
dolfin_navier_scipy Documentation

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CONTENTS

1	Code Reference	3
1.1	dolfin_to_sparrays	3
1.2	stokes_navier_utils	3
1.3	problem_setups	7
1.4	data_output_utils	12
1.5	time_int_utils	13
2	Indices and tables	15
	Python Module Index	17
	Index	19

The package *dolfin_navi_scipy (dns)* provides an interface between *scipy* and *FEniCS* in view of solving Navier-Stokes Equations. *FEniCS* is used to perform a Finite Element discretization of the equations. The assembled coefficients are exported as sparse matrices for use in *scipy*. Nonlinear and time-dependent parts are evaluated and assembled on demand. Visualization is done via the *FEniCS* interface to *paraview*.

Contents:

CODE REFERENCE

1.1 dolfin_to_sparrays

1.2 stokes_navier_utils

```
stokes_navier_utils.get_pfromv(v=None, V=None, M=None, A=None, J=None, fv=None,  
fp=None, decouplevp=False, solve_M=None, symmetric=False,  
cgtol=1e-08, stokes_flow=False, diribcs=None, dbcinds=None,  
dbcvals=None, invinds=None, **kwargs)
```

for a velocity v , get the corresponding p

Notes

Formula is only valid for constant rhs in the continuity equation

```
stokes_navier_utils.get_v_conv_consts(vvec=None, V=None, invinds=None, dbcvals=[],  
dbcinds=[], semi_explicit=False, Picard=False, retparts=False)
```

get and condense the linearized convection

to be used in a Newton scheme

$$(u \cdot \nabla)u \rightarrow (u_0 \cdot \nabla)u + (u \cdot \nabla)u_0 - (u_0 \cdot \nabla)u_0$$

or in a Picard scheme

$$(u \cdot \nabla)u \rightarrow (u_0 \cdot \nabla)u$$

Parameters

vvec [(N,1) ndarray] convection velocity

V [dolfin.VectorFunctionSpace] FEM space of the velocity

invinds [(N,) ndarray or list] indices of the inner nodes

Picard [Boolean] whether Picard linearization is applied, defaults to *False*

semi_explicit: Boolean, optional whether to return minus the convection vector, and zero convmats as needed for semi-explicit integration, defaults to *False*

retparts [Boolean, optional] whether to return both components of the matrices and contributions to the rhs through the boundary conditions, defaults to *False*

Returns

convc_mat [(N,N) sparse matrix] representing the linearized convection at the inner nodes

rhs_con [(N,1) array] representing $(u_0 \cdot \nabla)u_0$ at the inner nodes

rhsv_conbc [(N,1) ndarray] representing the boundary conditions

```
stokes_navi_utils.solve_nse(A=None, M=None, J=None, JT=None, fv=None, fp=None,
                            fvtv=None, fvss=0.0, fvtvd=None, fv_tmdp=None, iniv=None,
                            inip=None, lin_vel_point=None, stokes_flow=False,
                            trange=None, t0=None, tE=None, Nts=None,
                            time_int_scheme='cnab', V=None, Q=None, in-
                            vinds=None, diribcs=None, dbcinds=None, dbc-
                            vals=None, diricontbcinds=None, diricontbc-
                            vals=None, diricontfuncs=None, diricontfuncmems=None,
                            N=None, nu=None, ppin=None, closed_loop=False,
                            static_feedback=False, dynamic_feedback=False,
                            dyn_fb_dict={}, dyn_fb_disc='trapezoidal', b_mat=None,
                            cv_mat=None, vp_output=False, vp_out_fun=None,
                            vp_output_dict=None, vel_nwtn_stps=20, vel_nwtn_tol=5e-15,
                            nsects=1, loc_nwtn_tol=5e-15, loc_pcrd_stps=True, addfull-
                            sweep=False, vel_pcrd_stps=4, krylov=None, krpslvprms={},
                            krplsprms={}, clearprvdata=False, get_datastring=None,
                            data_prfx='', paraviewoutput=False, plttrange=None, prvoutp-
                            nts=None, vfileprfx='', pfileprfx='', return_dictofvelstrs=False,
                            return_dictofpstrs=False, dictkeysstr=False, dictkeyfor-
                            mat='.5f', treat_nonl_explicit=True, no_data_caching=True,
                            treat_nonl_explct=False, use_custom_nonlinearity=False,
                            custom_nonlinear_vel_function=None, datarange=None,
                            dataoutpnts=None, return_final_vp=False, re-
                            turn_as_list=False, return_vp_dict=False, return_y_list=False,
                            check_ff=False, check_ff_maxv=100000000.0, verbose=True,
                            start_ssstokes=False, **kw)
```

