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# **Development II seminar Documentation**

***Release 0.1***

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**Oct 04, 2018**



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Lecture Notes and materials for The Graduate Center's Economics 842 taught by Jonathan Conning

- [Reading list](#) (links to weekly reading assignments and slides)
- [Syllabus](#) (course details and longer bibliography)

These reading and lecture notes on development microeconomics are written mostly as interactive jupyter notebooks which are kept at this github repository.

- <https://github.com/jhconning/Dev-II>

See below for different ways to access and interact with this content.



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## Development Microeconomics with Jupyter Notebooks

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List of notebooks:

### 1.1 Economics with Jupyter Notebooks

- Jupyter Notebook is “a web application that allows you to create and share documents that contain live code, equations, visualizations and explanatory text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, machine learning and much more.”
- Open-source, browser-based
- Evolved from [ipython notebook](#) to leverage huge scientific python ecosystem.
- Now a ‘language agnostic’ platform so that you can use any of 50+ other kernels including MATLAB, Octave, [Stata](#), Julia, etc.
- **Open-source, fast evolving, large community:** Widely used in academic and scientific computing community.
- **Ties projects together:** Code, data, documentation, output and analysis all in one place.
- **Encourages reproducible science:**
- Easy workflow from exploratory analysis to publish.
- Works well with github and other open-source sharing tools.

#### 1.1.1 Ways to view and run jupyter notebooks

**Jupyter server for interactive computing** Run on a local machine or cloud server to modify code and results on the fly.

- On your computer:
  - [Jupyter notebook on your local machine](#). I recommend using [Anaconda](#) to install Jupyter and scientific python. A good instalation guide [here](#)

- [nteract](#). A good one-click install solution for running notebooks. Provides you with standalone program that installs scientific python and runs jupyter notebooks (not quite full functionality).
- Jupyter notebooks are big in the data science space. Lots of free cloud server solutions are emerging:
  - [Microsoft Azure notebooks](#): Setup a free account for cloud hosted jupyter notebooks.
  - [Google Colaboratory](#): Run notebooks stored on your google drive.
  - [Cocalc](#): jupyter notebooks, SageMath and other cloud hosted services.
  - [Try Jupyter](#): another cloud server, but you can't save work at all.

### Static rendering for presentations and publishing.

- Jupyter notebooks can be rendered in different ways for example as a styled HTML slideshow or page or as a PDF book by using tools and services such as github, nbconvert, [Sphinx](#) and [Read the Docs](#).
- **This very notebook is:**
  - hosted on [the Dev-II repository on github](#) where it is [rendered](#) in simple HTML (though underlying format is json).
  - viewable in HTML or as javascript slideshow [via nbviewer](#). To then see in slideshow mode click on 'present' icon on top right.
  - Tied together with other documents via Sphinx to create a [website on readthedocs](#)
  - Also viewable as a [PDF book on readthedocs](#)

## 1.1.2 A simple jupyter notebook example:

We can combine text, math, code and graph outputs in one place. Let's study a simple economics question:

### Are average incomes per capita converging?

#### Neoclassical growth theory:

- Solow-growth model with Cobb-Douglas technology  $f(k) = k^\alpha$ .
- Technology, saving rate  $s$ , capital depreciation rate  $\delta$ , population growth rate  $n$  and technological change rate  $g$  assumed same across countries.
- Steady-state capital per worker to which countries are converging:

$$k^* = (g/s)^{\frac{1}{\alpha-1}}$$

- Transitional dynamics:

$$\dot{k}(t) = sk(t)^t - (n + g + \delta)k(t)$$

- Diminishing returns to the accumulated factor  $k$  implies convergence:
- Lower initial capital stock implies lower initial per-capita GDP.
- Country that starts with lower capital stock and GDP per capita 'catches up' by growing faster.



## Convergence plots

### Did countries with low levels of income per capita in 1960 grow faster?

I found a dataset from World Penn Tables [on this website](#) (original data source [here](#)).

Let us import useful python libraries for data handling and plots and load the dataset into a pandas dataframe:

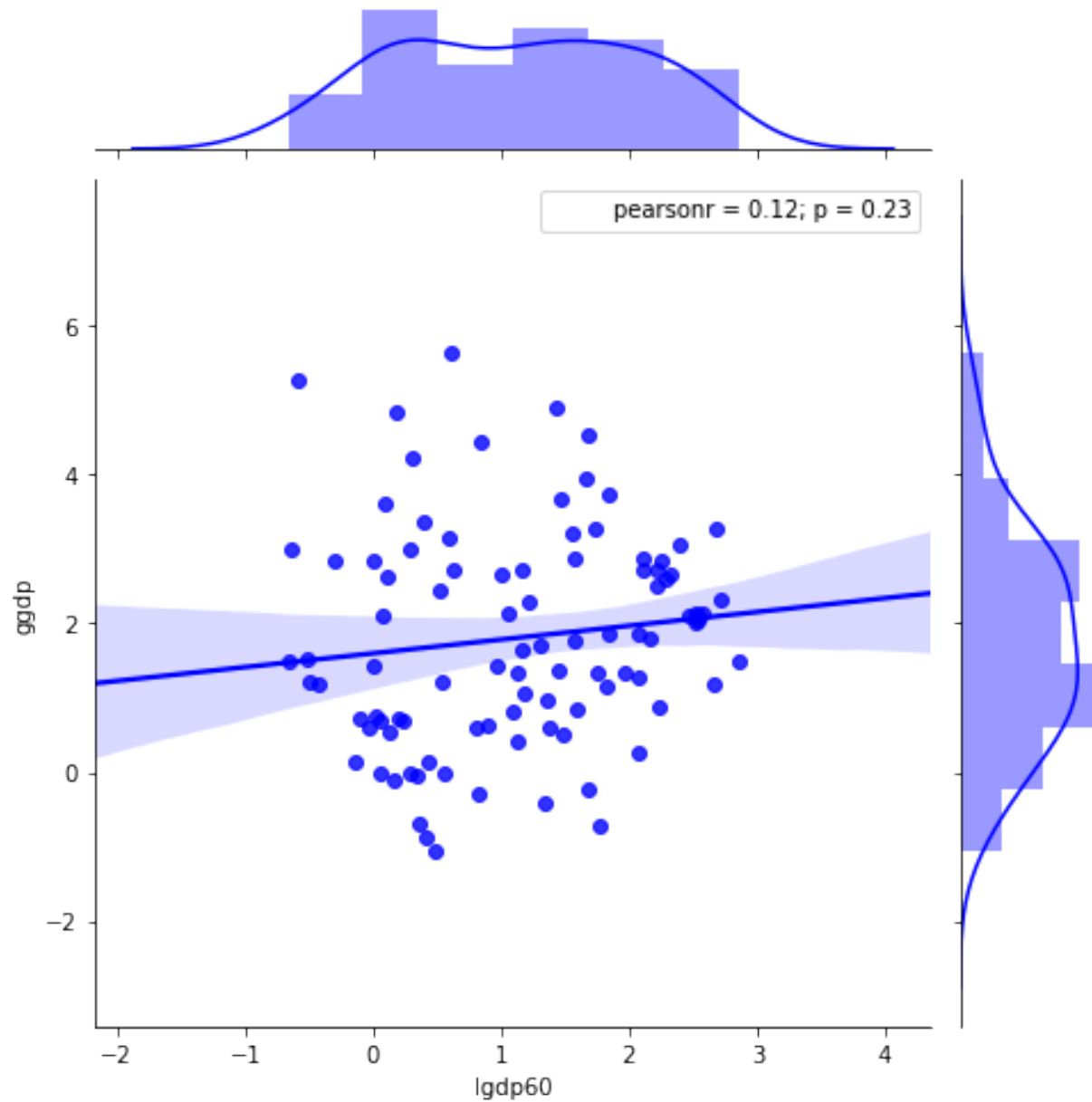
```
In [1]: import matplotlib.pyplot as plt
        %matplotlib inline
        import pandas as pd
        import seaborn as sns
        from ipywidgets import interact

        df = pd.read_stata("./data/country.dta")
```

(in the future we will follow best practice and place library imports at the top of our notebooks).

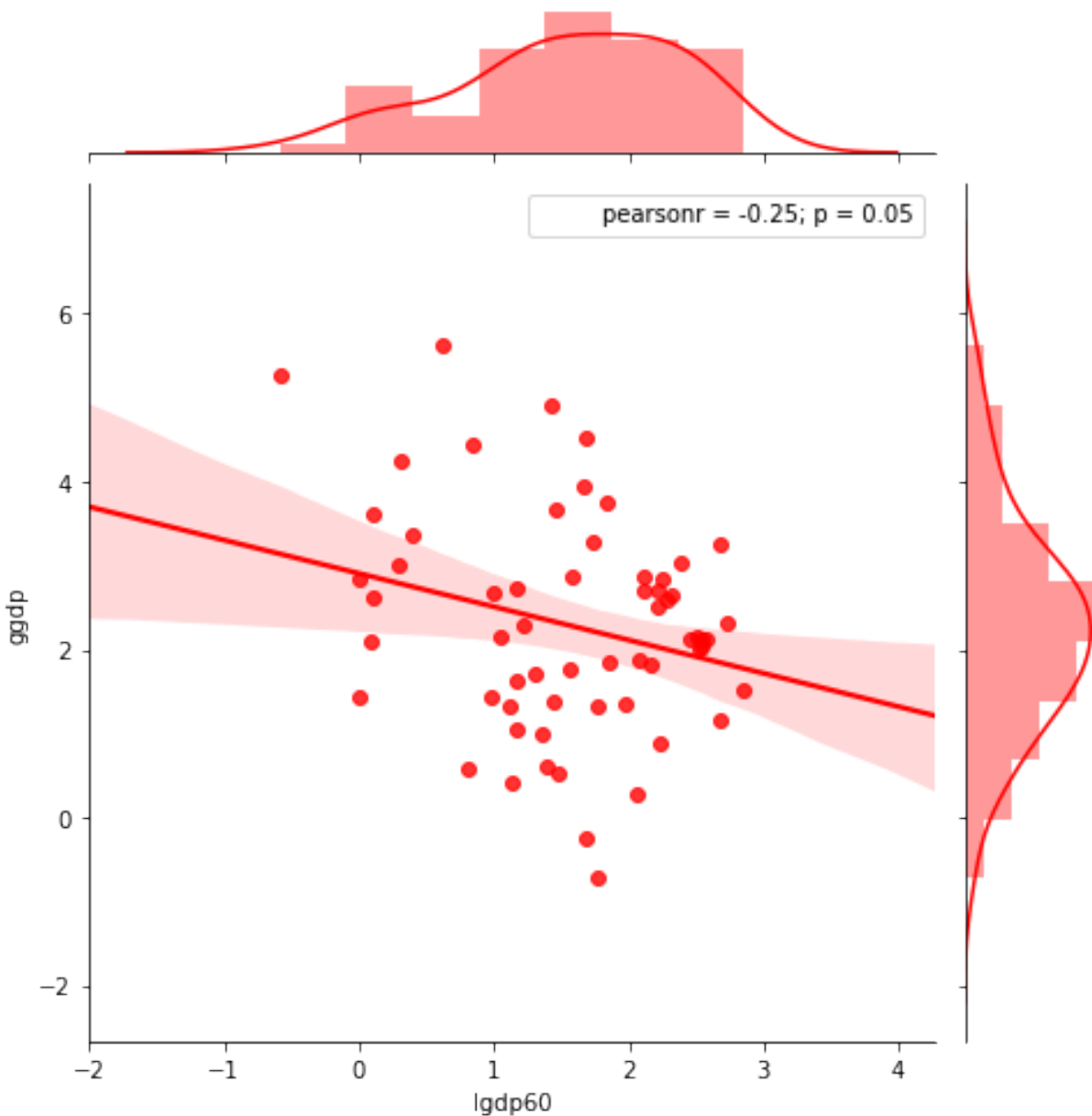
### Scatterplot of log GDP per capita and average growth 1960-2000:

```
In [2]: g = sns.jointplot("lgdp60", "ggdp", data=df, kind="reg",
                          color="b", size=7)
```



**Same plot but excluding African countries:**

```
In [3]: g = sns.jointplot("lgdp60", "ggdp", data=df[df.cont != "Africa"], kind="reg",  
                        color="r", size=7)
```



## Interactive plots

There are ways to make plots like these interactive.

\* On the next slide I use `ipywidgets`: When the notebook is run on a jupyter server 'radio buttons' above the plot allow quick re-plotting for selected country regions.

- Other libraries such as `Bokeh` and `Plotly` create plots with embedded javascript code that allow interactive plot elements even on HTML renderings (i.e. in most browsers even if you do not have a jupyter server running).

Here is how do do a **dropdown menu**. First we write a function to take a 'region' as argument that plots data only for that region (by specifying that filter in the pandas dataframe). We then use `interact` from the `ipywidgets` library to switch quickly between regions.

You'll only see this as truly interactive on a live notebook, not in a static HTML rendering of the same notebook.

```
In [4]: def jplot(region):
        sns.jointplot("lgdp60", "ggdp", data=df[df.cont == region],
                      kind="reg", color="g", size=7)
        plt.show();

In [5]: interact(jplot, region=list(df.cont.unique()))
Out[5]: <function __main__.jplot>
```

### 1.1.3 More on jupyter notebooks and scientific python

- An excellent introduction to Jupyter notebooks and scientific python for economists written by John Stachurski is at this link:

[http://quant-econ.net/py/learning\\_python.html](http://quant-econ.net/py/learning_python.html)

## 1.2 Lucas (1990) “Why Doesn’t Capital Flow from Rich to Poor Countries?”

Summary and analytical notes on the paper:

Lucas, Robert E. (1990) “Why Doesn’t Capital Flow from Rich to Poor Countries?” *American Economic Review*: 92-96.

