import hvplot.pandas
hv.extension('bokeh')

def plot_samples(samples):
    df = pd.DataFrame(samples, columns=['eep', 'age', 'feh', 'distance', 'AV'])
    df['mass'] = mod.ic.interp_value([df.eep, df.age, df.feh], ['mass'])
    return hv.Layout([df.hvplot.hist(c).options(width=300) for c in df.columns]).
↪cols(3)

plot_samples(samples)
```

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

[12]:
```
:Layout
   .Histogram.I   :Histogram   [eep]   (eep_frequency)
   .Histogram.II  :Histogram   [age]   (age_frequency)
   .Histogram.III :Histogram   [feh]   (feh_frequency)
   .Histogram.IV  :Histogram   [distance]   (distance_frequency)
   .Histogram.V   :Histogram   [AV]   (AV_frequency)
   .Histogram.VI  :Histogram   [mass]   (mass_frequency)
```

Note that there are some built-in defaults here to be aware of. The metallicity distribution is based on a local metallicity prior from SDSS, the distance prior has a maximum distance of 10kpc, and AV is flat from 0 to 1. Now, let's change our distance prior to only go out to 1000pc, and our metallicity distribution to be flat between -2 and 0.5.

[13]:
```
from isochrones.priors import FlatPrior, DistancePrior
mod.set_prior(feh=FlatPrior((-2, 0.5)), distance=DistancePrior(1000))
```

[14]:
```
plot_samples(mod.sample_from_prior(1000))
```

[14]:
```
:Layout
   .Histogram.I   :Histogram   [eep]   (eep_frequency)
   .Histogram.II  :Histogram   [age]   (age_frequency)
   .Histogram.III :Histogram   [feh]   (feh_frequency)
   .Histogram.IV  :Histogram   [distance]   (distance_frequency)
   .Histogram.V   :Histogram   [AV]   (AV_frequency)
   .Histogram.VI  :Histogram   [mass]   (mass_frequency)
```

Also note that the default mass prior is the Chabrier broken powerlaw, which is nifty:

[15]:
```
pd.Series(mod._priors['mass'].sample(10000), name='mass').hvplot.hist(bins=100, bin_
↪range=(0, 5))
```

[15]:
```
:Histogram   [mass]   (mass_frequency)
```

You can also define a metallicity prior to have a different mix of halo and (local) disk:

[16]:
```
from isochrones.priors import FehPrior

pd.Series(FehPrior(halo_fraction=0.5).sample(10000), name='feh').hvplot.hist()
```

[16]:
```
:Histogram   [feh]   (feh_frequency)
```

7.3 Sampling the posterior

Once you have defined your stellar model and are happy with your priors, you may either execute your optimization/sampling method of choice using the `.lnpost()` method as your posterior, or you may use the built-in **MultiNest** fitting routine with `.fit()`. One thing to note especially is that the **MultiNest** chains get automatically created in a `chains` subdirectory from wherever you execute `.fit()`, with a basename for the files that you can access with:

[17]:
```
mod.mnest_basename
```

[17]:
```
'./chains/demo-mist-single-'
```

This can be changed or overwritten in two ways, which is often a good idea to avoid clashes between different fits with the same default basename. You can either by pass an explicit `basename` keyword to `.fit()`, or you can set a name attribute, as we did when initializing this model. OK, now we will run the fit. This will typically take a few minutes (unless the chains for the fit have already completed, in which case it will be read in and finish quickly).

[18]:
```
mod.fit()
```

```
INFO:root:MultiNest basename: ./chains/demo-mist-single-
```

The posterior samples of the sampling parameters are available in the `.samples` attribute. Note that this is different from the original vanilla `StarModel` object (the one fully backward-compatible with **isochrones** v1), which contained both sampling parameters and derived parameters at the values of those samples.

[19]:
```
mod.samples.head()
```

[19]:

	eep	age	feh	distance	AV	lnprob
0	306.566211	8.867352	-0.084787	99.754595	0.128567	-51.124447
1	385.106002	9.744207	0.179155	99.818131	0.492972	-49.729296
2	301.018106	8.745846	-0.030370	100.473273	0.579248	-49.425361
3	259.680008	8.214644	0.010053	98.376332	0.363801	-48.479515
4	380.210824	9.700131	-0.178482	99.398149	0.633511	-48.453800

[20]:
```
mod.samples.describe()
```

[20]:

	eep	age	feh	distance	AV \
count	5344.000000	5344.000000	5344.000000	5344.000000	5344.000000
mean	373.193478	9.686096	-0.045856	100.021338	0.146828
std	19.800711	0.181541	0.076720	0.993574	0.111217
min	217.053516	7.653568	-0.301781	96.356886	0.000078
25%	359.164345	9.599845	-0.098619	99.355457	0.060500
50%	375.054046	9.712589	-0.045498	100.013786	0.124330
75%	387.980876	9.806347	0.007986	100.685282	0.208648
max	420.506604	10.089448	0.193737	103.660815	0.755101

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```
          lnprob
count  5344.000000
mean    -40.820994
std       1.525383
min     -51.124447
25%     -41.532015
50%     -40.468810
75%     -39.723250
max     -38.472525
```

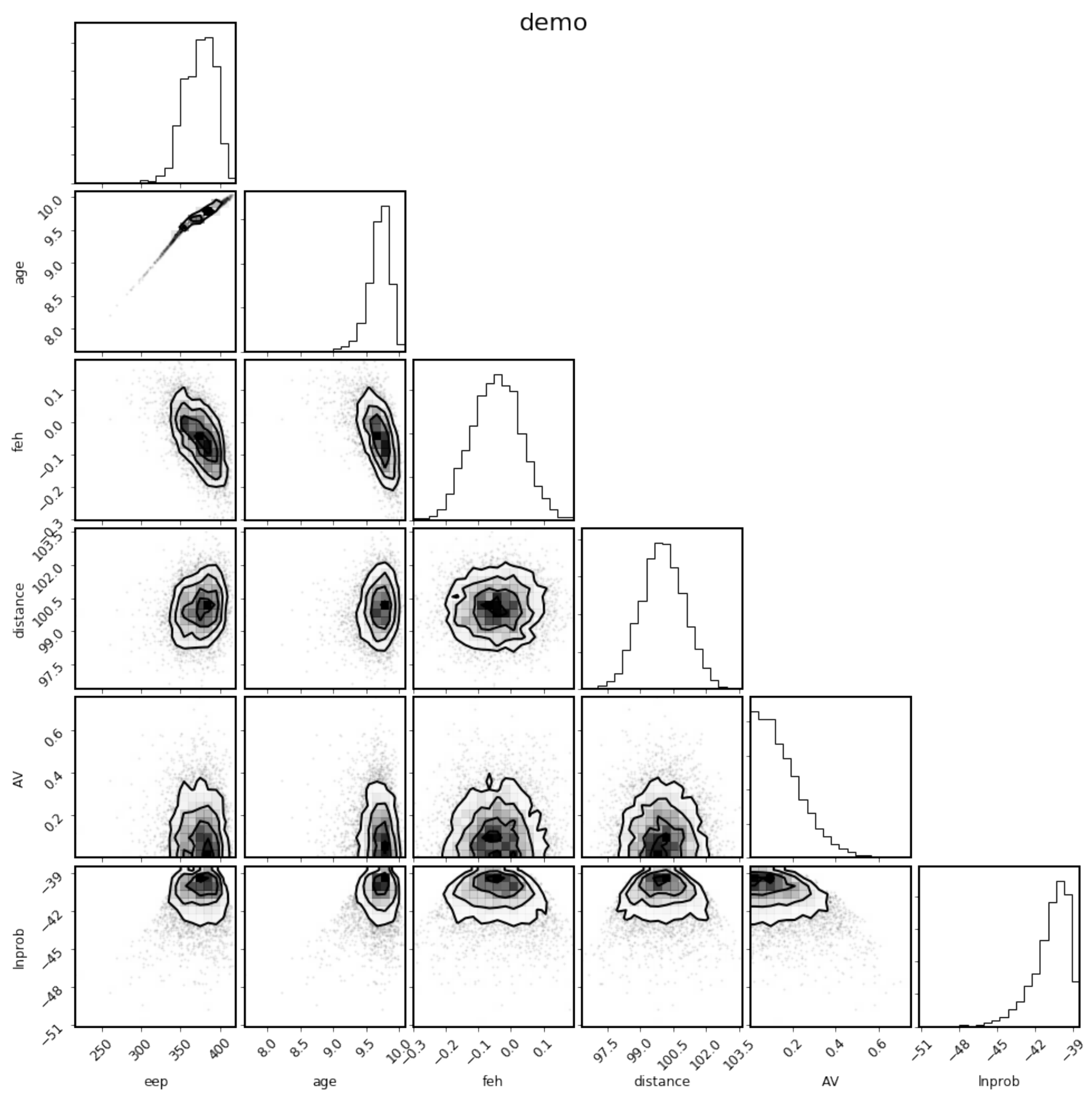
The derived parameters are available in `.derived_samples` (`StarModel` on its own does not have this attribute):