solution of the time-dependent nonlinear Navier-Stokes equation

$$\begin{aligned} M\dot{v} + Av + N(v)v + J^T p &= f_v \\ Jv &= f_p \end{aligned}$$

using a Newton scheme in function space, i.e. given v_k , we solve for the update like

$$M\dot{v} + Av + N(v_k)v + N(v)v_k + J^T p = N(v_k)v_k + f,$$

and trapezoidal rule in time. To solve an *Oseen* system (linearization about a steady state) or a *Stokes* system, set the number of Newton steps to one and provide a linearization point and an initial value.

Parameters

lin_vel_point [dictionary, optional] contains the linearization point for the first Newton iteration

- Steady State: `{None: 'path_to_nparray'}`, `{'None': nparray}`
- Newton: `{t: 'path_to_nparray'}`

defaults to `None`

dictkeysstr [boolean, optional] whether the *keys* of the result dictionaries are strings instead of floats, defaults to `False`

fvtv [callable f(t), optional] time dependent right hand side in momentum equation

fvtvd [callable f(t, v), optional] time and velocity dependent right hand side in momentum equation

fvss [array, optional] right hand side in momentum for steady state computation

fv_tmdp [callable f(t, v, dict), optional] XXX: this is deprecated

dbcinds: list, optional indices of the Dirichlet boundary conditions

dbcvals: list, optional values of the Dirichlet boundary conditions (as listed in *dbcinds*)

diricontbcinds: list, optional list of dirichlet indices that are to be controlled

diricontbcvals: list, optional list of the vals corresponding to *diricontbcinds*

diricontfuncs: list, optional list like [*ufunc*] where *ufunc*: (t, v) -> u where u is used to scale the corresponding *diricontbcvals*

check_ff: boolean, optional whether to check for failures and to invoke a premature break, defaults to *False*

check_ff_maxv: float, optional threshold for norm of velocity that indicates a failure, defaults to *1e8*

krylov [{None, ‘gmres’}, optional] whether or not to use an iterative solver, defaults to *None*

krpslvrms [dictionary, optional] v specify parameters of the linear solver for use in Krypy, e.g.,

- initial guess
- tolerance
- number of iterations

defaults to *None*

krplsprms [dictionary, optional] parameters to define the linear system like

- preconditioner

ppin [{int, None}, optional] which dof of *p* is used to pin the pressure, defaults to *None*

stokes_flow [boolean, optional] whether to consider the Stokes linearization, defaults to *False*

start_ssstokes [boolean, optional] for your convenience, compute and use the steady state stokes solution as initial value, defaults to *False*

treat_nonl_explicit= string, optional whether to treat the nonlinearity explicitly, defaults to *False*

use_custom_nonlinearity: boolean, optional whether to use a custom nonlinear velocity function to replace the convection part, defaults to *False*

custom_nonlinear_vel_function: callable f(v), optional the custom nonlinear function (cp. *use_custom_nonlinearity*), defaults to *None*

nsects: int, optional in how many segments the trange is split up. (The newton iteration will be confined to the segments and, probably, converge faster than the global iteration), defaults to *I*

loc_nwtn_tol: float, optional tolerance for the newton iteration on the segments, defaults to *1e-15*

loc_perd_stps: boolean, optional whether to init with *vel_pcrd_stps* Picard steps on every section, if *False*, Picard iterations are performed only on the first section, defaults to *True*

addfullsweep: boolean, optional whether to compute the newton iteration on the full *trange*, useful to check and for the plots, defaults to *False*

cv_mat: (Ny, Nv) sparse array, optional output matrix for velocity outputs, needed, e.g., for output dependent feedback control, defaults to *None*

dynamic_feedback: boolean whether to apply dynamic feedback, defaults to *False*

dyn_fb_dict, dictionary

that defines the dynamic observer via the keys

- *ha* observer dynamic matrix
- *hb* observer input matrix
- *hc* observer output matrix
- *drift* observer drift term, e.g., for nonzero setpoints

dyn_fb_disc: string

how to discretize the dynamic observer/controller

- *AB2* explicit scheme (Adams Bashforth)
- *trapezoidal* implicit scheme (Trapezoidal)