Note: To change or interact with the code and figures in this notebook first run the *code section* below the ‘Code Section’ below. Then run any code cells above.

### 1.2.1 Neoclassical trade, migration and growth model predictions:

- the convergence of factor prices and incomes per-capita over time (Factor Price Equalization Theorem)
- via substitute mechanisms: factor movement, trade in products, and/or capital accumulation (e.g. Solow).
- rests on assumptions about technology and market competition
  - in particular: diminishing returns to the accumulated factor capital (i.e.  $F_{KK} < 0$ ).

#### Simplest model

Assume two countries with same aggregate production function, where  $X$  is capital stock and  $L$  is labor force:

$$Y = AX^\beta L^{1-\beta}$$

in intensive or per-capita form:

$$y = Ax^\beta$$

where

$$y = \frac{Y}{L} \text{ and } x = \frac{X}{L}$$

Marginal Product of Capital is:

$$r = A\beta x^{\beta-1}$$

**Marginal Product of Capital (MPK) as a function of income per capita:**

$$r = \beta A^{\frac{1}{\beta}} y^{\frac{\beta-1}{\beta}}$$

or

$$r = A\beta \left(\frac{y}{A}\right)^{\frac{\beta-1}{\beta}}$$

Steps:

$$y = Ax^{\beta}$$

so

$$x = \left(\frac{y}{A}\right)^{\frac{1}{\beta}}$$

substitute this into  $r = A\beta x^{\beta-1}$

### The puzzle

- Let capital share  $\beta = 0.4$  (average of USA and India)
- Assume first that  $A$  is the *same* in both countries
- In 1988 income per capita in USA was 15 times higher than India:

$$\frac{y_{US}}{y_{IN}} = 15$$

- this implies MPK in India would have to be:

$$\frac{r_{IN}}{r_{US}} = \left[\frac{y_{US}}{y_{IN}}\right]^{\frac{1-\beta}{\beta}} = 58.1$$

times higher in India! Implausibly large.

What differences in capital per worker account for this large a gap?

As  $x = \left(\frac{y}{A}\right)^{\frac{1}{\beta}}$  this multiple of the amount of capital per worker compared to India:

So if in India the capital-labor ratio is 1 in the USA it must be 871.4 !

- With such huge differences in returns, capital would surely RUSH from USA to India quickly lower the gap in returns and incomes.
- Evidently it does not. So what's wrong with the model? Lucas walks through 4 alternate hypotheses.

### Some plots

```
In [9]: print('Return to capital in India relative to USA : {:.1f}'.format(r(y_IN, A)/r(y_US, A)))
```

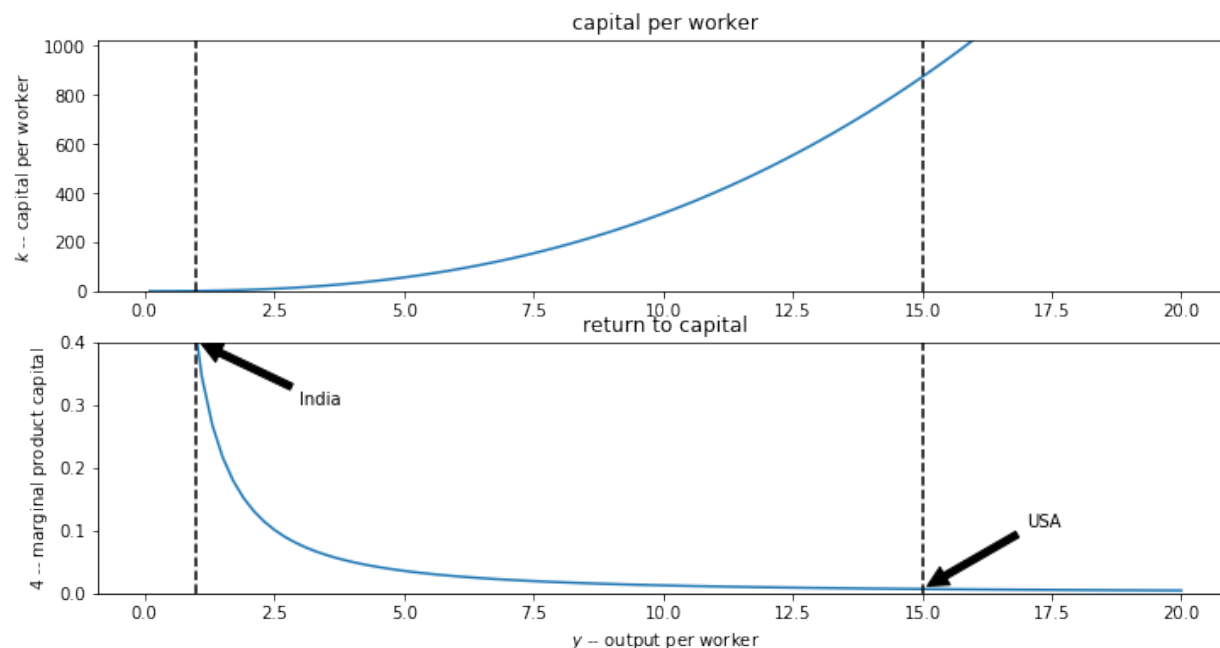
```
Return to capital in India relative to USA : 58.1
```

The ratio of capital stock per worker in the USA compared to India that is implied by this difference in incomes per capita is even more unbelievable:

```
In [10]: print('Capital per worker in USA relative to India : {:.1f}'.format(kap(y_US, A)/kap(y_IN, A)))
```

```
Capital per worker in USA relative to India : 871.4
```

```
In [11]: lucasplot()
```



## 1.2.2 The four hypotheses:

1. Differences in Human Capital (effective labor)
2. External Benefits of Human Capital (increasing returns)
3. Capital Market Imperfections I (political risk)
4. Capital Market Imperfections II (monopoly distortions)

### Hypothesis 1: Differences in Human Capital (or effective labor)

Human capital per worker differs substantially between USA and India. Let's adapt the framework to take this into account.

Redefine  $y$  and  $x$  to be output and capital per *effective* worker.

$$y = \frac{AX^\beta(E \cdot L)^{(1-\beta)}}{E \cdot L}$$

*or*

$$y = Ax^\beta \cdot E^{-\beta}$$

Lucas' cites evidence from Anne Krueger that leads him to suggest a value of  $E = 5$ , which is to say that for the same technology and capital a US worker produces five times as much as his/her Indian counterpart.

To see this first note that if we take the ratio of US to Indian income per effective worker we get:

$$\frac{y_{US}}{y_{IN}} = \frac{Y_{US}}{E \cdot L_{US}} \cdot \frac{L_{IN}}{Y_{IN}} = \frac{15}{5} = 3$$

With this we can easily calculate the implied difference in marginal products of capital:

$$\frac{r(y_{IN})}{r(y_{US})} = \left( \frac{y_{US}}{y_{IN}} \right)^{\frac{1-\beta}{\beta}}$$

$$= 3^{1.5} = 5.2$$

So this lowers the factor of proportionality from 58 to 5. As Lucas puts it: “This is a substantial revision but it leaves the original paradox very much alive: a factor of five difference in rates of return is still large enough to lead one to expect capital flows much larger than anything we observe (p. 93).”

## Hypothesis 2: External Benefits to Human Capital

We’ve assumed thus far that the total factor productivity parameter  $A$  is the same across countries. This is unlikely. An easy way to resolve the paradox is to simply solve for the level of  $A_{US}/A_{IN}$  that makes the gap disappear.

In this section Lucas isn’t quite doing that but he is in effect letting the values of  $A$  differ between the two countries. He motivates this with a stripped down version of his own Lucas (1988) paper on external economies, a model where human capital plays a role and where there is a positive external effect in human-capital accumulation in that he assumes that the marginal product of one’s human capital is augmented by the average level of human capital in the economy. In this paper he doesn’t work out this model in full but uses the story to rewrite the production function as:

$$y = Ah^\gamma x^\beta$$

and he then gives us some ‘guesstimates’ as to the differences in human capital per worker in each country. As it turns out this is in effect equivalent to sticking to the original model and just assuming that

$$\frac{A_{US}}{A_{IN}} = 5$$

A country with a higher level of  $A$  will have everywhere higher return on capital. After a little math the adjusted ratio now becomes

$$\begin{aligned} \frac{r(y_{IN})}{r(y_{US})} &= \left( \frac{y_{US}}{y_{IN}} \right)^{\frac{1-\beta}{\beta}} \\ &= \frac{3^{1.5}}{5} = 1.04 \end{aligned}$$

This would seem to almost resolve the paradox but Lucas in fact dismisses it as not entirely realistic. It assumes for example that knowledge spillovers across borders are zero.

### ### Hypothesis 3: Capital Market Imperfections I (political risk)

I haven’t written this up yet...

### ### Hypothesis 4: monopoly power distortions to capital market

This is the least often mentioned but possibly the most interesting of Lucas’ hypotheses. The hypothesis is that local elites (or ‘an imperial power’) are able to collude to control the entry of capital into India to drive up capital rents and drive down real wages in such a way that increase firm profits. Implicitly the story is that local elites control the organization of production and hence firm profits.

While at first it might seem far-fetched to believe a story like this in modern times (less far-fetched in the time of the East India Company) there is plenty of evidence that capital inflows into India and other developing countries were historically controlled in part to protect the rents of local elites. Rajan and Zingales’ (2013) book [Saving Capitalism from the Capitalists](#) is full of examples of local elites lobbying government bureaucrats to establish market power.

The elite is assumed to have access to international capital markets where they can borrow capital at the rate  $\rho$

If the elite ran the country as a monopoly they would choose the capital-labor ratio (by varying how much capital enters the country) to maximize profits per capita:

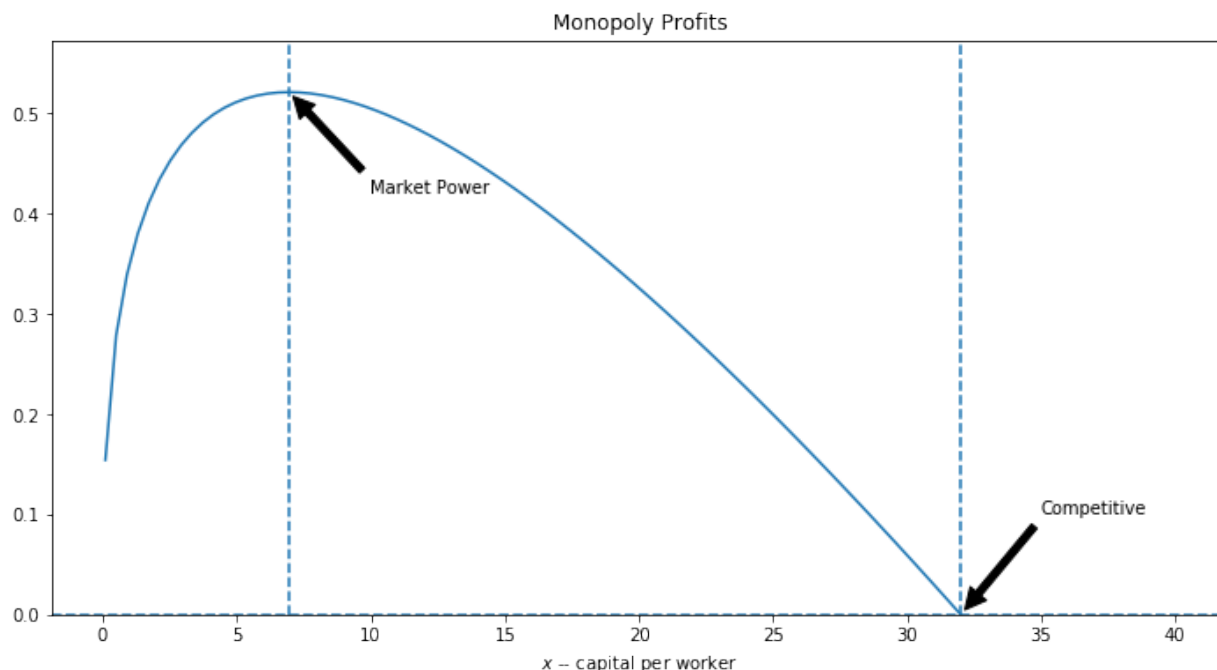
$$f(x) - [f(x) - xf'(x)] - rx$$

Here (by Euler's Theorem)  $f(x) - xf'(x)$  is the wage.

Implicit in Lucas' formulation is the idea that there is a large monopolist and a 'fringe' of smaller competitive firms, all producing the common output. The competitive fringe hires factors until the marginal value product equals the market factor price. By restricting capital into the economy the monopolists drives up the domestic rental price of capital. With less capital to work with the competitive fringe hires less labor and this drives down the market equilibrium real wage. Monopolist profits rise due to this monopsony power effect.

$$f(x) - [f(x) - xf'(x)] - rx$$

```
In [12]: monopolyplot(rho=0.05)
```



In an open competitive economy all firm profits would be zero and the capital-labor ratio  $x$  would be simply set from

$$f'(x) = \rho$$

where  $\rho$  is the world market price of capital. This implies we can find the capital labor ratio as a function of  $\rho$ :

$$\beta x^{\beta-1} = \rho$$

$$x = \left(\frac{\rho}{\beta}\right)^{\frac{1}{\beta-1}}$$

Suppose we normalize the world rate of return to capital to  $\rho = 0.05$ . Then equilibrium capital per worker in the competitive economy would be:

```
In [13]: rho = 0.05
         beta = 0.4
         xC, _ = xoptimum(rho, beta)
         print('%.1f' % xC)
```

```
32.0
```

The monopolist on the other hand sets  $x$  to:

$$f'(x) = \rho - xf''(x)$$



Note that  $f''(x)$  can be written:

$$f''(x) = (\beta - 1) \frac{f'(x)}{x}$$

which allows us to simplify the monopoly FOC to:

$$f'(x) = \frac{\rho}{\beta}$$

Given our assumed value of beta, this means a monopolist might maintain the rate of return on capital  $1/\beta$  times higher than the international market rate  $\rho$ , and resists capital inflows that might push this rate down.