[21]:
```
mod.derived_samples.head()
```

[21]:
```
          eep       age       feh      mass  initial_mass    radius   density  \
0  306.566211  8.867352 -0.068345  1.098369      1.098407  1.037567  1.391965
1  385.106002  9.744207  0.159727  1.057168      1.057394  1.141433  1.002494
2  301.018106  8.745846 -0.011900  1.152686      1.152721  1.096043  1.237448
3  259.680008  8.214644  0.048588  1.147012      1.147023  1.065911  1.338449
4  380.210824  9.700131 -0.257214  1.010634      1.010881  1.123263  1.005285

    logTeff         Teff      logg    ...        BP_mag    RP_mag    W1_mag  \
0  3.790660  6176.198702  4.447143    ...      9.673531  8.942160  8.124458
1  3.763116  5796.649384  4.347338    ...     10.181673  9.176220  8.004902
2  3.796728  6262.723700  4.420403    ...      9.982068  9.077651  8.030137
3  3.790769  6177.197873  4.442458    ...      9.830745  8.993534  8.044905
4  3.787708  6134.335252  4.341725    ...     10.085126  9.130129  7.986850

     W2_mag    W3_mag  TESS_mag  Kepler_mag   parallax    distance        AV
0  8.127637  8.109427  8.936103    9.307189  10.024601   99.754595  0.128567
1  8.018791  7.972552  9.164784    9.669073  10.018220   99.818131  0.492972
2  8.021413  7.994322  9.066848    9.529013   9.952896  100.473273  0.579248
3  8.045347  8.020660  8.985118    9.409833  10.165047   98.376332  0.363801
4  7.974103  7.941984  9.117792    9.608276  10.060550   99.398149  0.633511

[5 rows x 30 columns]
```

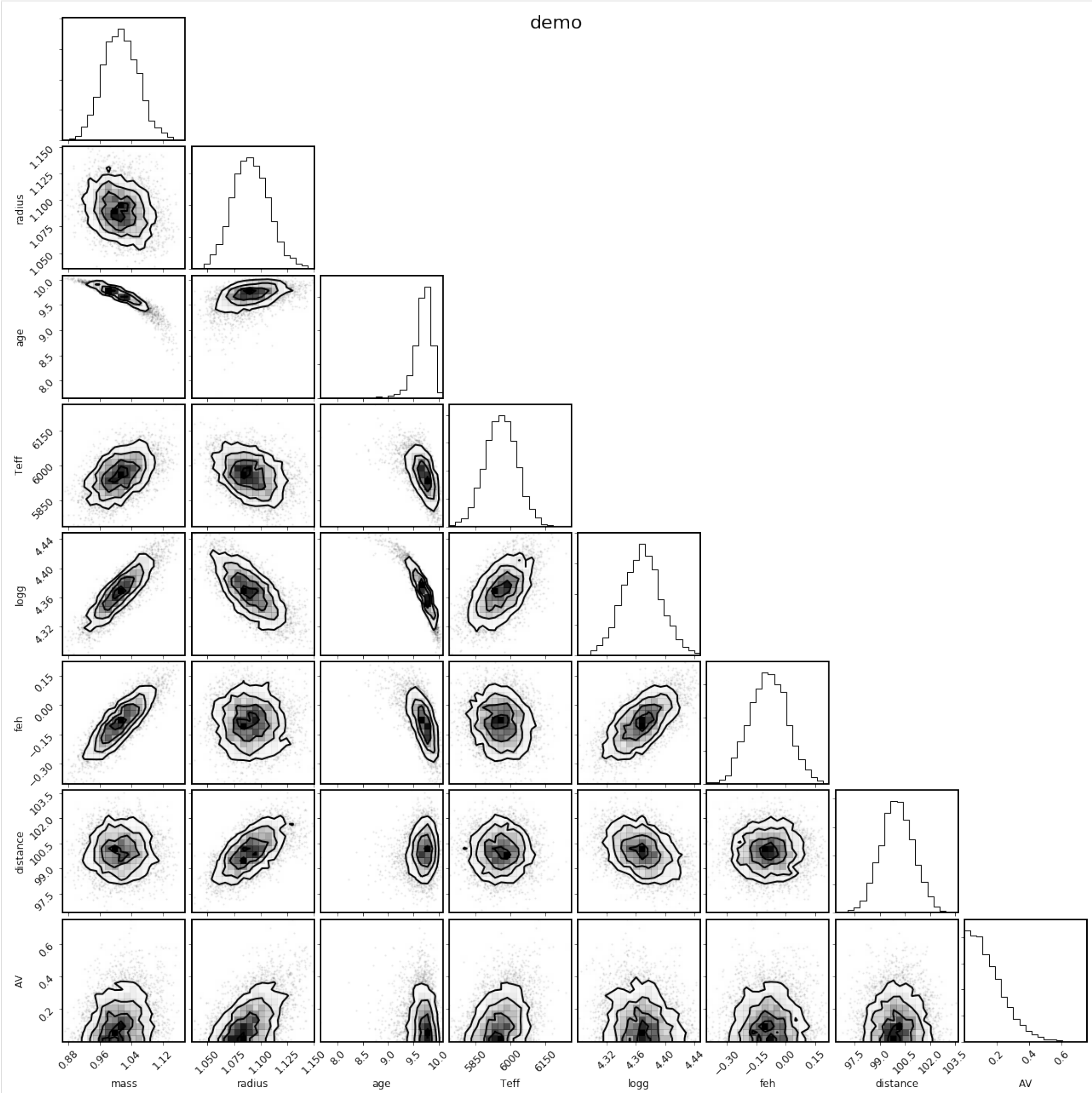
You can make a corner plot of the fit parameters as follows:

[22]:
```
%matplotlib inline

mod.corner_params();   # Note, this is also new in v2.0, for the SingleStarModel object
```