Returns

dictofvelstrs [dictionary, on demand] dictionary with time t as keys and path to velocity files as values

dictofpstrs [dictionary, on demand] dictionary with time t as keys and path to pressure files as values

vellist [list, on demand] list of the velocity solutions, deprecated use *ylist* with *C=I*

```
stokes_navi_utils.solve_steadystate_nse(A=None, J=None, JT=None, M=None,
                                         fv=None, fp=None, V=None, Q=None, in-
                                         vinds=None, diribcs=None, dbcvvals=None,
                                         dbcinds=None, diricontbcinds=None, diri-
                                         contbcvals=None, diricontfuncs=None,
                                         diricontfuncmems=None, return_yp=False,
                                         ppin=None, return_nwtnupd_norms=False,
                                         N=None, nu=None, only_stokes=False,
                                         vel_pcrd_stps=10, vel_pcrd_tol=0.0001,
                                         vel_nwtn_stps=20, vel_nwtn_tol=5e-15,
                                         clearprvdata=False, useolddata=False,
                                         vel_start_nwtn=None, get_datastring=None,
                                         data_prfx='', paraviewoutput=False,
                                         save_data=False, vfileprfx='', pfileprfx='',
                                         verbose=True, **kw)
```

Solution of the steady state nonlinear NSE Problem

using Newton's scheme. If no starting value is provided, the iteration is started with the steady state Stokes solution.

Parameters

A [(N,N) sparse matrix] stiffness matrix aka discrete Laplacian, note the sign!

M [(N,N) sparse matrix] mass matrix

J [(M,N) sparse matrix] discrete divergence operator

JT [(N,M) sparse matrix, optional] discrete gradient operator, set to *J.T* if not provided

fv, fp [(N,1), (M,1) ndarrays] right hand sides restricted via removing the boundary nodes in the momentum and the pressure freedom in the continuity equation

ppin [{int, None}, optional] which dof of p is used to pin the pressure, defaults to *None*

dbcinds: list, optional indices of the Dirichlet boundary conditions

dbcvals: list, optional values of the Dirichlet boundary conditions (as listed in *dbcinds*)

diricontbcinds: list, optional list of dirichlet indices that are to be controlled

diricontbcvals: list, optional list of the vals corresponding to *diricontbcinds*

diricontfuncs: list, optional list like [*ufunc*] where *ufunc*: $(t, v) \rightarrow u$ where *u* is used to scale the corresponding *diricontbcvals*

return_vp [boolean, optional] whether to return also the pressure, defaults to *False*

vel_pcrd_stps [int, optional] Number of Picard iterations when computing a starting value for the Newton scheme, cf. Elman, Silvester, Wathen: *FEM and fast iterative solvers*, 2005, defaults to 100

only_stokes: boolean, optional if only compute the stokes solution, defaults to *False*

vel_pcrd_tol [real, optional] tolerance for the size of the Picard update, defaults to 1e-4

vel_nwtn_stps [int, optional] Number of Newton iterations, defaults to 20

vel_nwtn_tol [real, optional] tolerance for the size of the Newton update, defaults to 5e-15

Returns:

—

vel_k [(N, 1) ndarray] the velocity vector, if not *return_vp*, else

(v, p) [tuple] of the velocity and the pressure vector

norm_nwtnupd_list [list, on demand] list of the newton upd errors

1.3 problem_setups

```
problem_setups.cyl3D_fems (refinement_level=2, scheme='TH', strtobcsobs='', strtomeshfile='', strtophysicalregions='', bccontrol=False, verbose=False)
dictionary for the fem items for the 3D cylinder wake
```

which is

- the 2D setup extruded in z-direction
- with symmetry BCs at the z-walls

Parameters

scheme [{None, ‘CR’, ‘TH’}] the finite element scheme to be applied, ‘CR’ for Crouzieux-Raviart, ‘TH’ for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*

bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to *False*

Returns

femp [a dictionary with the keys:]