Solving for  $x$  in this monopolized market gives:

$$x = \left( \frac{\rho}{\beta^2} \right)^{\frac{1}{\beta-1}}$$

```
In [14]: _ , xM = xoptimum(rho, beta)
         print('Capital per worker in the market-power distorted equilibrium: {:.1f}'.format(xM))
         print(' {:.0%} of the competitive level'.format(xM/xC))
```

```
Capital per worker in the market-power distorted equilibrium: 6.9
22% of the competitive level
```

Indian income per capita will be only

```
In [15]: print("{:5.0f} percent".format(100*f(xM)/f(xC)))
54 percent
```

as high as in a competitive market without barriers to capital inflows.

This is an inefficient outcome but the elites who capture the profits/rents do pretty well. They earn

```
In [16]: print('profits = {:.2f} or {:.0%} of total output'.format(profit(xM, rho), profit(xM, rho)/total_output))
profits = 0.52 or 24% of total output
```

### 1.2.3 Extensions

Lucas does not cite their work but models of this sort were explored by trade economists in the 1980s (e.g. Feenstra (1980).

In a model described later, Conning (2006) explores similar factor market power distortions in a model with heterogeneous agents to make predictions about the size distribution of firms within each sector.

#### Solving for maxima and roots numerically

This was an easy model to solve analytically, but let's solve it numerically to illustrate the use of the root solving (fsolve function from scipy optimize library) and minimization techniques (brentq from the same library). See the code section below first.

Let's setup first order condition functions to find the  $x$  that sets them to zero.

For the competitive case:  $f'(x) - \rho$  For the monopoly case:  $f'(x) - \rho - x f''(x)$

```
In [17]: def cfoc(x):
         return mpx(x) - rho

         def mfoc(x):
         return mpx(x) - rho + (beta-1)*mpx(x)
```

and solve for the root:

```
In [18]: from scipy.optimize import fsolve, brentq, minimize
        xC = fsolve(cfoc, 20)[0]
        xC
```

```
Out[18]: 32.0
```

Same as the analytical solution of course.

Now let's find the  $x$  that maximizes monopoly profits (we need to provide a guess value):

```
In [19]: xM = fsolve(mfoc, 5)[0]
        xM
```

```
Out[19]: 6.9489090984826332
```

We could have instead directly maximized profits (minimized negative profits) with an optimization routine:

```
In [20]: def negprofit(x):
        return - profit(x, rho)

        res = minimize(negprofit, 5, method='Nelder-Mead')
        res.x[0]
```

```
Out[20]: 6.94891357421875
```

### ## Code Section

To unclutter the notebook above much of the code has been moved to this section. To re-create or modify any content go to the 'Cell' menu above run all code cells below by choosing 'Run All Below'. Then 'Run all Above' to recreate all output above (or go to the top and step through each code cell manually).

```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt

In [2]: A = 1
        rho = 0.05
        beta = 0.4
        y_US = 15
        y_IN = 1

        def f(x):
            return x**beta

        def mpx(x):
            return beta*x**(beta-1)

        def r(y, A):
            return beta * (A**(1-beta)) * y**((beta-1)/beta)

        def kap(y, A):
            return (y/A)**(1/beta)

In [3]: def xoptimum(rho, beta):
        xC = (rho/beta)**(1/(beta-1))
        xM = (rho/beta**2)**(1/(beta-1))
        return xC, xM

In [4]: def lucasplot():
        y = np.linspace(0.1, 20, 100)
        plt.figure(figsize=(12, 6))
        plt.subplot(2, 1, 1)
        plt.plot(y, kap(y, A))
```

```

plt.ylabel("$k$ -- capital per worker")
plt.ylim(0, kap(y_US +1, A))
plt.axvline(y_IN, color='k', ls='dashed')
plt.axvline(y_US, color='k', ls='dashed')
plt.title("capital per worker")

plt.subplot(2,1,2)
plt.plot(y, r(y,A))
plt.xlabel("$y$ -- output per worker")
plt.ylabel("$4$ -- marginal product capital")
plt.title("return to capital")
plt.ylim(0, r(y_IN,A))
plt.axvline(y_IN, color='k', ls='dashed')
plt.axvline(y_US, color='k', ls='dashed')
plt.annotate('India', xy=(y_IN, r(y_IN,A)), xytext=(y_IN +2, r(y_IN,A)*0.75),
             arrowprops=dict(facecolor='black', shrink=0.05),)
plt.annotate('USA', xy=(y_US, r(y_US,A)), xytext=(y_US +2, r(y_US,A)+0.1),
             arrowprops=dict(facecolor='black', shrink=0.05),);

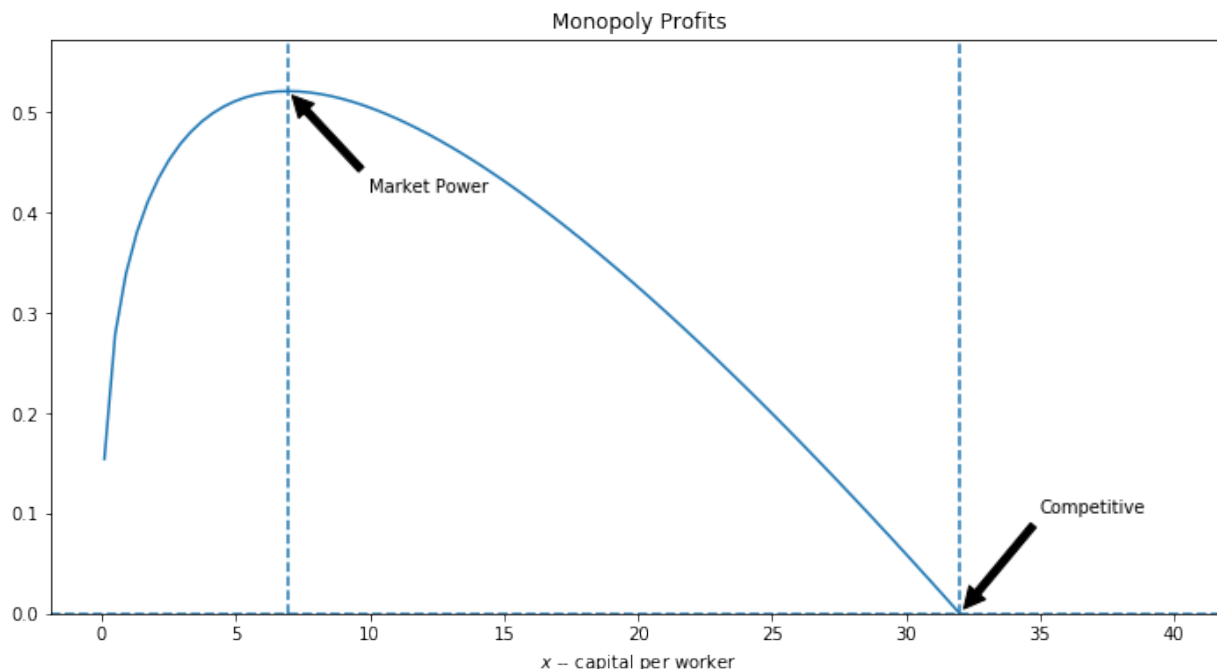
In [5]: def profit(x, rho):
        return f(x) -(f(x) -mpx(x)*x) - rho*x

In [6]: x = np.linspace(0.1,40,100)

def monopolyplot(rho):
    prf = profit(x, rho)
    xC, xM = xoptimum(rho, beta)
    fig, ax = plt.subplots(1, 1, figsize=(12, 6))
    plt.plot(x,prf)
    plt.ylim(0,max(prf)*1.1)
    plt.title('Monopoly Profits')
    plt.axvline(xC,ls='dashed')
    plt.axvline(xM,ls='dashed')
    plt.axhline(0,ls='dashed')
    plt.xlabel("$x$ -- capital per worker")
    plt.annotate('Competitive', xy=(xC, 0), xytext=(xC+3, 0.1),
                arrowprops=dict(facecolor='black', shrink=0.05),)
    plt.annotate('Market Power', xy=(xM, profit(xM,rho)), xytext=(xM+3, profit(xM,rho)-0.1),
                arrowprops=dict(facecolor='black', shrink=0.05),)
    plt.show()

In [7]: monopolyplot(0.05)

```



## 1.3 Edgeworth Box: Efficiency in production allocation

### 1.3.1 Efficiency in production

Consider a small-open economy with two production sectors – agriculture and manufacturing – with production in each sector taking place with constant returns to scale production functions. Producers in the agricultural sector maximize profits

$$\max_{K_A, L_A} p_A F(K_A, L_A) - wL_A - rK_A$$

And producers in manufacturing similarly maximize

$$\max_{K_M, L_M} p_M F(K_M, L_M) - wL_M - rK_M$$

In equilibrium total factor demands must equal total supplies:

$$K_A + K_M = \bar{K}$$

$$L_A + L_M = \bar{L}$$

The first order necessary conditions for an interior optimum in each sector lead to an equilibrium where the following condition must hold:

$$\frac{F_L(K_A, L_A)}{F_K(K_A, L_A)} = \frac{w}{r} = \frac{G_L(\bar{K} - K_A, \bar{L} - L_A)}{F_K(\bar{K} - K_A, \bar{L} - L_A)}$$

Efficiency requires that the marginal rates of technical substitutions (MRTS) be equalized across sectors (and across firms within a sector which is being assumed here). In an Edgeworth box, isoquants from each sector will be tangent to a common wage-rental ratio line.

If we assume Cobb-Douglas forms  $F(K, L) = K^\alpha L^{1-\alpha}$  and  $G(K, L) = K^\beta L^{1-\beta}$  the efficiency condition can be used to find a closed form solution for  $K_A$  in terms of  $L_A$ :

$$\frac{(1-\alpha) K_A}{\alpha L_A} = \frac{w}{r} = \frac{(1-\beta) \bar{K} - K_A}{\beta \bar{L} - L_A}$$

Rearranging the expression above we can get a closed-form expression for the efficiency locus  $K_A(L_A)$ :

$$K_A(L_A) = \frac{L_A \cdot \bar{K}}{\frac{\beta(1-\alpha)}{\alpha(1-\beta)}(\bar{L} - L_A) + L_A}$$

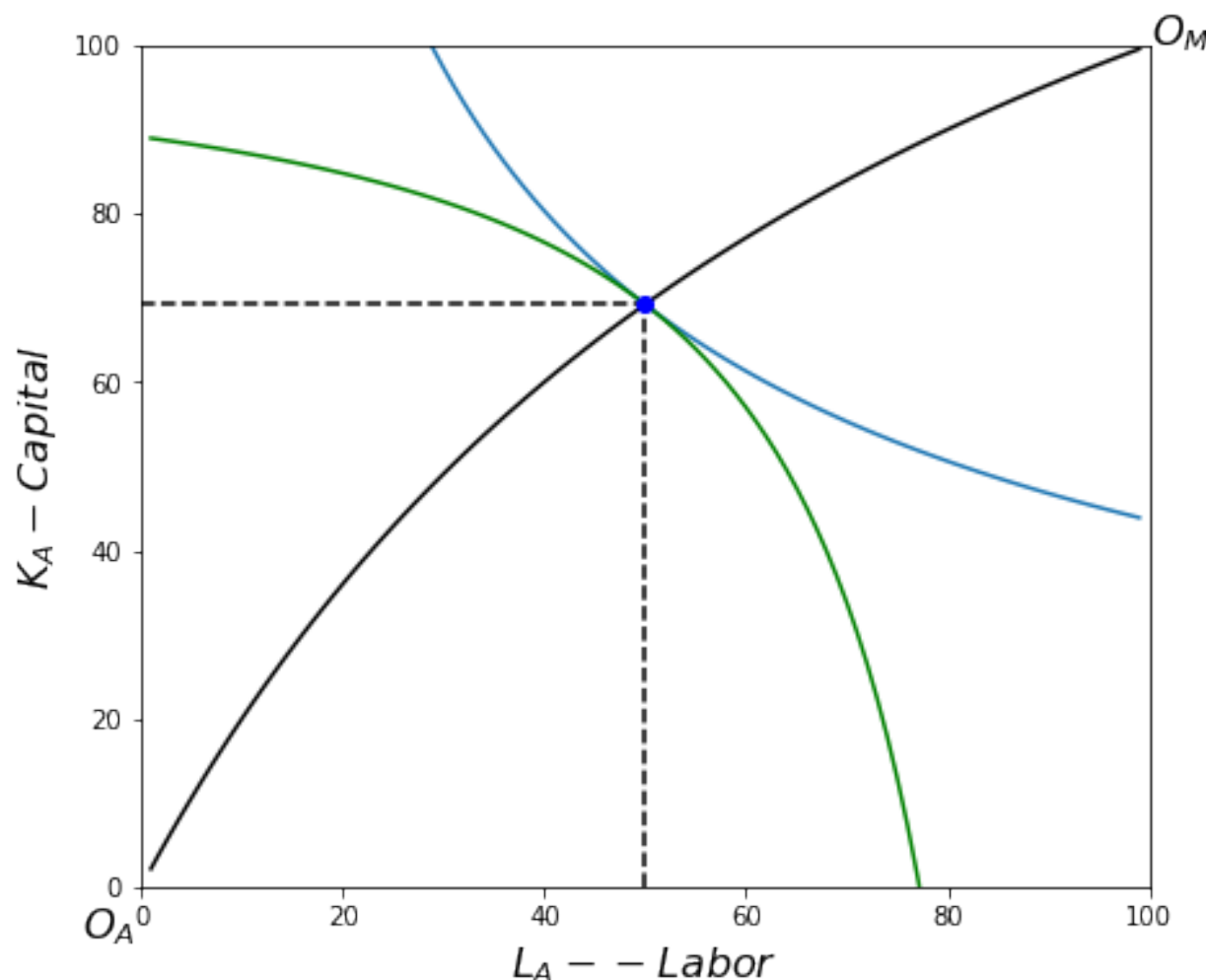
With this we can now plot the *efficiency locus* curve in an Edgeworth box.