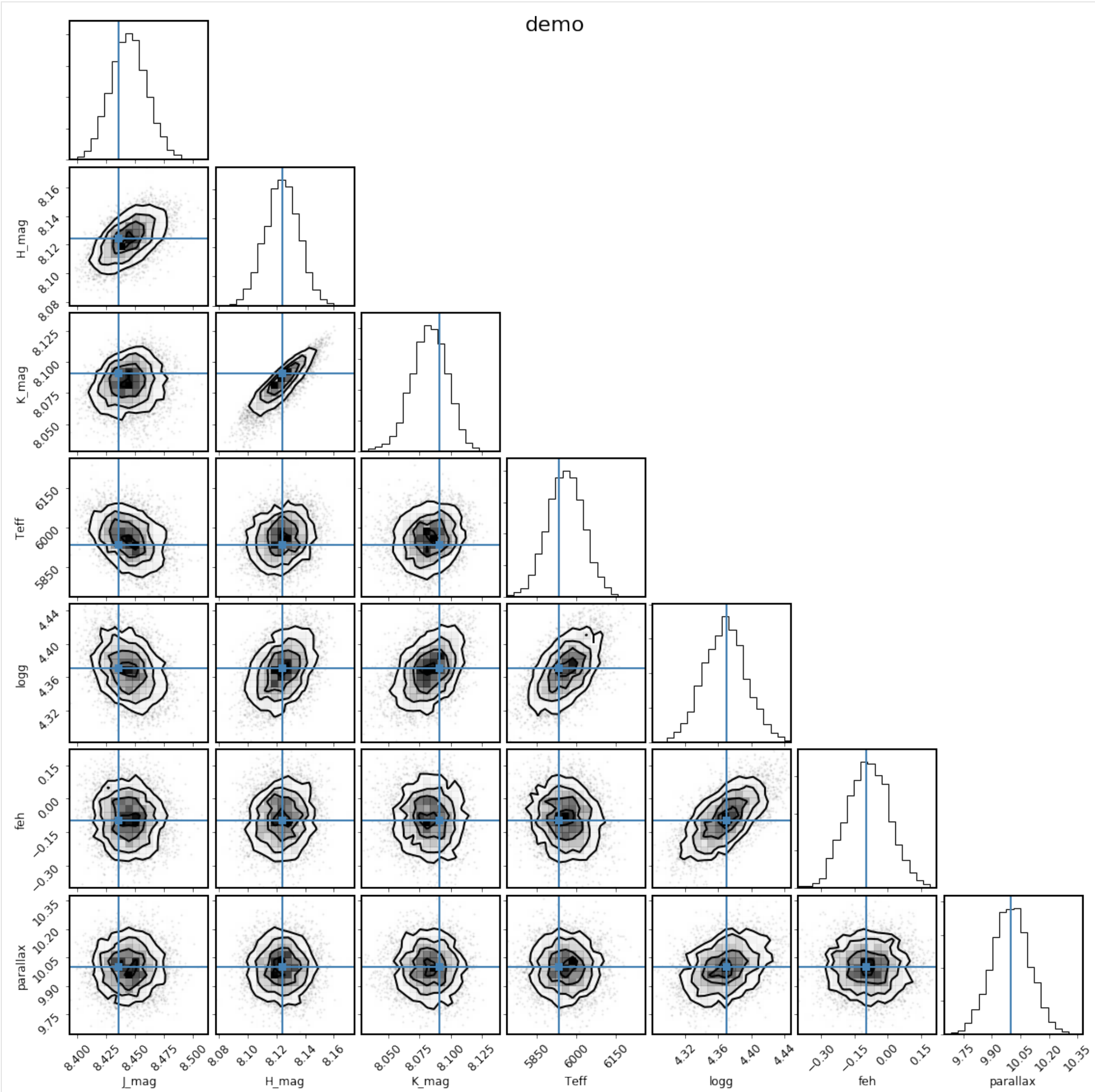
There is also a convenience method to select the parameters of physical interest.

[23]:
```
mod.corner_physical();
```



It can also be instructive to see how the derived samples of the observed parameters compare to the observations themselves; the shortcut to this is with the `.corner_observed()` convenience method:

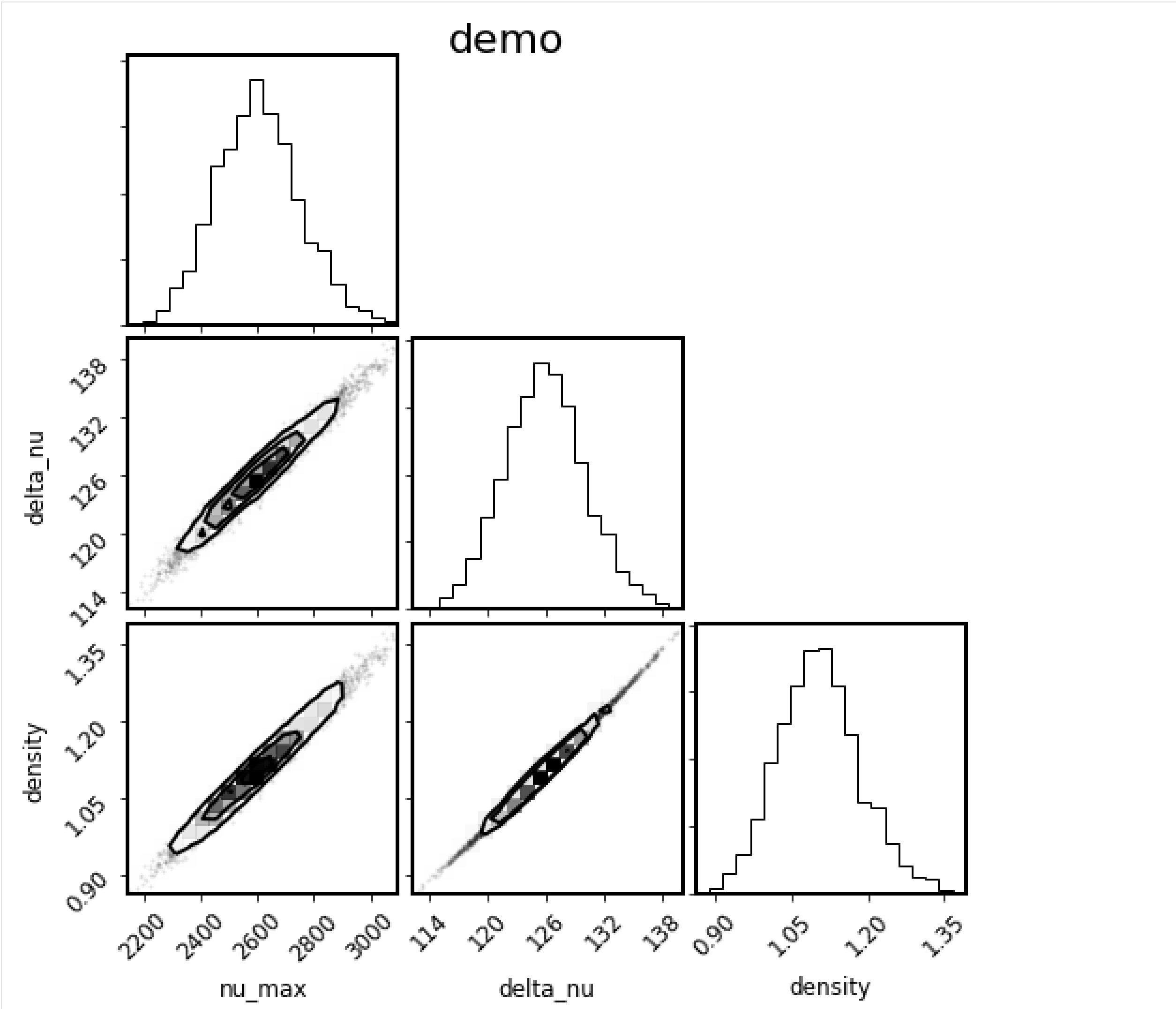
[24]:
```
mod.corner_observed();
```



This looks good, because we generated the synthetic observations directly from the same stellar model grids that we used to fit. For real data, this is an important figure to look at to see if any of the observations appear to be inconsistent with the others, and to see if the model is a good fit to the observations.