- V : FEM space of the velocity

- Q : FEM space of the pressure
- $diribcs$: list of the (Dirichlet) boundary conditions
- $dirip$: list of the (Dirichlet) boundary conditions for the pressure
- f_v : right hand side of the momentum equation
- f_p : right hand side of the continuity equation
- $charlen$: characteristic length of the setup
- $odcoo$: dictionary with the coordinates of the domain of observation
- $cdcoo$: dictionary with the coordinates of the domain of control
- $uspacedep$: int that specifies in what spatial direction B_u changes. The remaining is constant
- $bcsubdoms$: list of subdomains that define the segments where the boundary control is applied

Notes

parts of the code were taken from the NSbench collection <https://launchpad.net/nsbench>

```
__author__ = "Kristian Valen-Sendstad <kvs@simula.no>"  
__date__ = "2009-10-01"  
__copyright__ = "Copyright (C) 2009-2010 " + __author__  
__license__ = "GNU GPL version 3 or any later version"
```

```
problem_setups.cyl_fems (refinement_level=2, vdgree=2, pdgree=1, scheme=None, inflowvel=1.0,  
                         bccontrol=False, verbose=False)
```

dictionary for the fem items for the cylinder wake

Parameters

vdgree [polynomial degree of the velocity basis functions,] defaults to 2
pdgree [polynomial degree of the pressure basis functions,] defaults to 1
scheme [{None, ‘CR’, ‘TH’}] the finite element scheme to be applied, ‘CR’ for Crouzieux-Raviart, ‘TH’ for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*
bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to *False*

Returns

femp [a dictionary with the keys:]

- V : FEM space of the velocity
- Q : FEM space of the pressure
- $diribcs$: list of the (Dirichlet) boundary conditions
- $dirip$: list of the (Dirichlet) boundary conditions for the pressure
- f_v : right hand side of the momentum equation
- f_p : right hand side of the continuity equation

- *charlen*: characteristic length of the setup
- *odcoo*: dictionary with the coordinates of the domain of observation
- *cdcoo*: dictionary with the coordinates of the domain of control
- *uspacedep*: int that specifies in what spatial direction B_u changes. The remaining is constant
- *bcsubdoms*: list of subdomains that define the segments where the boundary control is applied

Notes

TODO: *inflowvel* as input is there for consistency but not processed

parts of the code were taken from the NSbench collection <https://launchpad.net/nsbench>

```
__author__ = "Kristian Valen-Sendstad <kvs@simula.no>"  
__date__ = "2009-10-01"  
__copyright__ = "Copyright (C) 2009-2010 " + __author__  
__license__ = "GNU GPL version 3 or any later version"
```

problem_setups.**drivcav_fems** (*N*=10, *vdgree*=2, *pdgree*=1, *scheme*=None, *bccontrol*=None)
dictionary for the fem items of the (unit) driven cavity

Parameters

N [int] mesh parameter for the unitsquare (N gives 2^*N*N triangles)
vdgree [int, optional] polynomial degree of the velocity basis functions, defaults to 2
pdgree [int, optional] polynomial degree of the pressure basis functions, defaults to 1
scheme [{None, ‘CR’, ‘TH’}] the finite element scheme to be applied, ‘CR’ for Crouzeix-Raviart, ‘TH’ for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*
bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to false. TODO: not implemented yet but we need it here for consistency

Returns

femp [a dict]

of problem FEM description with the keys:

- V : FEM space of the velocity
- Q : FEM space of the pressure
- *diribcs*: list of the (Dirichlet) boundary conditions
- *fv*: right hand side of the momentum equation
- *fp*: right hand side of the continuity equation
- *charlen*: characteristic length of the setup
- *odcoo*: dictionary with the coordinates of the domain of observation
- *cdcoo*: dictionary with the coordinates of the domain of control

```
problem_setups.gen_bccont_fems (scheme='TH', bccontrol=True, verbose=False, strtomeshfile=",
                                strtophysicalregions=", inflowvel=1.0, inflowprofile='parabola',
                                movingwallcntrl=False, strtobcsobs=")
```

dictionary for the fem items for a general 2D flow setup

with

- inflow/outflow
- boundary control

Parameters

scheme [{None, 'CR', 'TH'}]] the finite element scheme to be applied, 'CR' for Crouzeix-Raviart, 'TH' for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*

bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to *True*

movingwallcntrl [boolean, optional] whether control is via moving boundaries

Returns

femp [a dictionary with the keys:]

- *V*: FEM space of the velocity
- *Q*: FEM space of the pressure
- *diribcs*: list of the (Dirichlet) boundary conditions
- *dbcinds*: list vortex indices with (Dirichlet) boundary conditions
- *dbcsvs*: list of values of the (Dirichlet) boundary conditions
- *dirip*: list of the (Dirichlet) boundary conditions for the pressure
- *fv*: right hand side of the momentum equation
- *fp*: right hand side of the continuity equation
- *charlen*: characteristic length of the setup
- *odcoo*: dictionary with the coordinates of the domain of observation

```
problem_setups.gen_bccont_fems_3D (scheme='TH', bccontrol=True, verbose=False, strtomeshfile=",
                                         strtophysicalregions=", inflowvel=1.0, inflowprofile='parabola',
                                         movingwallcntrl=False, strtobcsobs=")
```

dictionary for the fem items for a general 3D flow setup

with

- inflow/outflow
- boundary control

Parameters

scheme [{None, 'CR', 'TH'}]] the finite element scheme to be applied, 'CR' for Crouzeix-Raviart, 'TH' for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*

bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to *True*

movingwallcntrl [boolean, optional] whether control is via moving boundaries

Returns

femp [a dictionary with the keys:]

- V : FEM space of the velocity
- Q : FEM space of the pressure
- $diribcs$: list of the (Dirichlet) boundary conditions
- $dbcinds$: list vortex indices with (Dirichlet) boundary conditions
- $dbcvals$: list of values of the (Dirichlet) boundary conditions
- $dirip$: list of the (Dirichlet) boundary conditions for the pressure
- fv : right hand side of the momentum equation
- fp : right hand side of the continuity equation
- $charlen$: characteristic length of the setup
- $odcoo$: dictionary with the coordinates of the domain of observation

```
problem_setups.get_sysmats(problem='gen_bccont', scheme=None, ppin=None, Re=None,
                           nu=None, charvel=1.0, gradvsysmmtrc=True, bccontrol=False,
                           mergerhs=False, onlymesh=False, meshparams={})
```

retrieve the system matrices for stokes flow

Parameters

problem [{‘drivencavity’, ‘cylinderwake’, ‘gen_bccont’, ‘cylinder_rot’}] problem class
N [int] mesh parameter
nu [real, optional] kinematic viscosity, is set to L/Re if Re is provided
Re [real, optional] Reynoldsnumber, is set to L/nu if nu is provided
bccontrol [boolean, optional] whether to consider boundary control via penalized Robin defaults to *False*
mergerhs [boolean, optional] whether to merge the actual rhs and the contribution from the boundary conditions into one rhs, defaults to *False*
onlymesh [boolean, optional] whether to only return *femp*, containing the mesh and FEM spaces, defaults to *False*

Returns

femp [dict]

with the keys:

- V : FEM space of the velocity
- Q : FEM space of the pressure
- $diribcs$: list of the (Dirichlet) boundary conditions
- $dbcinds$: indices of the boundary nodes
- $dbcvals$: values of the boundary nodes
- $invinds$: indices of the inner nodes
- fv : right hand side of the momentum equation
- fp : right hand side of the continuity equation
- $charlen$: characteristic length of the setup

- ν : the kinematic viscosity
- Re : the Reynolds number
- $odcoo$: dictionary with the coordinates of the domain of observation
- **$cdcoo$: dictionary with the coordinates of the domain of *ppin [{int, None}]**
which dof of p is used to pin the pressure, typically **-1** for internal flows, and **None** for flows with outlet control

stokesmatsc [dict]

a dictionary of the condensed matrices:

- M : the mass matrix of the velocity space,
- MP : the mass matrix of the pressure space,
- A : the stiffness matrix,
- JT : the gradient matrix, and
- J : the divergence matrix
- $Jfull$: the uncondensed divergence matrix

and, if **bccontrol=True**, the boundary control matrices that weakly impose $Arob^*v = Brob^*u$, where

- $Arob$: contribution to A
- $Brob$: input operator

if mergerhs

rhsd [dict] $rhsd_vfrc$ and $rhsd_stbc$ merged

else

rhsd_vfrc [dict] of the dirichlet and pressure fix reduced right hand sides

rhsd_stbc [dict]

of the contributions of the boundary data to the rhs:

- fv : contribution to momentum equation,
- fp : contribution to continuity equation

Examples

```
femp, stokesmatsc, rhsd_vfrc, rhsd_stbc = get_sysmats(problem='drivencavity', N=10, nu=1e-2)
```

1.4 data_output_utils

```
data_output_utils.load_or_comp(filestr=None, comprtn=None, comprtnargs={}, arraytype=None, debug=False, loadrtn=None, loadmsg='loaded', savertn=None, savemsg='saved', itsadict=False, numthings=1)
```

routine for caching computation results on disc

Parameters

filestr: {string, list of strings, `None`} where to load/store the computed things, if *None* nothing is loaded or stored

arraytype: {`None`, ‘sparse’, ‘dense’} if not None, then it sets the default routines to save/load dense or sparse arrays

itsadict: **boolean, optional** whether it is *python dictionary* that can be JSON serialized, overrides all other options concerning arrays

savertn: **fun(), optional** routine for saving the computed results, defaults to None, i.e. no saving here

debug: **boolean, optional** no saving or loading, defaults to *False*

```
data_output_utils.output_paraview(V=None, Q=None, VS=None, fstring='nn', invinds=None,  
diribcs=None, dbcinds=None, dbcvvals=None, vp=None,  
vc=None, pc=None, sc=None, sname='nn', ppin=None,  
t=None, tfilter=None, writeoutput=True, vfile=None,  
pfile=None, sfile=None)
```

write the paraview output for a solution (*v,p*) or a scalar *s*

given as coefficients

```
data_output_utils.plot_outp_sig(str_to_json=None, tmeshkey='tmesh', sigkey='outsig', out-  
sig=None, tmesh=None, fignum=222, reference=None,  
tikzstr=None, compress=5, tikzplease=False)
```

plotting output signals

```
data_output_utils.save_output_json(datadict=None, fstring='unspecified_outputfile', mod-  
ule='dolfin_navi_scipy.data_output_utils', plotrou-  
tine='plot_outp_sig')
```

save output to json for postprocessing

1.5 time_int_utils

**CHAPTER
TWO**

INDICES AND TABLES

- genindex
- modindex
- search

PYTHON MODULE INDEX

d

`data_output_utils`, 12

p

`problem_setups`, 7

s

`stokes_navier_utils`, 3

INDEX

C

cyl13D_fems () (*in module problem_setups*), 7
cyl_fems () (*in module problem_setups*), 8

D

data_output_utils
 module, 12
drivcav_fems () (*in module problem_setups*), 9

G

gen_bccont_fems () (*in module problem_setups*), 9
gen_bccont_fems_3D () (*in module problem_setups*), 10
get_pffrommv () (*in module stokes_navier_utils*), 3
get_sysmats () (*in module problem_setups*), 11
get_v_conv_consts () (*in module stokes_navier_utils*), 3

L

load_or_comp () (*in module data_output_utils*), 12

M

module
 data_output_utils, 12
 problem_setups, 7
 stokes_navier_utils, 3

O

output_paraview () (*in module data_output_utils*),
 13

P

plot_outp_sig () (*in module data_output_utils*), 13
problem_setups
 module, 7

S

save_output_json () (*in module data_output_utils*), 13
solve_nse () (*in module stokes_navier_utils*), 4

solve_steadystate_nse () (*in module stokes_navier_utils*), 6
stokes_navier_utils
 module, 3