### 1.3.2 Edgeworth Box plots

Here is an Edgeworth Box depicting the situation where  $L_A = 50$  units of labor are allocated to the agricultural sector and all other allocations are efficient (along the efficiency locus).

```
In [76]: edgeplot(50)
```

```
(LA, KA) = (50.0, 69.2)    (QA, QM) = (60.8, 41.2)    RTS = 2.1
```



If you're reading this using a jupyter server you can interact with the following plot, changing the technology parameters and position of the isoquant. If you are not this may appear blank or static.

```
In [75]: LA = 50
        interact(edgeplot, LA=(10, LBAR-10,1),
                  Kbar=fixed(KBAR), Lbar=fixed(LBAR), z
                  alpha=(0.1,0.9,0.1),beta=(0.1,0.9,0.1));

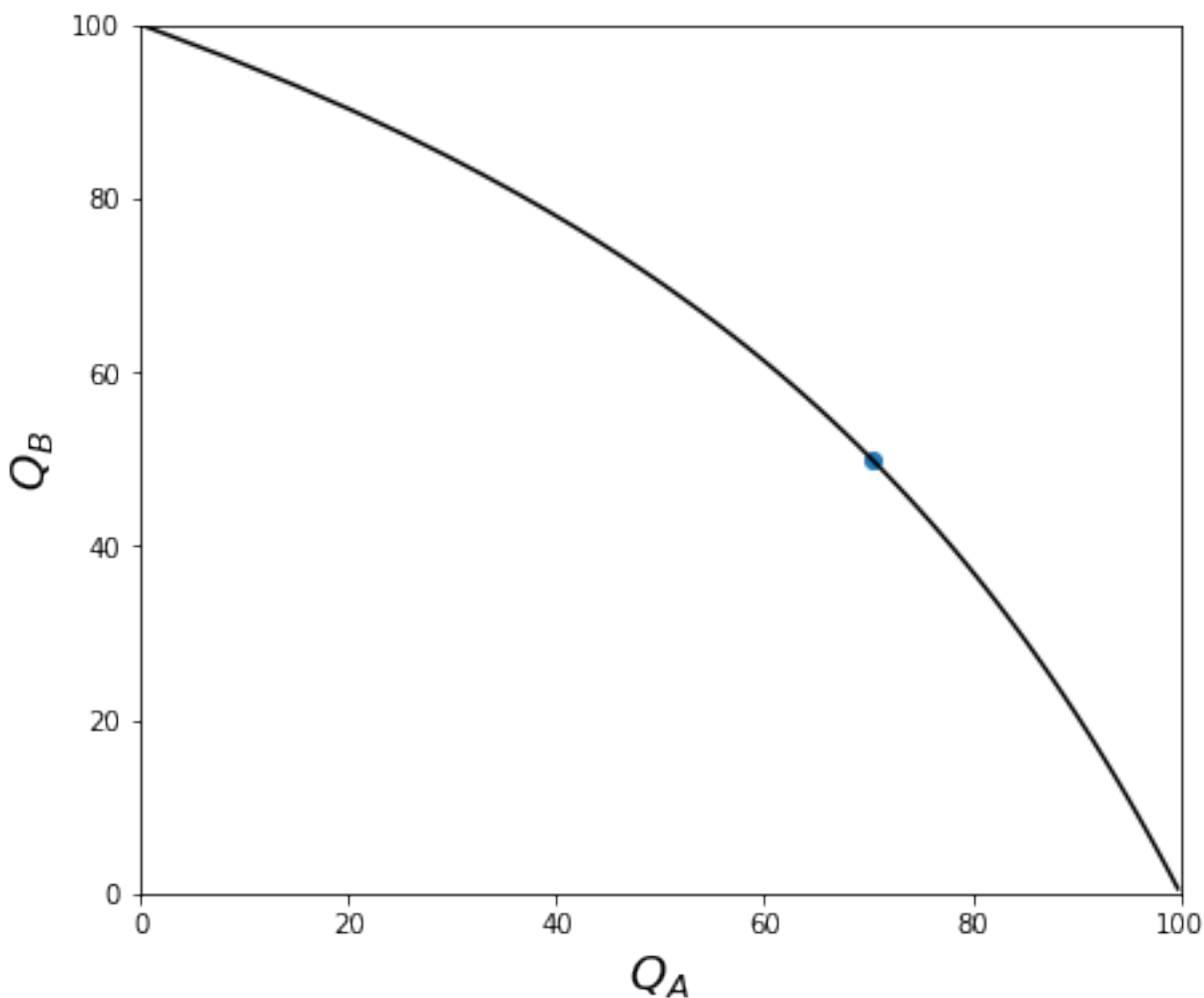
interactive(children=(IntSlider(value=50, description='LA', max=90, min=10), FloatSlider(value=0.6, o
```

## The Production Possibility Frontier

The **efficiency locus** also allows us to trace out the production possibility frontier: by varying  $L_A$  from 0 to  $\bar{L}$  and, for every  $L_A$ , calculating  $K_A(L_A)$  and with that efficient production  $(q_A, q_B)$  where  $q_A = F(K_A(L_A), L_A)$  and  $q_B = F(\bar{K} - K_A(L_A), \bar{L} - L_A)$ .

For Cobb-Douglas technologies the PPF will be quite straight unless  $\beta$  and  $\alpha$  are very different from each other.

```
In [37]: fig, ax = plt.subplots(figsize=(7,6))
        ppf(30,alpha =0.8, beta=0.2)
```



### 1.3.3 Efficient resource allocation and comparative advantage in a small open economy

We have a production possibility frontier which also tells us the opportunity cost of producing different amounts of good  $A$  in terms of how much of good  $B$  (via its slope or the Rate of Product Transformation (RPT)  $\frac{MC_A}{MC_B}$ ). This is given by the slope of the PPF. The bowed out shape of the PPF tells us that the opportunity cost of producing either good is rising in its quantity.

How much of each good will the economy produce? If this is a competitive small open economy then product prices will be given by world prices. Each firm maximizes profits which leads every firm in the  $A$  sector to increase output until  $MC_X(q_a) = P_A$  and similarly in the  $B$  sector so that in equilibrium we must have

$$\frac{MC_A}{MC_B} = \frac{P_A}{P_B}$$

and the economy will produce where the slope of the PPF exactly equals the world relative price. This is where national income valued at world prices is maximized, and the country is producing according to comparative advantage.

Consumers take this income as given and maximize utility. If we make heroic assumptions about preferences (preferences are identical and homothetic) then we can represent consumer preferences on the same diagram and we would have consumers choosing a consumption basket somewhere along the *consumption possibility frontier* given by the world price line passing through the production point.

If the economy is instead assumed to be closed then product prices must be calculated alongside the resource allocation. The PPF itself becomes the economy's budget constraint and we find an optimum (and equilibrium autarky domestic prices) where the community indifference curve is tangent to the PPF.

As previously noted, given our linear homogenous production technology, profit maximization in agriculture will lead firms to choose inputs to satisfy  $\frac{(1-\alpha)K_A}{\alpha L_A} = \frac{w}{r}$ . This implies a relationship between the optimal production technique or capital-labor intensity  $\frac{K_A}{L_A}$  in agriculture and the factor price ratio  $\frac{w}{r}$ :

$$\frac{K_A}{L_A} = \frac{\alpha}{1-\alpha} \frac{w}{r}$$

and similarly in manufacturing

$$\frac{K_M}{L_M} = \frac{\beta}{1-\beta} \frac{w}{r}$$

From the first order conditions we also have:

$$P_A F_L(K_A, L_A) = w = P_M G_L(K_M, L_M)$$

Note this condition states that competition has driven firms to price at marginal cost in each industry or  $P_A = MC_A = w \frac{1}{F_L}$  and  $P_M = MC_M = w \frac{1}{G_L}$  which in turn implies that at a market equilibrium optimum

$$\frac{P_A}{P_M} = \frac{G_L(K_M, L_M)}{F_L(K_A, L_A)}$$

This states that the world price line with slope (negative)  $\frac{P_A}{P_M}$  will be tangent to the production possibility frontier which as a slope (negative)  $\frac{MC_A}{MC_M} = \frac{P_A}{P_M}$  which can also be written as  $\frac{G_L}{F_L}$  or equivalently  $\frac{G_K}{F_K}$ . The competitive market leads producers to move resources across sectors to maximize the value of GDP at world prices.

With the Cobb Douglas technology we can write:

$$F_L = (1-\alpha) \left[ \frac{K_A}{L_A} \right]^\alpha$$

$$G_L = (1-\beta) \left[ \frac{K_M}{L_M} \right]^\beta$$

Using these expressions and the earlier expression relating  $\frac{K_A}{L_A}$  and  $\frac{K_M}{L_M}$  to  $\frac{w}{r}$  we have:

$$\frac{P_A}{P_M} = \frac{1 - \alpha}{1 - \beta} \frac{\left[ \frac{(1-\beta)w}{\beta r} \right]^\beta}{\left[ \frac{(1-\alpha)w}{\alpha r} \right]^\alpha}$$

or

$$\frac{P_A}{P_M} = \Gamma \left[ \frac{w}{r} \right]^{\beta - \alpha}$$

where

$$\Gamma = \frac{1 - \alpha}{1 - \beta} \left( \frac{\alpha}{1 - \alpha} \right)^\alpha \left( \frac{1 - \beta}{\beta} \right)^\beta$$

Solving for  $\frac{w}{r}$  as a function of the world prices we find an expression for the ‘Stolper-Samuelson’ (SS) line:

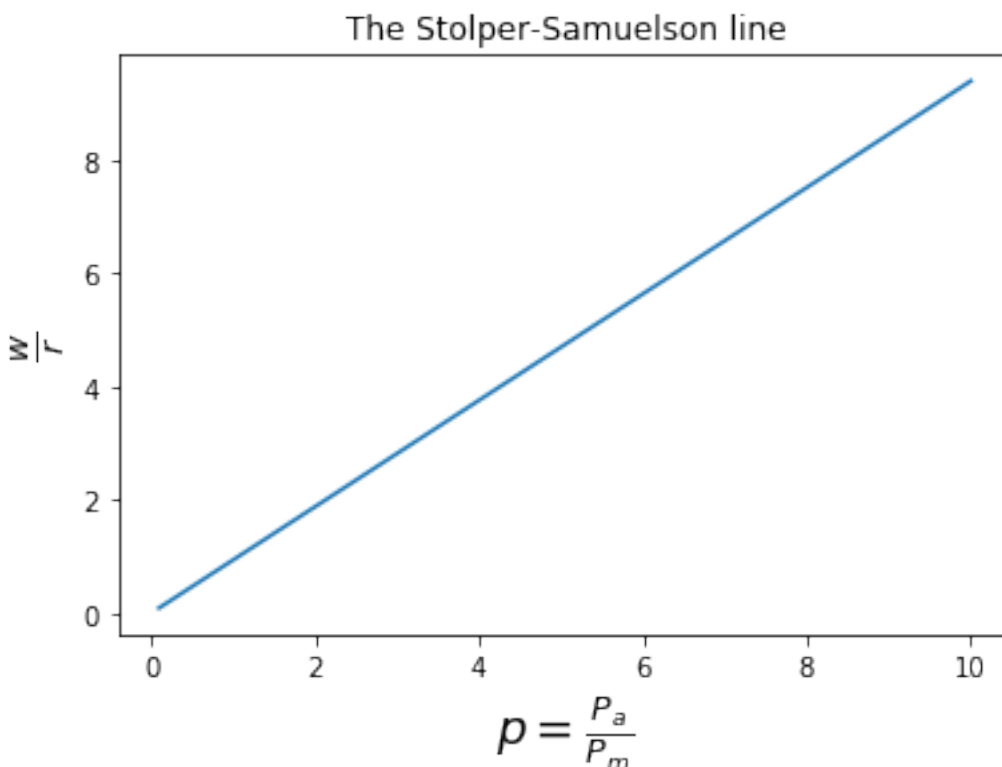
$$\frac{w}{r} = \frac{1}{\Gamma} \left[ \frac{P_A}{P_M} \right]^{\frac{1}{\beta - \alpha}}$$

## The Stolper Samuelson Theorem

The Stolper Samuelson theorem tells us how changes in the world relative price of products translates into changes in the relative price of factors and therefore in the distribution of income in society.

The theorem states that an increase in the relative price of a good will lead to an increase in both the relative and the real price of the factor used intensively in the production of that good (and conversely to a decline in both the real and the relative price of the other factor).