Generically, you can also make a corner plot of arbitrary derived parameters as follows:

```
[25]: mod.corner_derived(['nu_max', 'delta_nu', 'density']);
```


demo
delta_nu
density
nu_max
delta_nu
density
138
132
126
120
114
1.35
1.20
1.05
0.90
2200
2400
2600
2800
3000
114
120
126
132
138
0.90
1.05
1.20
1.35

CHAPTER 8

Multiple star systems

One of the signature capabilities of **isochrones** is the ability to fit multiple star systems to observational data. This works by providing a `StarModel` with more detailed information about the observational data, and about how many stars you wish to fit. There are several layers of potential intricacy here, which we will walk through in stages.

8.1 Unresolved multiple systems

Often it is of interest to know what potential binary star configurations are consistent with observations of a star. For most stars the best available observational data is a combination of broadband magnitudes from various all-sky catalogs and parallax measurements from *Gaia*. Let's first generate synthetic observations of such a star, and then see what we can recover with a binary or triple star model, and also what inference of this system under a single star model would tell us.

Note here that for this simplest of multiple star scenarios—unresolved, physically associated, binary or triple-star systems—there are special `StarModel` objects available that have more highly optimized likelihood calculations, analogous to the `SingleStarModel` that is available for a simple single-star fit. `BinaryStarModel` and `TripleStarModel` are these special objects. In order to accommodate more complex scenarios, such as fitting resolved steller companions, it is necessary to use the vanilla `StarModel` object.

First, we will initialize the isochrone interpolator. Note that we actually *require* the isochrone interpolator here, rather than the evolution track interpolator, because the model requires the primary and secondary components to have the same age, so that age must be a sampling paramter.

[1]:
```
from isochrones import get_ichrone

mist = get_ichrone('mist')
```

Now, define the "true" system parameters and initialize the `StarModel` accordingly, with two model stars. Remember that even though we need to use an isochrone interpolator to fit the model, we have to use the evolution tracks to generate synthetic data; this here shows that you can actually do this by using the `.track` complementary attribute. Note also the use of the utility function `addmags` to combine the magnitudes of the two stars.

[2]:
```
from isochrones import BinaryStarModel
from isochrones.utils import addmags

distance = 500  # pc
AV = 0.2
mass_A = 1.0
mass_B = 0.5
age = 9.6
feh = 0.0

# Synthetic 2MASS and Gaia magnitudes
bands = ['J', 'H', 'K', 'BP', 'RP', 'G']
props_A = mist.track.generate(mass_A, age, feh, distance=distance, AV=AV,
                              bands=bands, return_dict=True, accurate=True)
props_B = mist.track.generate(mass_B, age, feh, distance=distance, AV=AV,
                              bands=bands, return_dict=True, accurate=True)

unc = dict(J=0.02, H=0.02, K=0.02, BP=0.002, RP=0.002, G=0.001)
mags_tot = {b: (addmags(props_A[b], props_B[b]), unc[b]) for b in bands}

# Gaia parallax in mas for a system at 500 pc
parallax = (2, 0.05)

mod_binary = BinaryStarModel(mist, **mags_tot, parallax=parallax, name='demo_binary')
```

This model has the following parameters; `eep_0` and `eep_1` correspond to the primary and secondary components, respectively. All the other parameters are assumed to be the same between the two components; that is, they are assumed to be co-eval and co-located.

[3]:
```
mod_binary.param_names
```

[3]:
```
('eep_0', 'eep_1', 'age', 'feh', 'distance', 'AV')
```

Let's also restrict the prior ranges for the parameters, to help with convergence.

[4]:
```
mod_binary.set_bounds(eep=(1, 600), age=(8, 10))
```

Let's test out the posterior computation, and then run a fit to see if we can recover the true parameters.

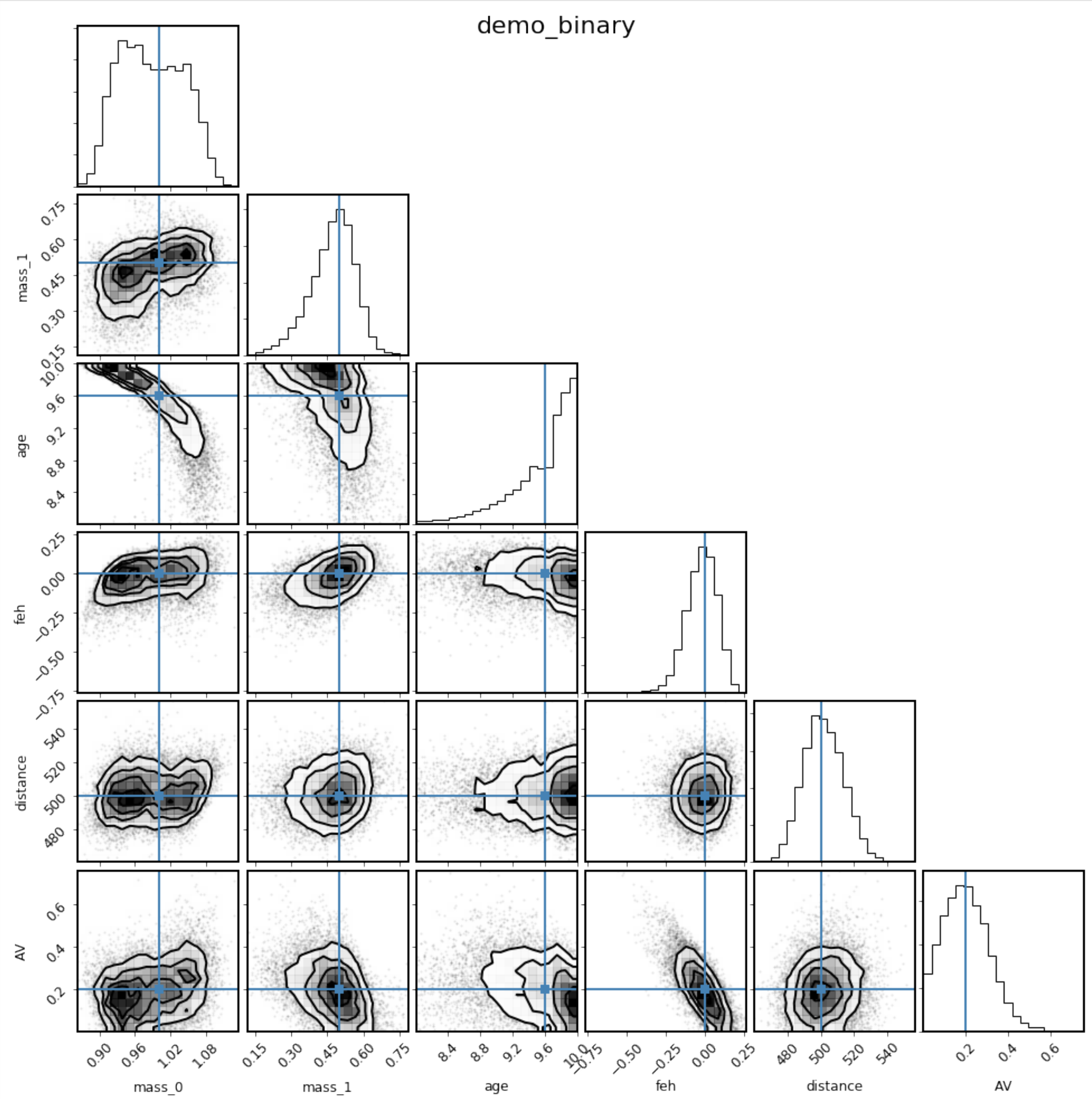
[5]:
```
pars = [350., 300., 9.7, 0.0, 300., 0.1]
print(mod_binary.lnpost(pars))
%timeit mod_binary.lnpost(pars)
```

```
-645802.2025506602
1000 loops, best of 3: 719 µs per loop
```

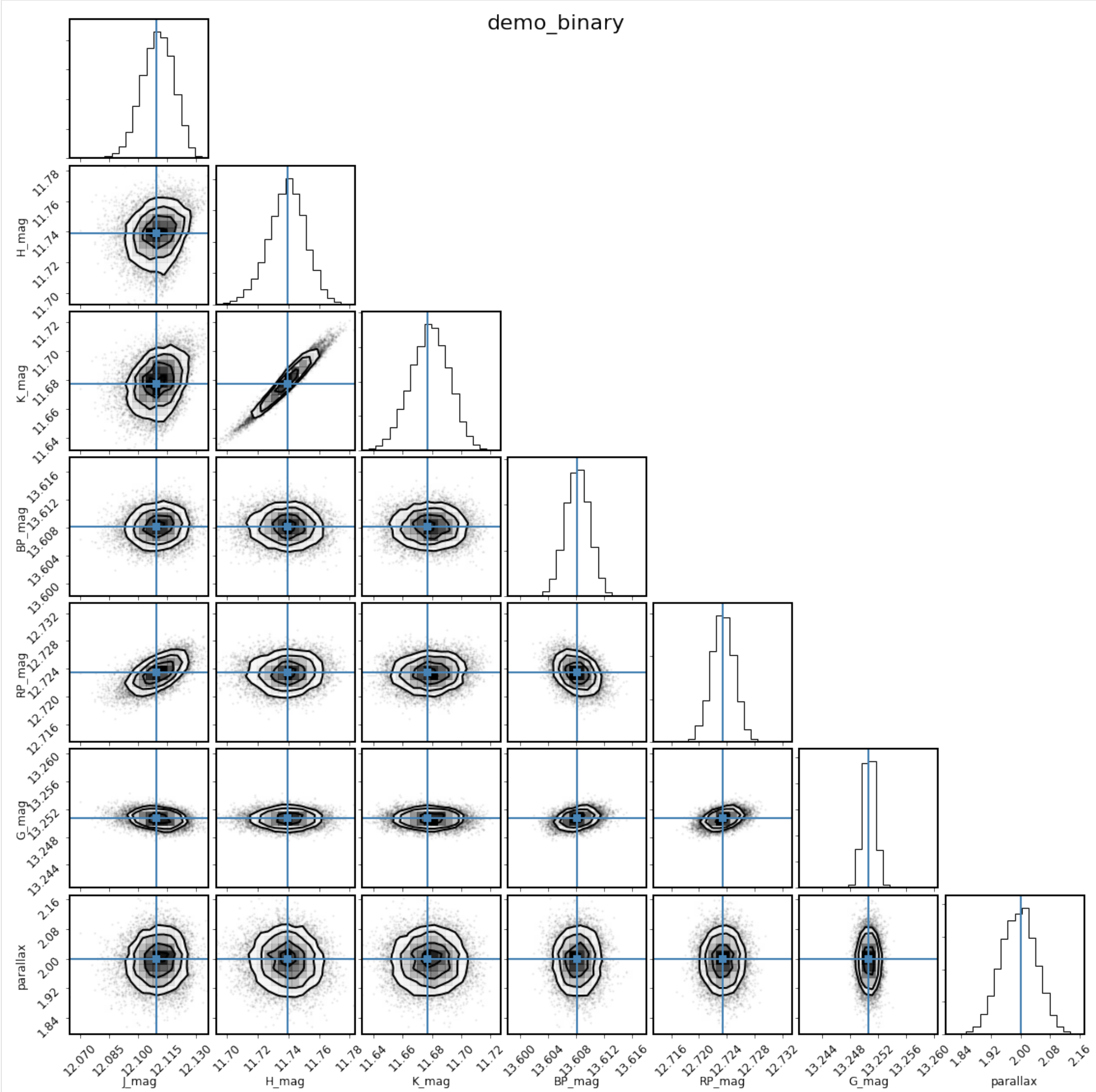
For a binary fit, it is often desirable to run with more than the default number of live points; here we double from 1000 to 2000.

[6]:
```
mod_binary.fit(n_live_points=2000)  # takes about 14 minutes on my laptop
```

[7]:
```
%matplotlib inline

columns = ['mass_0', 'mass_1', 'age', 'feh', 'distance', 'AV']
truths = [mass_A, mass_B, age, feh, distance, AV]
mod_binary.corner_derived(columns, truths=truths);
```




```
[8]: mod_binary.corner_observed();
```



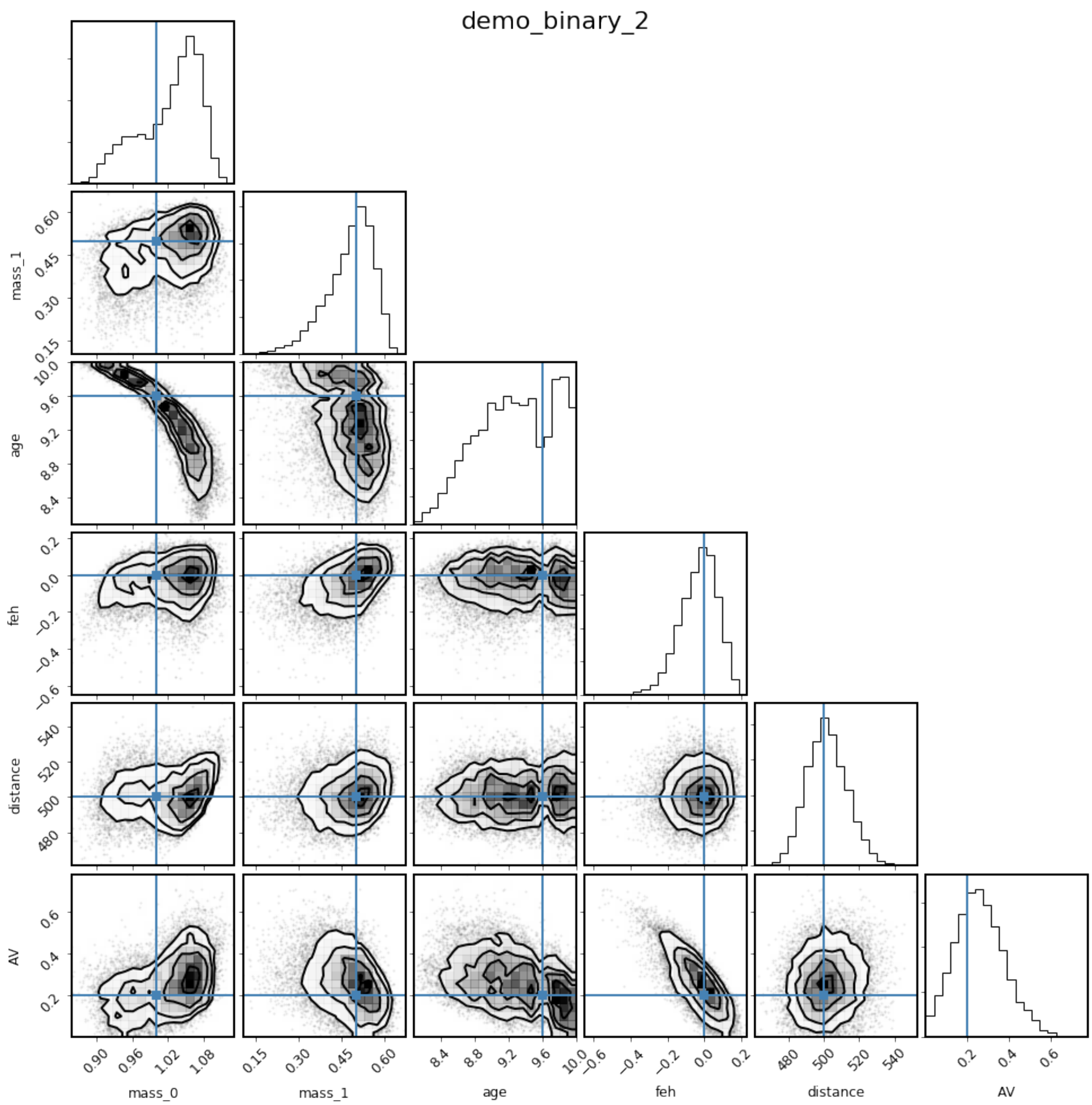
Looks like this recovers the injected parameters pretty well, though not exactly. It looks like the flat-linear age prior (which weights the fit significantly to older ages) is biasing the masses somewhat low. Let's explore what happens if we change the prior and try again, imagining we have some other indicaton the log(age) should be around 9.6.