In [55]: `ssline(a=0.6, b=0.3);`





This relationship can be seen from the formula. If agriculture were more labor intensive than manufacturing, so  $\alpha < \beta$ , then an increase in the relative price of agricultural goods creates an incipient excess demand for labor and an excess supply of capital (as firms try to expand production in the now more profitable labor-intensive agricultural sector and cut back production in the now relatively less profitable and capital-intensive manufacturing sector). Equilibrium will only be restored if the equilibrium wage-rental ratio falls which in turn leads firms in both sectors to adopt more capital-intensive techniques. With more capital per worker in each sector output per worker and hence real wages per worker  $\frac{w}{P_a}$  and  $\frac{w}{P_m}$  increase.

### To be completed

- Code to solve for unique HOS equilibrium as a function of world relative price  $\frac{P_A}{P_M}$
- Interactive plot with  $\frac{P_A}{P_M}$  slider that plots equilibrium in Edgeworth box and PPF

### 1.3.4 Code section

To keep the presentation tidy I've put the code that is used in the notebook above at the end of the notebook. Run all code below this cell first. Then return and run all code above.

```
In [10]: %matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
from ipywidgets import interact, fixed

In [11]: ALPHA = 0.6    # capital share in agriculture
BETA = 0.4    #

KBAR = 100
LBAR = 100

p = 1        # =Pa/Pm  relative price of ag goods

def F(K,L,alpha=ALPHA):
    """Agriculture Production function"""
    return (K**alpha)*(L**(1-alpha))

def G(K,L,beta=BETA):
    """Manufacturing Production function"""
    return (K**beta)*(L**(1-beta))

def budgetc(c1, p1, p2, I):
    return (I/p2)-(p1/p2)*c1

def isoq(L, Q, mu):
    return (Q/(L**(1-mu)))**(1/mu)

In [12]: def edgeworth(L, Kbar=KBAR, Lbar=LBAR,alpha=ALPHA, beta=BETA):
    """efficiency locus: """
    a = (1-alpha)/alpha
    b = (1-beta)/beta
    return b*L*Kbar/(a*(Lbar-L)+b*L)

In [13]: def edgeplot(LA, Kbar=KBAR, Lbar=LBAR,alpha=ALPHA,beta=BETA):
    """Draw an edgeworth box

    arguments:
    LA -- labor allocated to ag, from which calculate QA(Ka(La),La)
```

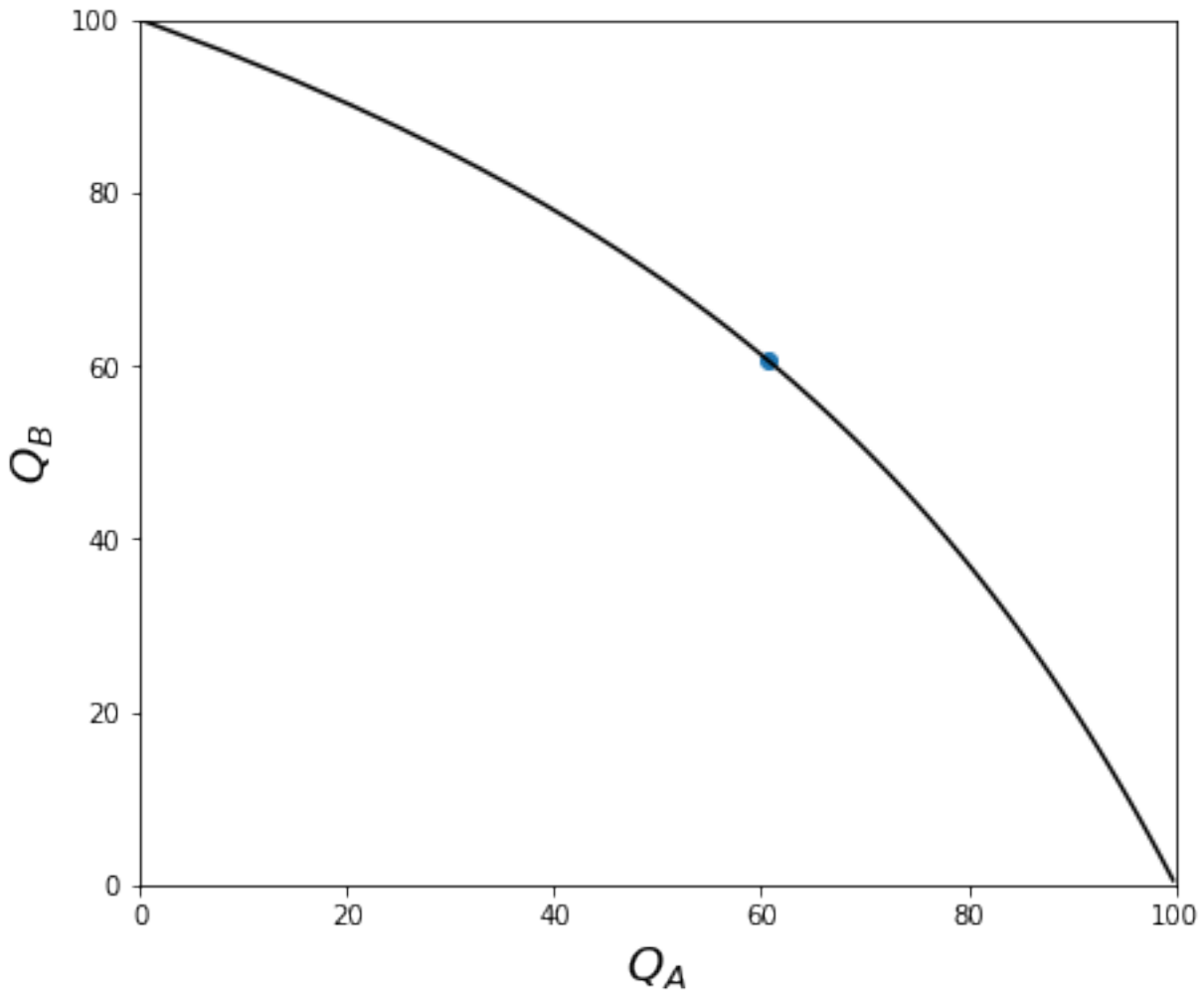
```
"""
KA = edgeworth(LA, Kbar, Lbar,alpha, beta)
RTS = (alpha/(1-alpha))*(KA/LA)
QA = F(KA,LA,alpha)
QM = G(Kbar-KA,Lbar-LA,beta)
print("(LA,KA)=({:4.1f}, {:4.1f})  (QA, QM)=({:4.1f}, {:4.1f})  RTS={:4.1f}"
      .format(LA,KA,QA,QM,RTS))
La = np.arange(1,Lbar)
fig, ax = plt.subplots(figsize=(7,6))
ax.set_xlim(0, Lbar)
ax.set_ylim(0, Kbar)
ax.plot(La, edgeworth(La,Kbar,Lbar,alpha,beta),'k-')
#ax.plot(La, La, 'k--')
ax.plot(La, isoq(La, QA, alpha))
ax.plot(La, Kbar-isoq(Lbar-La, QM, beta),'g-')
ax.plot(LA, KA, 'ob')
ax.vlines(LA,0,KA, linestyle="dashed")
ax.hlines(KA,0,LA, linestyle="dashed")
ax.text(-6,-6,r'$O_A$', fontsize=16)
ax.text(Lbar,Kbar,r'$O_M$', fontsize=16)
ax.set_xlabel(r'$L_A$ -- Labor$', fontsize=16)
ax.set_ylabel(r'$K_A$ - Capital$', fontsize=16)
plt.show()
```

```
In [14]: def ppf(LA,Kbar=KBAR, Lbar=LBAR,alpha=ALPHA,beta=BETA):
        """Draw a production possibility frontier

        arguments:
        LA -- labor allocated to ag, from which calculate QA(Ka(La),La)
        """
        KA = edgeworth(LA, Kbar, Lbar,alpha, beta)
        RTS = (alpha/(1-alpha))*(KA/LA)
        QA = F(KA,LA,alpha)
        QM = G(Kbar-KA,Lbar-LA,beta)
        ax.scatter(QA,QM)
        La = np.arange(0,Lbar)
        Ka = edgeworth(La, Kbar, Lbar,alpha, beta)
        Qa = F(Ka,La,alpha)
        Qm = G(Kbar-Ka,Lbar-La,beta)
        ax.set_xlim(0, Lbar)
        ax.set_ylim(0, Kbar)
        ax.plot(Qa, Qm, 'k-')
        ax.set_xlabel(r'$Q_A$', fontsize=18)
        ax.set_ylabel(r'$Q_B$', fontsize=18)
        plt.show()
```

It's interesting to note that for Cobb-Douglas technologies you really need quite a difference in capital-intensities between the two technologies in order to get much curvature to the production function.

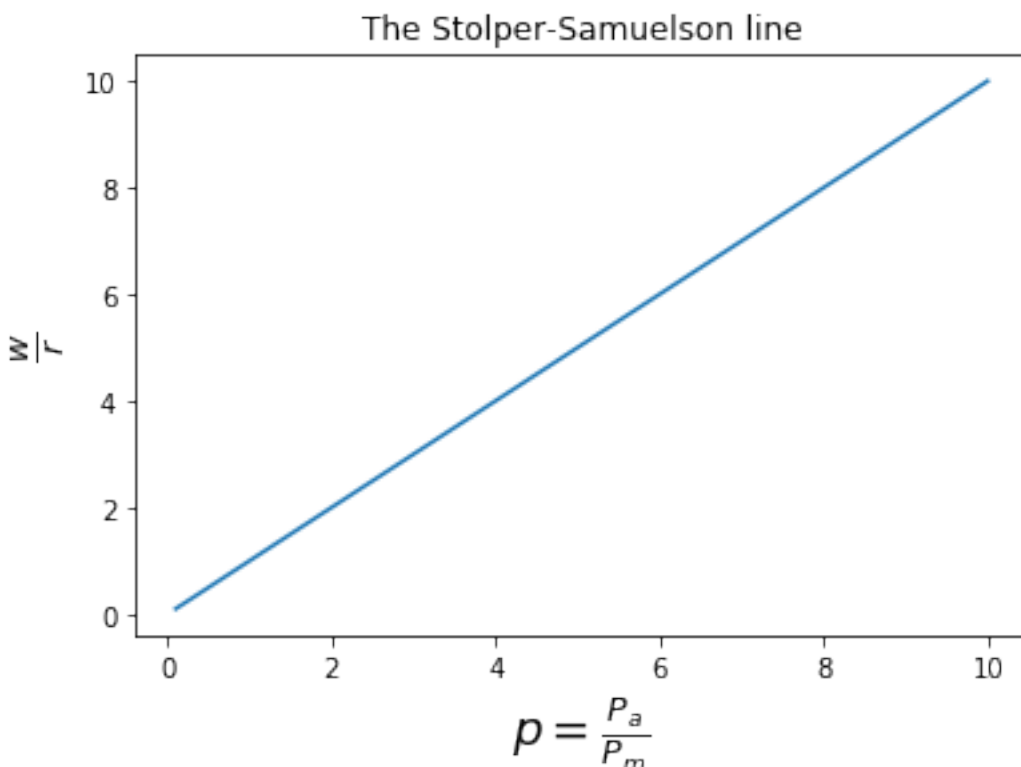
```
In [15]: fig, ax = plt.subplots(figsize=(7,6))
        ppf(20,alpha =0.8, beta=0.2)
```



```
In [16]: def wreq(p,a=ALPHA, b=BETA):
          B = ((1-a)/(1-b))*(a/(1-a))**a * ((1-b)/b)**b
          return B*p

In [17]: def ssline(a=ALPHA, b=BETA):
          p = np.linspace(0.1,10,100)
          plt.title('The Stolper-Samuelson line')
          plt.xlabel(r'$p = \frac{P_a}{P_m}$', fontsize=18)
          plt.ylabel(r'$\frac{w}{r}$', fontsize=18)
          plt.plot(p,wreq(p, a, b));

In [18]: ssline()
```



## 1.4 The Specific Factors or Ricardo-Viner Model

### 1.4.1 Background

The SF model is a workhorse model in trade, growth, political economy and development. We will see variants of the model used to describe rural to urban migration, the Lewis model and other dual sector models of sectoral misallocation, models such as the Harris-Todaro model that explain migration and the urban informal sector. The specific factors model predicts that, in the absence of political redistribution mechanisms, specific factors in declining sectors will organize strong opposition to policies that might otherwise raise growth. The initial distribution of factor endowments may therefore make a big difference in terms of what types of political coalitions mobilize for and against different policies. This is the basic driving force in Moav-Galor's (2006) growth model on why some regions made public investments in human capital which sped the transition from agriculture to manufacturing and enhanced growth, whereas similar policies were delayed in other regions where political/economic resistance was stronger, for instance where landlords had stronger voice in political decisions.

These are just a few of the applications. The model is relatively easy to analyze – it can be described compactly in terms of diagrams and yet is very rich in predictions.

The Specific Factors (SF) or Ricardo-Viner model is a close relative of the Heckscher-Ohlin-Samuelson (HOS) neoclassical trade model. The 2x2 HOS model assumes production in each of two sectors takes place by firms using constant returns to scale technologies with capital and labor as inputs and that both capital and labor are mobile across sectors. In the SF model only labor is inter-sectorally mobile and the fixed amounts of capital become 'specific' to the sector they are trapped within.

In effect the SF model therefore consists of three factors of production: mobile labor and two types of capital, one specific to each sector. Let's label the two sectors are Agriculture and Manufacturing. In agriculture competitive firms

bid to hire land and labor. In manufacturing competitive firms bid to hire capital and labor. Each factor of production will be competitively priced in equilibrium but only labor is priced on a national labor market.