[9]:
```
from isochrones.priors import GaussianPrior

mod_binary_2 = BinaryStarModel(mist, **mags_tot, parallax=parallax, name='demo_binary_
↪2')
mod_binary_2.set_bounds(eep=(1, 600))
mod_binary_2.set_prior(age=GaussianPrior(9.6, 1, bounds=(8,10)))
mod_binary_2.lnpost(pars)
```

[9]:
```
-645802.7700077017
```

```
[10]: mod_binary_2.fit(n_live_points=2000)
```

```
[11]: mod_binary_2.corner_derived(columns, truths=truths);
```



Hmm, doesn't seem to be much different. Looks like this needs more exploration!

8.2 Resolved multiple system

Another useful capability of **isochrones** is the ability to fit binary (or higher-order multiple) systems that are resolved in high-resolution imaging but blended in catalog photometry. This is done by using the `StarModel` object directly (instead of the optimized models) and explicitly passing the observations.

As before, let's begin by using simulating data. Let's pretend that the same binary system from above is resolved in AO K-band imaging, but blended in *2MASS* catalog data. Let's say this time that we also have spectroscopic constraints of the primary properties.

Inspecting this tree to make sure it accurately represents the desired model becomes more important if the model is more complicated, but this simple case is a good example to review. Each node named with a bandpass represents an observation, with some magnitude and uncertainty (at some separatrion and position angle—irrelevant for the unresolved case). The model nodes here are named `0_0` and `0_1`, with the first index representing the system, and the second index the star number within that system. All stars in the same system share the same age, metallicity, distance, and extinction. In the computation of the likelihood, the apparent magnitude in each observed node is compared with a model-based magnitude that is computed from the *sum of the fluxes of all model nodes underneath that observed node in the tree*. In the unresolved case, this is trivial, but this structure becomes important when a binary is resolved. This model, because the two model stars share all attributes except mass, has the following parameters:

[12]:
```
from isochrones import StarModel
from isochrones.observation import ObservationTree, Observation, Source

def build_obstree(name):
    obs = ObservationTree(name=name)
    for band in 'JHK':
        o = Observation('2MASS', band, 4)  # Name, band, resolution (in arcsec)
        s = Source(addmags(props_A[band], props_B[band]), 0.02)
        o.add_source(s)
        obs.add_observation(o)

    o = Observation('AO', 'K', 0.1)
    s_A = Source(0., 0.02, separation=0, pa=0,
                 relative=True, is_reference=True)
    s_B = Source(props_B['K'] - props_A['K'], 0.02, separation=0.2, pa=100,
                 relative=True, is_reference=False)
    o.add_source(s_A)
    o.add_source(s_B)

    obs.add_observation(o)
    return obs

obs = build_obstree('demo_resolved')
mod_resolved = StarModel(mist, obs=obs,
                         parallax=parallax, Teff=(props_A['Teff'], 100),
                         logg=(props_A['logg'], 0.15), feh=(props_A['feh'], 0.1))
mod_resolved.print_ascii()
```

```
demo_resolved
  2MASS J=(12.11, 0.02) @(0.00, 0 [4.00])
     2MASS H=(11.74, 0.02) @(0.00, 0 [4.00])
        2MASS K=(11.68, 0.02) @(0.00, 0 [4.00])
           AO delta-K=(0.00, 0.02) @(0.00, 0 [0.10])
             0_0, Teff=(5834.782979719397, 100), logg=(4.435999146983706, 0.15),␣
↪feh=(-0.012519050601435218, 0.1), parallax=(2, 0.05)
           AO delta-K=(2.43, 0.02) @(0.20, 100 [0.10])
              0_1, parallax=(2, 0.05)
```

[13]:
```
pars = [300, 280, 9.6, 0.0, 400, 0.1]
mod_resolved.lnpost(pars)
```

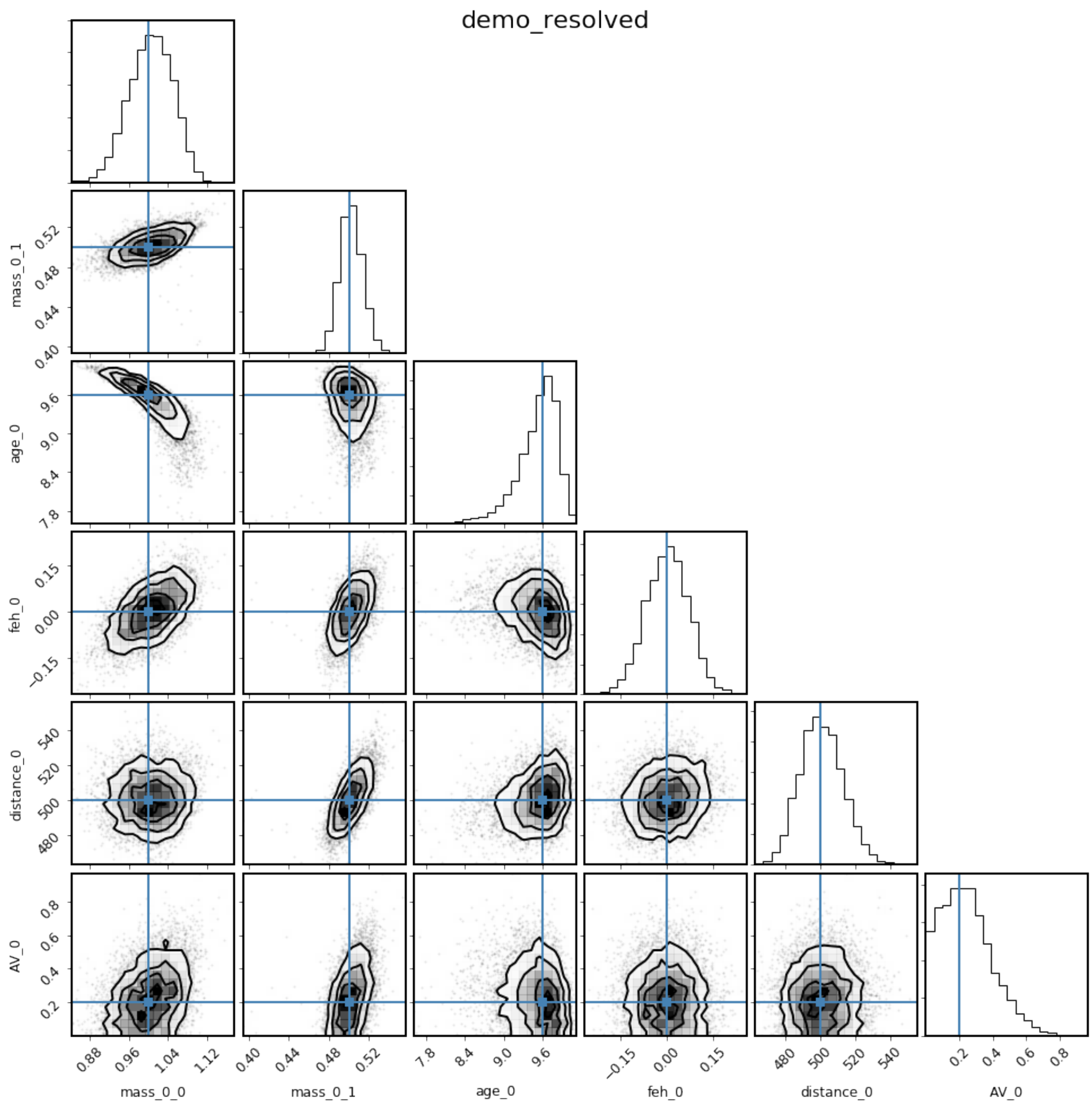
[13]:
```
-8443.175970078633
```

[14]:
```
%timeit mod_resolved.lnpost(pars)
```

```
100 loops, best of 3: 1.23 ms per loop
```

[15]:
```
mod_resolved.fit()
```

[16]:
```
%matplotlib inline

columns = ['mass_0_0', 'mass_0_1', 'age_0', 'feh_0', 'distance_0', 'AV_0']
truths = [mass_A, mass_B, age, feh, distance, AV]
mod_resolved.corner(columns, truths=truths);
```



Nailed it! Looks like the spectroscopy was very helpful in getting the fit correct (age in particular).

8.3 Unassociated companions

The previous two examples model a binary star system in which the two components are co-located and co-eval; that is, they have the same age, metallicity, distance, and extinction.

One can imagine, however, wanting to model a scenario in which the two components are *not* physically associated, but rather just chance-aligned in the plane of the sky. In this case, you can set up the `StarModel` with just a small difference:

[17]:
```
obs = build_obstree('demo_resolved_unassoc')  # N.B., running this again, because the␣
↪old "obs" was changed by the previous model
mod_resolved_unassoc = StarModel(mist, obs=obs,
                         parallax=parallax, Teff=(props_A['Teff'], 100),
                         logg=(props_A['logg'], 0.15), feh=(props_A['feh'], 0.1),
                         index=[0, 1])
mod_resolved_unassoc.print_ascii()
```

```
demo_resolved_unassoc
  2MASS J=(12.11, 0.02) @(0.00, 0 [4.00])
     2MASS H=(11.74, 0.02) @(0.00, 0 [4.00])
        2MASS K=(11.68, 0.02) @(0.00, 0 [4.00])
           AO delta-K=(0.00, 0.02) @(0.00, 0 [0.10])
             0_0, Teff=(5834.782979719397, 100), logg=(4.435999146983706, 0.15),␣
↪feh=(-0.012519050601435218, 0.1), parallax=(2, 0.05)
           AO delta-K=(2.43, 0.02) @(0.20, 100 [0.10])
              1_0
```

Note that this model now has ten parameters, since the two systems are now decoupled, so we will not run the fit for this example, but it is in principle possible. (Note that you would probably want to run this with MPI for this number of parameters.)

[18]:
```
mod_resolved_unassoc.param_names
```

[18]:
```
['eep_0_0',
 'age_0',
 'feh_0',
 'distance_0',
 'AV_0',
 'eep_1_0',
 'age_1',
 'feh_1',
 'distance_1',
 'AV_1']
```

8.4 More complex models

You can define arbitrarily complex models, by explicitly defining the model nodes by hand, using the `N` and `index` keywords. Below are some examples.

This is a physically associated hierarchical triple, where the bright star from AO is an unresolved binary:

[19]:
```
obs = build_obstree('triple1')
StarModel(mist, obs=obs, N=[2, 1], index=[0, 0]).print_ascii()
```

```
triple1
  2MASS J=(12.11, 0.02) @(0.00, 0 [4.00])
```

(continues on next page)

(continued from previous page)

```
      2MASS H=(11.74, 0.02) @(0.00, 0 [4.00])
         2MASS K=(11.68, 0.02) @(0.00, 0 [4.00])
            AO delta-K=(0.00, 0.02) @(0.00, 0 [0.10])
              0_0
              0_1
            AO delta-K=(2.43, 0.02) @(0.20, 100 [0.10])
               0_2
```

Here is a situation where the faint visual binary is an unrelated binary star:

```
[20]: obs = build_obstree('triple2')
StarModel(mist, obs=obs, N=[1, 2], index=[0, 1]).print_ascii()
```

```
triple2
  2MASS J=(12.11, 0.02) @(0.00, 0 [4.00])
     2MASS H=(11.74, 0.02) @(0.00, 0 [4.00])
        2MASS K=(11.68, 0.02) @(0.00, 0 [4.00])
           AO delta-K=(0.00, 0.02) @(0.00, 0 [0.10])
             0_0
           AO delta-K=(2.43, 0.02) @(0.20, 100 [0.10])
              1_0
              1_1
```

Here, both AO stars are unresolved binaries:

```
[21]: obs = build_obstree('double_binary')
StarModel(mist, obs=obs, N=2, index=[0, 1]).print_ascii()
```

```
double_binary
  2MASS J=(12.11, 0.02) @(0.00, 0 [4.00])
     2MASS H=(11.74, 0.02) @(0.00, 0 [4.00])
        2MASS K=(11.68, 0.02) @(0.00, 0 [4.00])
           AO delta-K=(0.00, 0.02) @(0.00, 0 [0.10])
             0_0
             0_1
           AO delta-K=(2.43, 0.02) @(0.20, 100 [0.10])
              1_0
              1_1
```

You can in principle create even more crazy models, but I don’t recommend it. . .

CHAPTER 9

Simulating stellar populations

Many astronomical investigations require simulating populations of stars, and **isochrones** contains some utilities to help enable this. Given population distributions of the quantities required to simulate individual stars, a `StarPopulation` object can be defined and used to generate sample populations following this distribution. Binary stars, ubiquitous as they are, are necessarily built into this framework, so the parameters needed to simulate an individual stellar observation are the following:

$$M_A, M_B, T, [Fe/H], d, A_V$$

where M_A, M_B are the primary and (if present) secondary masses, T is age, $[Fe/H]$ is the metalicity, d is distance, and A_V is the V-band extinction, quantifying the effect of dust along the line of sight. Generating a population of such stars then requires sampling from distributions of each of the above quantities. A `StarPopulation` takes metallicity, distance, and extinction distributions as arguments, and samples from each of those distributions when generating a sample population.

Sampling primary/secondary masses and ages is a bit less straightforward. For M_A, M_B, **isochrones** parametrizes the distribution with a primary initial mass function (IMF), binary fraction f_B, and mass-ratio ($q = M_B/M_A$) distribution $p(q) \propto q^\gamma$. The age distribution of stars in a population is often described as a "star-formation history" (SFH)—sampling a population with a given SFH is the same as treating the SFH as the probability distribution function of stellar age, sampling ages from this distribution, and then truncating any stars that have reached the end of their evolution. Practically, this truncation happens by rejection sampling: evaluating the `ModelGridInterpolator` at each sampled set of parameters, and rejecting samples for which the interpolator returns `np.nan` values for the observed stellar properties (which will happen when trying to interpolate out-of-bounds, which happens when a star is requested beyond the end of its lifetime).