The SF model is often described as a short-run version of the HOS model. For example suppose we start with a HOS model equilibrium where labor wage and rental rate of capital have equalized across sectors (which also implies marginal products are equalized across sectors – no productivity differences across sectors). Now suppose that the relative price of manufacturing products suddenly rises (due to a change of world price, or government trade protection or other policies that favor manufacturing). The higher relative product price should lead firms in the booming manufacturing sector to demand both more capital and labor. Correspondingly, demand for land and labor decline in the agricultural sector. In the short run however only labor can be move from agriculture to manufacturing. Agricultural workers can become factory workers but agricultural capital (say ‘land’ or ‘tractors’) cannot be easily converted to manufacturing capital (say ‘weaving machines’). So labor moves from manufacturing to the agriculture, lowering the capital labor ratio in manufacturing and raising the land to labor ratio in agriculture. The model thus predicts a surge in the real return to capital in the expanding sector and a real decline in the real return to capital in agriculture (land). Hence the measured average and marginal product of capital will now diverge across sectors. What happens to the real wage is more ambiguous: it rises measured in terms of purchasing power over agricultural goods but falls in terms of purchasing power over manufacturing goods. Whether workers are better off or worse off following this price/policy change therefore comes down to how important agricultural and manufacturing goods are in their consumption basket. This result is labeled the neo-classical ambiguity.

Over the longer-term weaving machines cannot be transformed into tractors but over time new capital accumulation will build tractors and old weaving machines can be sold overseas or as scrap metal. Hence over time more capital will arrive into the manufacturing sector and leave the agricultural sector, which in turn will lead to even more movement of labor to manufacturing. If this process continues capital has, in effect become mobile over time, and we end up getting closer to the predictions of the HOS model.

### 1.4.2 Technology and Endowments

There are two sectors Agriculture and Manufacturing. Production in each sector takes place with a linear homogenous (constant returns to scale) production functions. Agricultural production requires land which is non-mobile or specific to the sector and mobile labor.

$$Q_a = F(\bar{T}, L_a)$$

Manufacturing production requires specific capital  $K$  and mobile labor.

$$Q_m = G(\bar{K}, L_m)$$

The quantity of land in the agricultural sector and the quantity of capital in the manufacturing sector are in fixed supply during the period of analysis. That means that firms within the agricultural (manufacturing) sector may compete with one another for the limited supply of the factor, but no new land (capital) can be supplied in the short-run period of analysis. This of course means that the price of the factor will rise (or fall) quickly in response to swings in factor demand compared to the wage of labor whose supply is more elastic.

The market for mobile labor is competitive and the market clears at a wage where the sum of labor demands from each sector equals total labor supply. While the total labor supply in the economy is inelastic, the supply of labor to each sector will be elastic, since a rise in the wage in one sector will attract workers from the other sector.

$$L_a + L_m = \bar{L}$$

Notice that we can invert the two production function to get minimum labor requirement functions  $L_a(Q_a)$  and  $L_m(Q_m)$  which tell us the minimum amount of labor  $L_i$  required in sector  $i$  to produce quantity  $Q_i$ . If we take these expressions and substitute them into the labor resource constraint we get an expression for the **production possibility frontier (PPF)** which summarizes the tradeoffs between sectors.

### 1.4.3 Assumptions and parameters for visualizations

Let's get concrete and assume each sector employs a CRS Cobb-Douglas production function:

$$F(\bar{T}, L_a) = \bar{T}^{1-\alpha} \cdot L_a^\alpha$$

$$G(\bar{K}, L_m) = \bar{K}^{1-\beta} \cdot L_m^\beta$$

If  $\alpha = \beta = \frac{1}{2}$  and  $\bar{T} = \bar{K} = 100$ . Then

$$Q_a = \sqrt{\bar{T}} \sqrt{L_a}$$

$$Q_m = \sqrt{\bar{K}} \sqrt{L_m}$$

Substituting these into the labor resource constraint yields:

$$\frac{Q_a^2}{\bar{T}} + \frac{Q_m^2}{\bar{K}} = \bar{L}$$

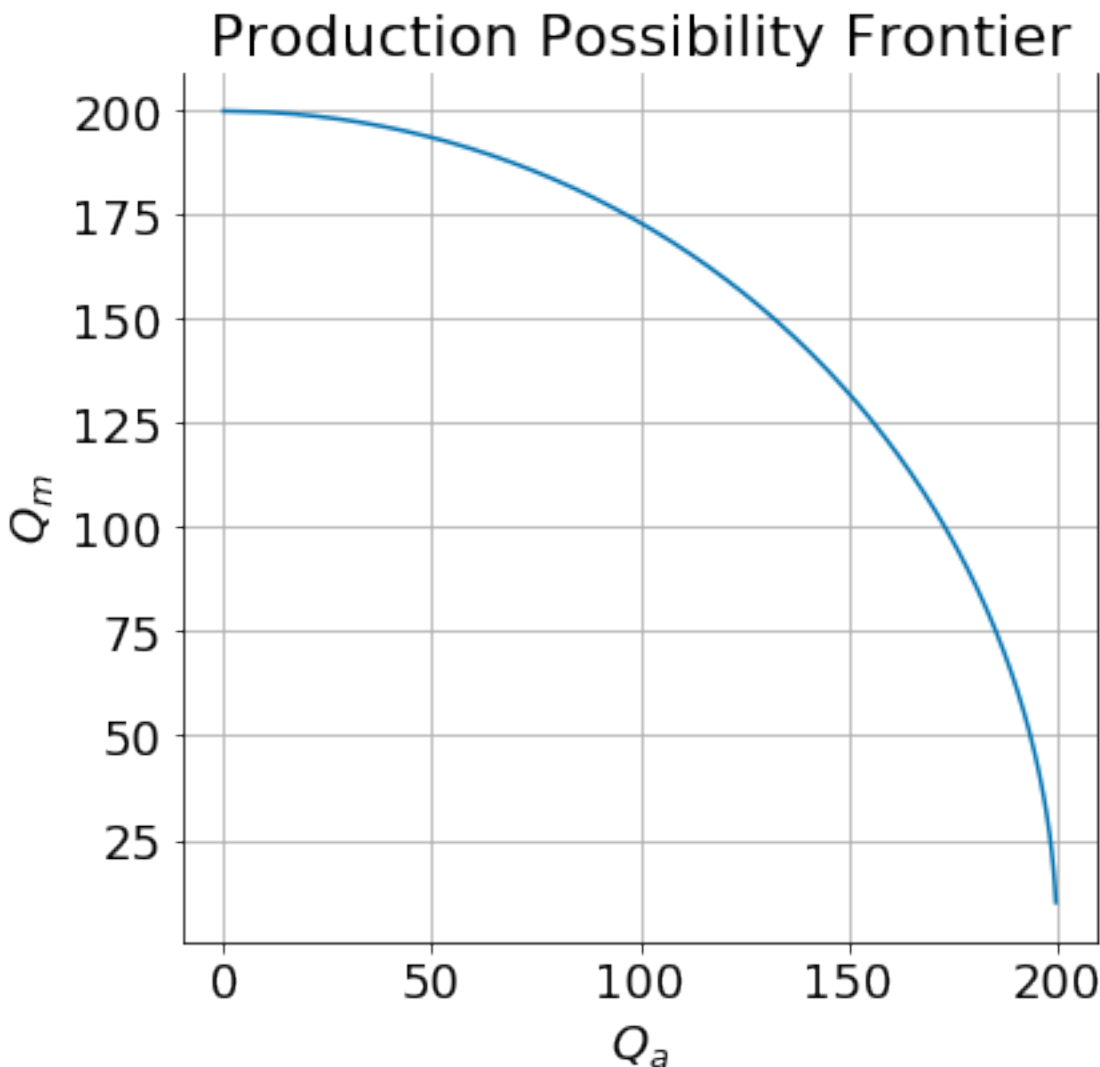
or

$$Q_m = \sqrt{\bar{K}\bar{L} - \frac{\bar{K}Q_a^2}{\bar{T}}}$$

If we make the further assumption that  $\bar{K} = \bar{L} = 100$  and  $\bar{L} = 400$  then the PPF would look like this:

**NOTE:** If you are running this as a live jupyter notebook please first go to the *code section* below and execute all the code cells there. Then return and run the code cells that follow sequentially.

```
In [22]: ppf(Tbar=100, Kbar=100, Lbar=400)
```



#### Labor market equilibrium

Profit maximizing firms in each sector will hire labor up to the point where the marginal value product of labor (MVPL) equals the market wage. Since labor is mobile across sectors in equilibrium workers must be paid the same nominal wage  $w$  in either sector:

$$P_a \cdot MPL_a(\bar{T}, L_a) = w = P_m \cdot MPL_m(\bar{K}, L_m)$$

It will be useful to express the wage in real terms. Divide each expression above by  $P_m$  to get

$$p \cdot MPL_a = \frac{w}{P_m} = MPL_m$$

$$\text{where } p = \frac{P_a}{P_m}$$

In the plots below we will place the real wage measured in terms of manufactured goods on the vertical axis of the labor demand and supply diagrams.

### 1.4.4 Labor allocation across sectors

In a competitive market firms take the market prices of goods  $p$  and wages  $w$  as given. Since in equilibrium the labor market clears we can write  $L_m = \bar{L} - L_a$  and substitute into the equilibrium condition above to get:

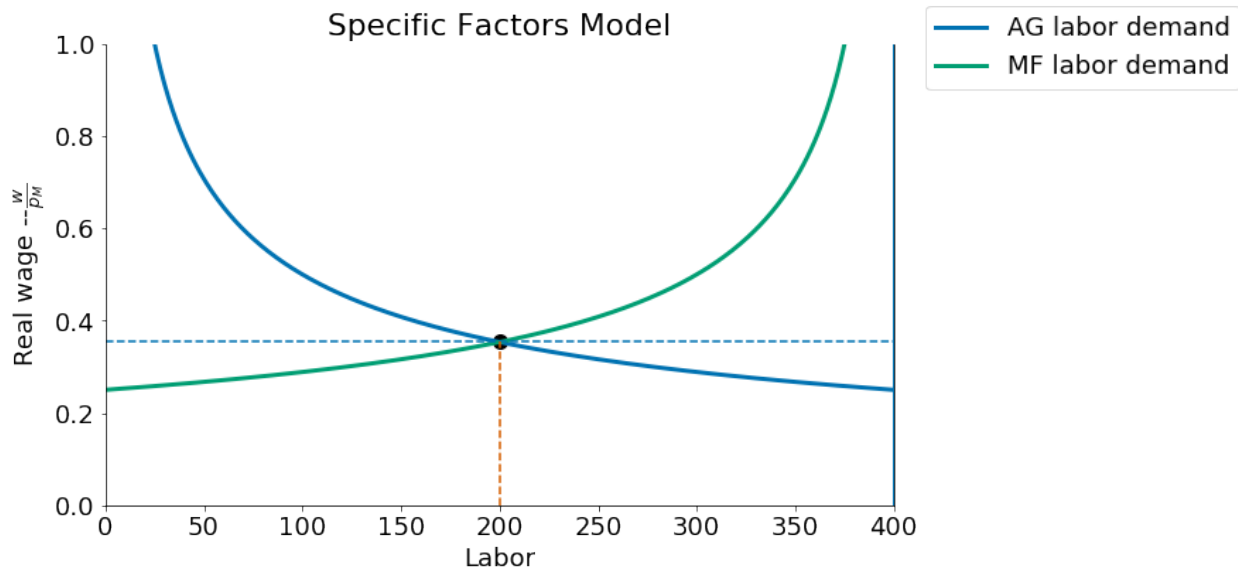
$$p \cdot MPL_a(L_a) = w = MPL_m(\bar{L} - L_a)$$

The left hand side  $p \cdot MPL_a(L_a) = w$  can be solved to give us demand for labor in the agricultural sector as a function of the wage  $L_a(w/p)$  and is plotted below. The right hand side  $w = MPL_m(\bar{L} - L_a)$  gives us demand for labor in the manufacturing sector  $L_m$  as a function of the real wage but this in turn also gives us the supply of labor to the agricultural sector  $L_a^s = \bar{L} - L_m(w/p)$  since firms in the agricultural sector can only attract workers to their sector by paying those workers just a bit above what they would be paid for the jobs they have to leave in the manufacturing sector.

The diagram below can therefore be interpreted as showing labor demand and supply in the agricultural sector and their intersection gives us the equilibrium real wage (measured on the vertical in terms of purchasing power over manufactured goods).

In [23]: `sfmplot(p=1)`

```
(La, Lm) = (200, 200)    (w/Pm, w/Pa) = (0.35, 0.35)
(v/Pa, v/Pm) = (0.7, 0.7)    (r/Pa, r/Pm) = (0.7, 0.7)
```



We can solve for the equilibrium labor allocation  $L_a$  as a function of the relative price  $p$ , and this in turn also gives us the equilibrium real wage. When  $\alpha = \beta = \frac{1}{2}$  the equilibrium condition  $p \cdot MPL_a(L_a) = MPL_m(\bar{L} - L_a)$  becomes:

$$p \cdot \frac{\sqrt{\bar{T}}}{\sqrt{L_a}} = \frac{\sqrt{\bar{K}}}{\sqrt{\bar{L} - L_a}}$$

which we solve to find:

$$L_a^e = \frac{p^2 \bar{T} \bar{L}}{\bar{K} - p^2 \bar{T}}$$

For cases where  $\alpha \neq \beta$  we find the optimal  $L_a$  that solves this equilibrium condition numerically.



## Autarky prices

Thus far we have a pretty complete model of how production allocations and real wages would be determined in a small open economy where producers face world price ratio  $p$ . we explore comparative statics in this economy in more detail below.