9.1 StarPopulation object

Here is an example of `StarPopulation` usage:

[1]:
```
from scipy.stats import uniform, norm
from isochrones import get_ichrone
from isochrones.priors import GaussianPrior, SalpeterPrior, DistancePrior, FlatPrior
```

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```
from isochrones.populations import StarFormationHistory, StarPopulation

# Initialize interpolator
mist = get_ichrone('mist')

# Initialize distributions

# Ingredients required to generate primary & secondary masses
imf = SalpeterPrior(bounds=(1, 10))  # minimum 1 Msun
fB = 0.4
gamma = 0.3

# SFH distribution takes a scipy stats distribution, of age in Gyr
sfh = StarFormationHistory(dist=uniform(0, 10))

# The following are all isochrones.priors.Prior objects,
# or anything with a .sample(N) method
feh = GaussianPrior(-0.2, 0.2)
distance = DistancePrior(max_distance=3000)
AV = FlatPrior(bounds=[0, 1])

pop = StarPopulation(mist, imf=imf, fB=fB, gamma=gamma, sfh=sfh, feh=feh,␣
↪distance=distance, AV=AV)
```

Once the object is created, it can be used to generate a population of stars.

[2]:
```
df = pop.generate(1000)
df.head()
```

[2]:
```
     mass_0    logg_0  delta_nu_0  initial_mass_0  phase_0         eep_0  \
0  1.553332  4.380359  102.670924        1.553406      0.0  299.894473
1  1.549665  4.211669   78.515862        1.549955      0.0  340.291660
2  1.127802  4.009138   66.783488        1.128399      0.0  447.067891
3  1.046129  4.299633  109.308356        1.046413      0.0  384.695516
4  1.267605  3.757970   42.600593        1.268362      2.0  460.286164

   radius_0    Mbol_0  logTeff_0     feh_0  ...     W1_mag      A_W1  \
0  1.332984  2.451403   3.927937 -0.327345  ...  14.208519  0.014534
1  1.617098  2.454235   3.885739 -0.111990  ...  13.706165  0.048358
2  1.740697  3.206514   3.794381 -0.366979  ...  14.346362  0.010542
3  1.199744  3.803016   3.815517 -0.668881  ...  14.342954  0.028781
4  2.463683  2.543589   3.785193 -0.307018  ...  13.221399  0.041585

      W2_mag      A_W2     W3_mag      A_W3   TESS_mag    A_TESS  Kepler_mag  \
0  14.203228  0.008647  14.194746  0.002359  14.477501  0.161784   14.609700
1  13.686343  0.028772  13.662330  0.007847  14.462563  0.529039   14.812933
2  14.338646  0.006274  14.318684  0.001710  15.193251  0.114144   15.567852
3  14.327035  0.017125  14.301296  0.004667  15.269802  0.311150   15.679683
4  13.205129  0.024749  13.170042  0.006745  14.403408  0.445389   14.910002

   A_Kepler
0  0.225833
1  0.730147
2  0.155572
3  0.424941
4  0.604260
```

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```
[5 rows x 110 columns]
```

Note that this operation is not nearly as fast as directly interpolating an isochrone or evolution track grid (since generating properites given mass, age, and metallicity necessarily involves solving for EEP first):

[3]:
```
%timeit pop.generate(1000)
```

```
1.24 s ± 152 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
```

Also, this can be made **much** faster if you loosen the requirement on getting *exactly* a particularly desired number of stars (as part of the generating algorithm involves replacing stars that come out as nan until no nans are left):

[4]:
```
print(len(pop.generate(1000, exact_N=False)))

%timeit pop.generate(1000, exact_N=False)
```

```
255
64.9 ms ± 381 µs per loop (mean ± std. dev. of 7 runs, 10 loops each)
```

The full column list of this table of simulated stars is the following:

[5]:
```
', '.join(df.columns)
```

[5]:
```
'mass_0, logg_0, delta_nu_0, initial_mass_0, phase_0, eep_0, radius_0, Mbol_0,␣
↪logTeff_0, feh_0, density_0, nu_max_0, logL_0, Teff_0, interpolated_0, star_age_0,␣
↪age_0, dt_deep_0, J_mag_0, H_mag_0, K_mag_0, G_mag_0, BP_mag_0, RP_mag_0, W1_mag_0,␣
↪W2_mag_0, W3_mag_0, TESS_mag_0, Kepler_mag_0, distance_0, AV_0, initial_feh_0,␣
↪requested_age_0, A_J_0, A_H_0, A_K_0, A_G_0, A_BP_0, A_RP_0, A_W1_0, A_W2_0, A_W3_0,
↪ A_TESS_0, A_Kepler_0, mass_1, logg_1, delta_nu_1, initial_mass_1, phase_1, eep_1,␣
↪radius_1, Mbol_1, logTeff_1, feh_1, density_1, nu_max_1, logL_1, Teff_1,␣
↪interpolated_1, star_age_1, age_1, dt_deep_1, J_mag_1, H_mag_1, K_mag_1, G_mag_1,␣
↪BP_mag_1, RP_mag_1, W1_mag_1, W2_mag_1, W3_mag_1, TESS_mag_1, Kepler_mag_1,␣
↪distance_1, AV_1, initial_feh_1, requested_age_1, A_J_1, A_H_1, A_K_1, A_G_1, A_BP_
↪1, A_RP_1, A_W1_1, A_W2_1, A_W3_1, A_TESS_1, A_Kepler_1, J_mag, A_J, H_mag, A_H, K_
↪mag, A_K, G_mag, A_G, BP_mag, A_BP, RP_mag, A_RP, W1_mag, A_W1, W2_mag, A_W2, W3_
↪mag, A_W3, TESS_mag, A_TESS, Kepler_mag, A_Kepler'
```

All quantities with a tag `_0` refer to the primary star; all quantities with `_1` refer to the secondary. Columns ending in just `_mag` represent the *combined* magnitude of both primary and secondary component. Let's look the *Gaia* color-magnitude diagram for this simulated population. Note also the `A_[x]` columns, which give the specific extinction per band for each system (and for the individual components of the binary).

[6]:
```
import holoviews as hv
hv.extension('bokeh')
import hvplot.pandas

def hr_plot(df):
    df['BpRp'] = df.BP_mag - df.RP_mag
    hr = df.hvplot.scatter('BpRp', 'G_mag',
                           hover_cols=['mass_0', 'mass_1', 'age_0', 'AV_0'],
                           color='feh_0')
    return hr.options(height=400, width=500, invert_yaxis=True)

hr_plot(df)
```

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

Data type cannot be displayed: application/javascript, application/vnd.holoviews_load.v0+json

[6]:
```
:Scatter   [BpRp]   (G_mag,feh_0,mass_0,mass_1,age_0,AV_0)
```

There is also a simple utility function that can "deredden" a generated population dataframe (e.g., recover the true intrinsic magnitudes of each star in the absence of dust), by subtracting off the `A_x` extinction values from the magnitudes, and setting all extinctions to zero. Let's use this to deredden the above hr diagram:

[7]:
```
from isochrones.populations import deredden

dereddened = deredden(df)

hr_plot(df).options(size=3, alpha=0.2, color='red') * hr_plot(dereddened).
↪options(alpha=0.2, color='black', size=3)
```

[7]:
```
:Overlay
   .Scatter.I  :Scatter   [BpRp]   (G_mag,feh_0,mass_0,mass_1,age_0,AV_0)
   .Scatter.II :Scatter   [BpRp]   (G_mag,feh_0,mass_0,mass_1,age_0,AV_0)
```

See how the dust (reddened points) moves each star down (fainter) and to the right (redder).

9.2 ModelGridInterpolator.generate_binary

The above-used `StarPopulation.generate` method is a wrapper around the `.generate_binary` method of a `ModelGridInterpolator`, which can also be used directly, if you wish to simulate observations of binary stars with specific properties:

[8]:
```
mass_A = 1.0
mass_B = [0.8, 0.6, 0.4, 0.2]
age, feh, distance, AV = (9.6, 0.02, 100, 0.1)

mist.generate_binary(mass_A, mass_B, age, feh, distance=distance, AV=AV)[['G_mag',
↪'BP_mag', 'RP_mag']]
```

[8]:

	G_mag	BP_mag	RP_mag
0	9.459204	9.822055	8.928917
1	9.669059	10.018673	9.150093
2	9.709775	10.046349	9.204878
3	9.718787	10.050889	9.219106