If however the economy is initially in autarky or closed to the world then we must also consider the role of domestic consumer preferences in the determination of domestic equilibrium product prices.

With Cobb-Douglas preferences  $u(x, y) = \gamma \ln(x) + (1 - \gamma) \ln(y)$  consumers demands can be written as a function of relative price  $p = \frac{P_a}{P_m}$ . Setting  $P_m = 1$  to make manufacturing the numeraire good, this can be written:

$$\begin{aligned} C_a(p) &= \gamma \cdot \frac{I(p)}{p} \\ C_m(p) &= (1 - \gamma) \cdot I(p) \end{aligned} \quad (1.1)$$

Income  $I$  in the expressions is given by the value of national production or GDP at these prices. Measured in manufactured goods:

$$I(p) = p \cdot F(\bar{T}, L_a(p)) + G(\bar{K}, \bar{L} - L_a(p))$$

By Walras' law we only need to find the relative price at which output equals demand in one of the two product markets so in the code below we solve for equilibrium domestic prices from the condition  $Q_a(p) = C_a(p)$ .

For parameters  $\alpha = \beta = \gamma = \frac{1}{2}$ ,  $\bar{T} = \bar{K} = 100$  and  $\bar{L} = 400$  the domestic equilibrium prices are unitary:

```
In [24]: p_autarky(Lbar, Tbar, Kbar)
```

```
Out[24]: 1.0
```

And this in turn leads to an equilibrium autarky allocation with  $L_a = L_m = 200$  and a real wage  $\frac{w}{p} = 0.35$ .

```
In [25]: eqn(p_autarky())
```

```
Out[25]: (200.0, 0.35355339059327373)
```

The plot below shows the autarky production and consumption point (marked by an 'X') as well as the new production point (marked by a circle) and consumption point (marked by a square) if the country opened to trade with a world relative price  $p = \frac{7}{4}$ .

Opening to trade leads the country to expand agricultural production by re-allocating labor from manufacturing to agriculture. We confirm the expected **neoclassical ambiguity** result which is that an increase in the relative price of agricultural goods will lead to an *increase* in the real wage measured in terms of manufactured goods  $\frac{w}{P_m}$  ( $= w$  since we have  $P_m = 1$ ) but a *decrease* in real wage measured in terms of agricultural goods or  $\frac{w}{P_a} = \frac{w}{p}$  in our notation (since  $\frac{w}{P_a} = \frac{w}{P_m} \cdot \frac{P_m}{P_a}$ ).

Diagrammatically

$$p \uparrow \rightarrow \frac{w}{p_m} \uparrow, \frac{w}{p_a} \downarrow$$

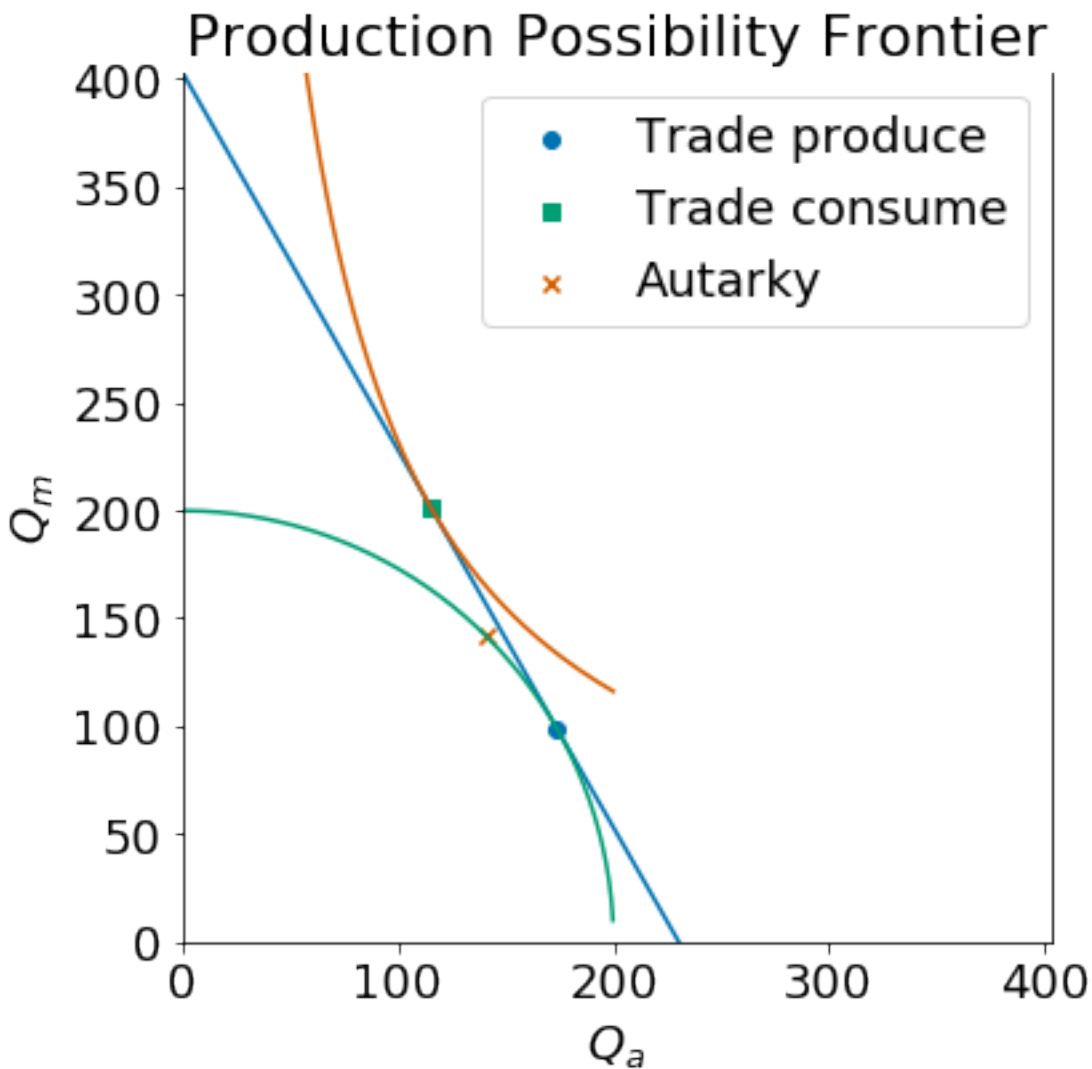
```
In [26]: pw = 7/4
```

```
Lao, wo = eqn(p=pw)
```

```
Lao, wo, wo/pw
```

```
Out[26]: (301.53846153846149, 0.50389110926865943, 0.28793777672494825)
```

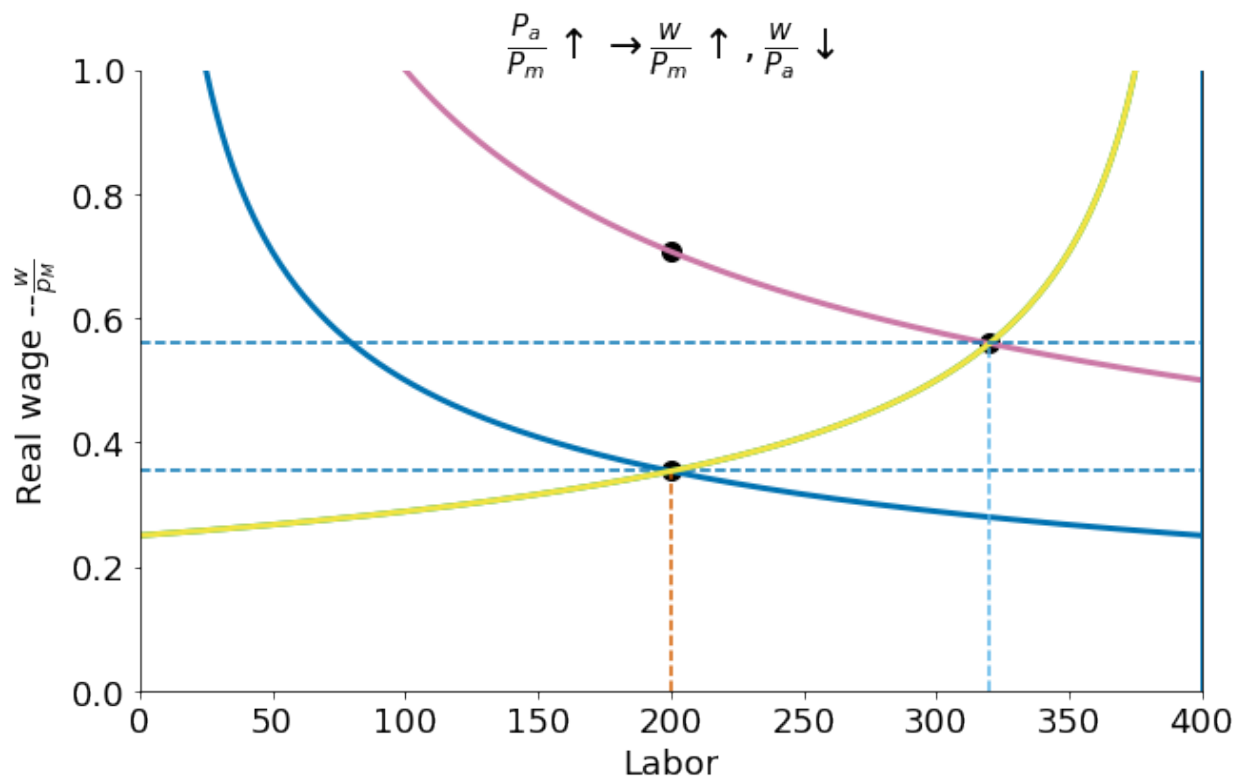
```
In [27]: sfmtrade(p=7/4)
```



#### Effects of a price change on the income distribution

More explanations to be placed here...

```
In [28]: sfmplot2(2)
```



## Code Section

Make sure you run the cells below FIRST. Then run the cells above

### Python simulation and plots

```
In [1]: import numpy as np
        from scipy.optimize import fsolve
        np.seterr(divide='ignore', invalid='ignore')
        import matplotlib.pyplot as plt
        from ipywidgets import interact, fixed
        import seaborn
        %matplotlib inline

In [2]: plt.style.use('seaborn-colorblind')
        plt.rcParams["figure.figsize"] = [7,7]
        plt.rcParams["axes.spines.right"] = True
        plt.rcParams["axes.spines.top"] = False
        plt.rcParams["font.size"] = 18
        plt.rcParams['figure.figsize'] = (10, 6)
        plt.rcParams['axes.grid']=True

In [3]: Tbar = 100      # Fixed specific land in ag.
        Kbar = 100      # Fixed specific capital in manuf
        Lbar = 400      # Total number of mobile workers
        LbarMax = 400   # Lbar will be on slider, max value.

        p = 1.00        # initial rel price of ag goods, p = Pa/Pm
        alpha, beta = 0.5, 0.5 # labor share in ag, manuf
```

For the plots we want to plot over  $L_a$  and  $L_m = \bar{L} - l_a$ :

```
In [4]: La = np.linspace(1, LbarMax-1, LbarMax)
        Lm = Lbar - La
```

The production functions in each sector:

```
In [5]: def F(La, Tbar = Tbar):
        return (Tbar**(1-alpha) * La**alpha)

        def G(Lm, Kbar =Kbar):
            return (Kbar**(1-beta) * Lm**beta)

In [6]: def MPLa(La):
        return alpha*Tbar**(1-alpha) * La**(alpha-1)

        def MPLm(Lm):
            return beta*Kbar**(1-beta) * Lm**(beta-1)

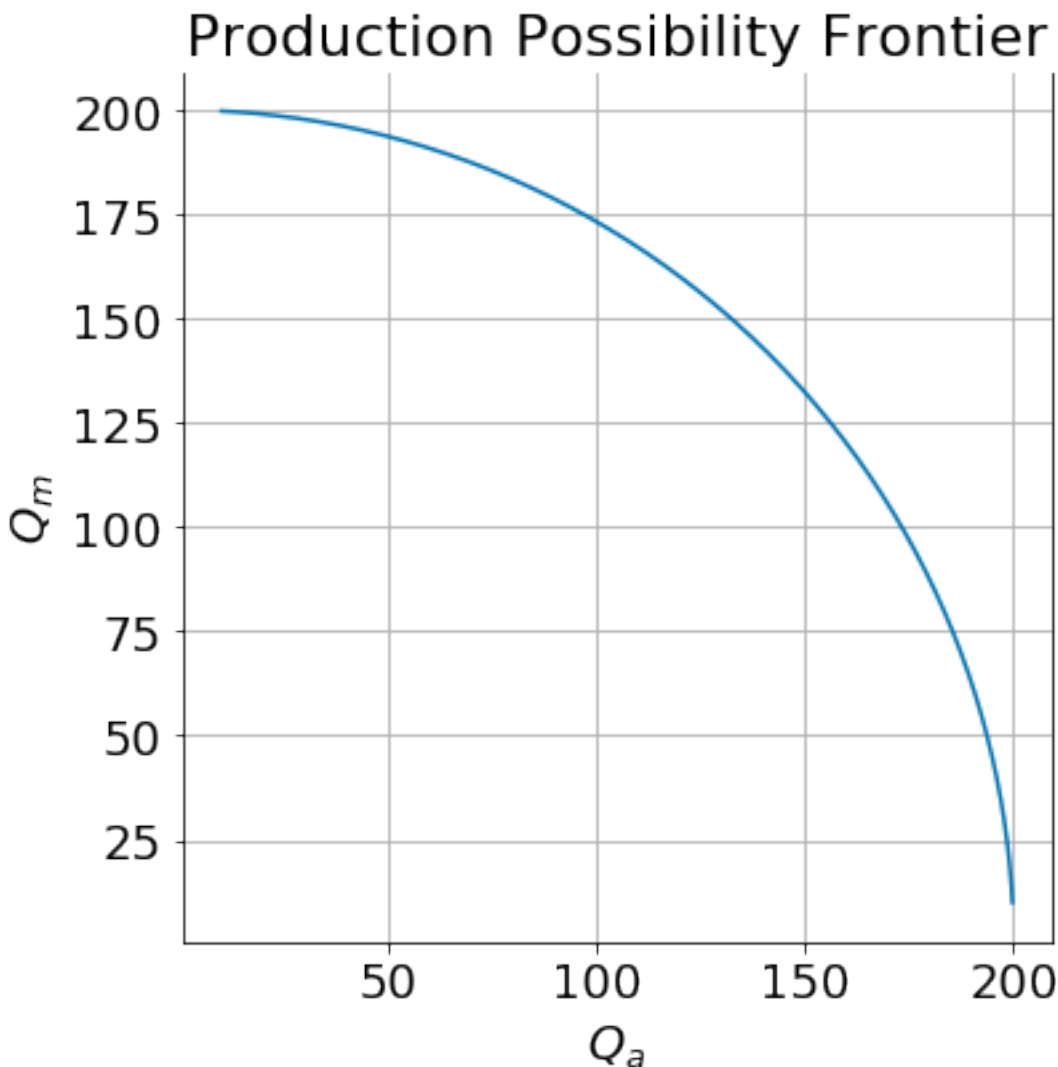
        def MPT(La, Tbar=Tbar):
            return (1-alpha)*Tbar**(-alpha) * La**alpha

        def MPK(Lm, Kbar=Kbar):
            return (1-beta)*Kbar**(-beta) * Lm**beta
```

We have enough to plot a production possibility Frontier (and how it varies with factor supplies):

```
In [7]: def ppf(Tbar=Tbar, Kbar=Kbar, Lbar=Lbar):
        Qa = F(La, Tbar) * (La<Lbar)
        Qm = G(Lm, Kbar)
        plt.title('Production Possibility Frontier')
        plt.xlabel(r'$Q_a$')
        plt.ylabel(r'$Q_m$')
        plt.plot(Qa, Qm)
        plt.gca().set_aspect('equal');

In [8]: ppf(Tbar=100, Kbar=100, Lbar=400)
```



Labor demand in each sector as a function of  $p = \frac{P_a}{P_m}$ :

```
In [9]: LDa = p * MPLa(La) * (La < Lbar)          # for Cobb-Douglas MPL can be written this way
        LDm = MPLm(Lbar - La)
```

The following function returns the equilibrium allocation of labor to agriculture and equilibrium nominal wage for any initial relative price of agricultural goods  $p = \frac{P_a}{P_m}$

```
In [10]: def eqn(p, Lbar=Lbar):
    '''returns numerically found equilibrium labor allocation and wage'''
    def func(La):
        return p * MPLa(La) - MPLm(Lbar - La)
    Laeq = fsolve(func, 1)[0]
    return Laeq, p * MPLa(Laeq)

In [11]: def u(x, y):
    '''Utility function'''
    return x * y

    def XD(p, Lbar = Lbar):
        '''Cobb-Douglas demand for goods given world prices (national income computed)'''
        LAe, we = eqn(p)
```

```
# gdp at world prices measured in manuf goods
gdp = p * F(LAe, Tbar = Tbar) + G(Lbar - LAe, Kbar = Kbar)
return (1/2) * gdp / p, (1/2) * gdp

def indif(x, ubar):
    return ubar / x

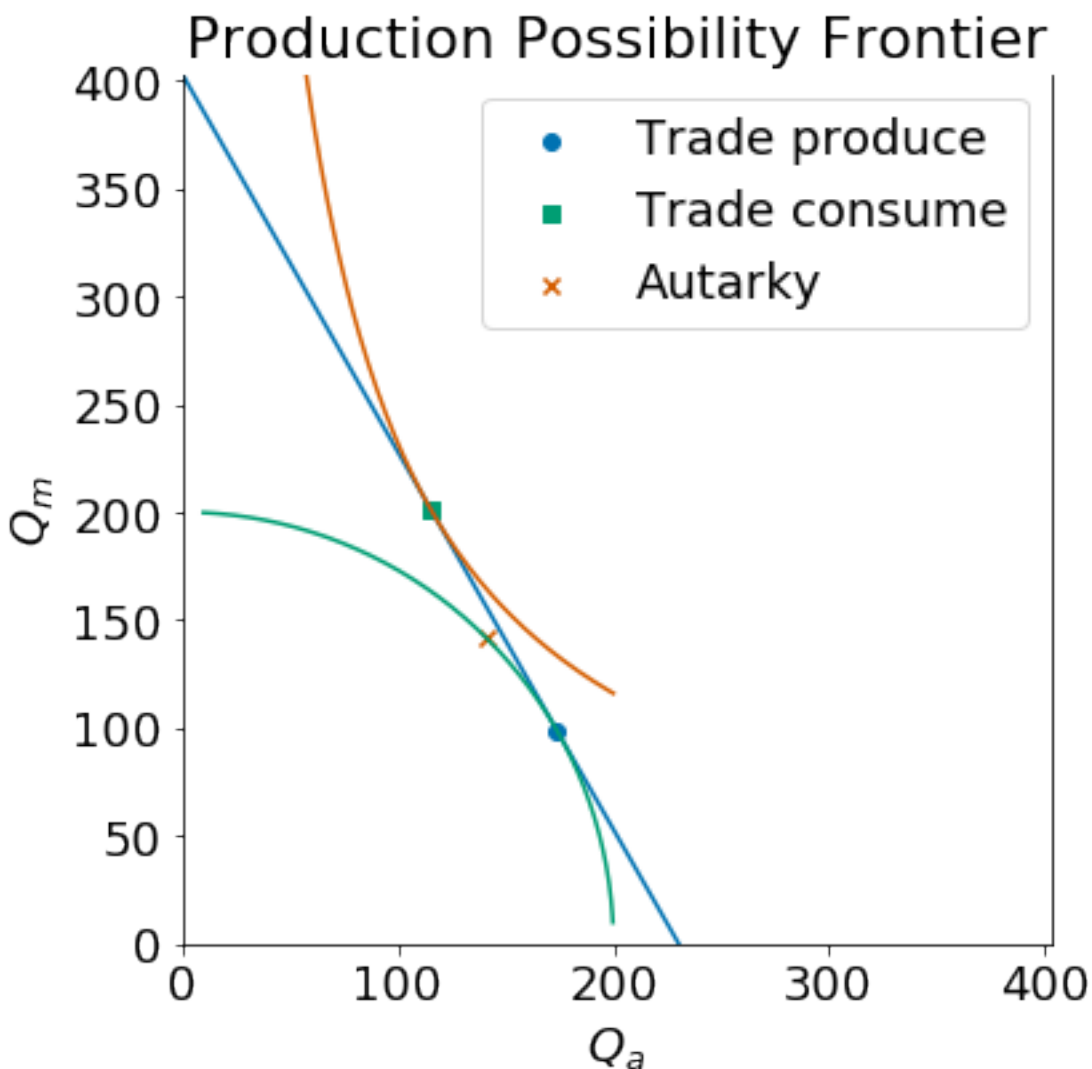
In [12]: def p_autarky(Lbar=Lbar, Tbar=Tbar, Kbar=Kbar):
        '''Find autarky product prices. By Walras' law enough to find price that
        sets excess demand in just one market'''
        def excessdemandA(p):
            LAe, _ = eqn(p)
            QA = F(LAe, Tbar = Tbar)
            CA, CM = XD(p, Lbar=Lbar)
            return QA - CA
        peq = fsolve(excessdemandA, 1)[0]
        return peq

In [13]: p_autarky()

Out[13]: 1.0

In [14]: def sfmtrade(p):
        Ca = np.linspace(0, 200, 200)
        LAe, we = eqn(p)
        X, Y = F(LAe, Tbar = Tbar), G(Lbar - LAe)
        gdp = p * X + Y
        plt.scatter(F(LAe, Tbar), G(Lbar - LAe, Tbar), label='Trade produce')
        plt.scatter(*XD(p), label='Trade consume', marker='s')
        plt.scatter(*XD(p_autarky()), marker='x', label='Autarky')
        plt.plot([0, gdp / p], [gdp, 0])
        ppf(100)
        ub = u(*XD(p))
        plt.ylim(0, gdp)
        plt.xlim(0, gdp)
        plt.plot(Ca, indif(Ca, ub))
        plt.grid(False)
        plt.legend()
        plt.gca().spines['bottom'].set_position('zero')
        plt.gca().spines['left'].set_position('zero')

In [15]: sfmtrade(p=7/4)
```



Simple example with  $\bar{L}$  at  $\bar{L}^{max}$

```
In [16]: La = np.arange(0, LbarMax) # this is always over the LbarMax range
```

```
def sfmplot(p, Lbar=LbarMax, show=True):
    Lm = Lbar - La
    Qa = (Tbar**(1-alpha) * La**alpha) * (La<Lbar)
    Qm = Kbar**(1-beta) * (Lbar - La)**beta
    pMPLa = (p * alpha * Qa/La)*(La<Lbar) # for Cobb-Douglass MPL can be written th
    MPLm = beta * Qm/(Lbar-La)
    LA, weq = eqn(p)
    ymax = 1.0
    plt.ylim(0,ymax)
    plt.xlim(0,LbarMax)
    plt.title('Specific Factors Model')
    plt.plot(La, pMPLa, linewidth = 3, label='AG labor demand')
    plt.plot(La, MPLm, linewidth=3, label='MF labor demand')
    plt.scatter(LA, weq, s=100, color='black')
    plt.axhline(weq, linestyle='dashed')
    plt.axvline(Lbar, linewidth = 3)
    plt.plot([LA, LA],[0, weq], linestyle='dashed')
```

```

plt.xlabel('Labor')
plt.ylabel('Real wage --' + r'$\frac{w}{p_M}$')
plt.grid()
if show:
    plt.legend(loc=(1.04,0.9))
    print("(La, Lm) = ({0:3.0f}, {1:3.0f})    (w/Pm, w/Pa) = ({2:3.2f}, {3:3.2f})"
          .format(LA, Lbar-LA, weq, weq/p))
    print("(v/Pa, v/Pm) = ({0:3.1f}, {1:3.1f})    (r/Pa, r/Pm) = ({2:3.1f}, {3:3.1f})"
          .format(MPT(LA), p*MPT(LA), MPK(Lbar-LA)/p, MPK(Lbar-LA))
    plt.show()

```

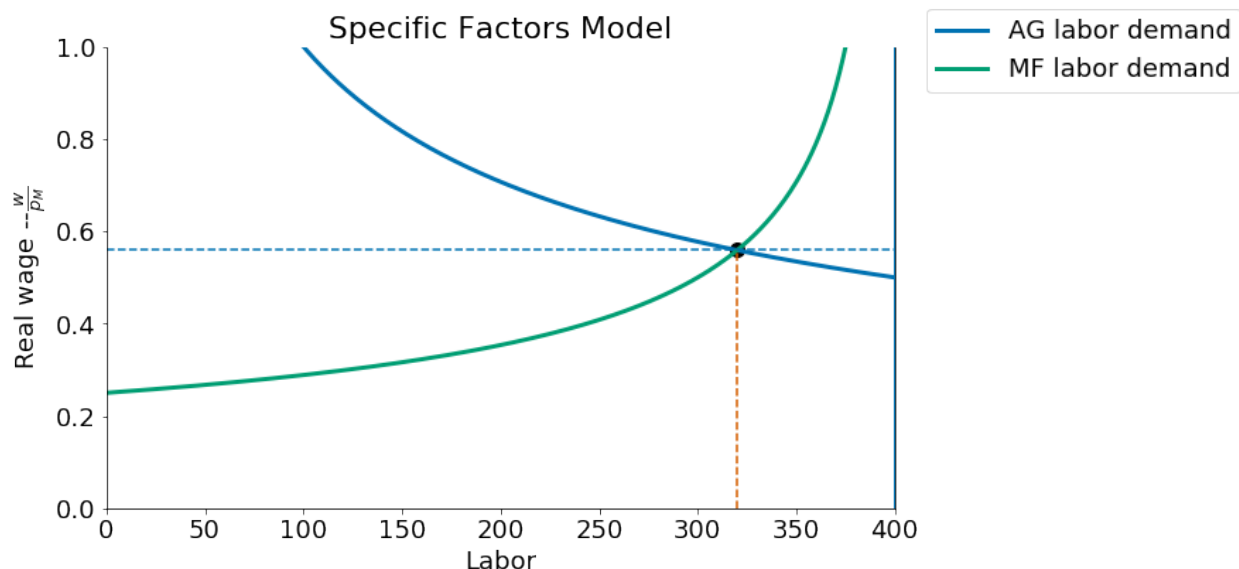
If you are wondering about why we skip the `plt.show()` when `show=True`, it's because for figures that use the `ipywidgets` `interact` function to allow parameters to change with sliders we will need to have a `plt.show()` to redraw the plot in every function call (otherwise we would end up with a stack of plots rather than a changing plot).

In [17]: `sfmplot(p=2)`

```

(La, Lm) = (320, 80)    (w/Pm, w/Pa) = (0.56, 0.28)
(v/Pa, v/Pm) = (0.9, 1.8)    (r/Pa, r/Pm) = (0.2, 0.4)

```



Suppose that by opening to trade the relative price of agricultural goods  $p$  doubles from  $p = 1$  to  $p = 2$ . This greatly shifts the demand for agricultural workers:

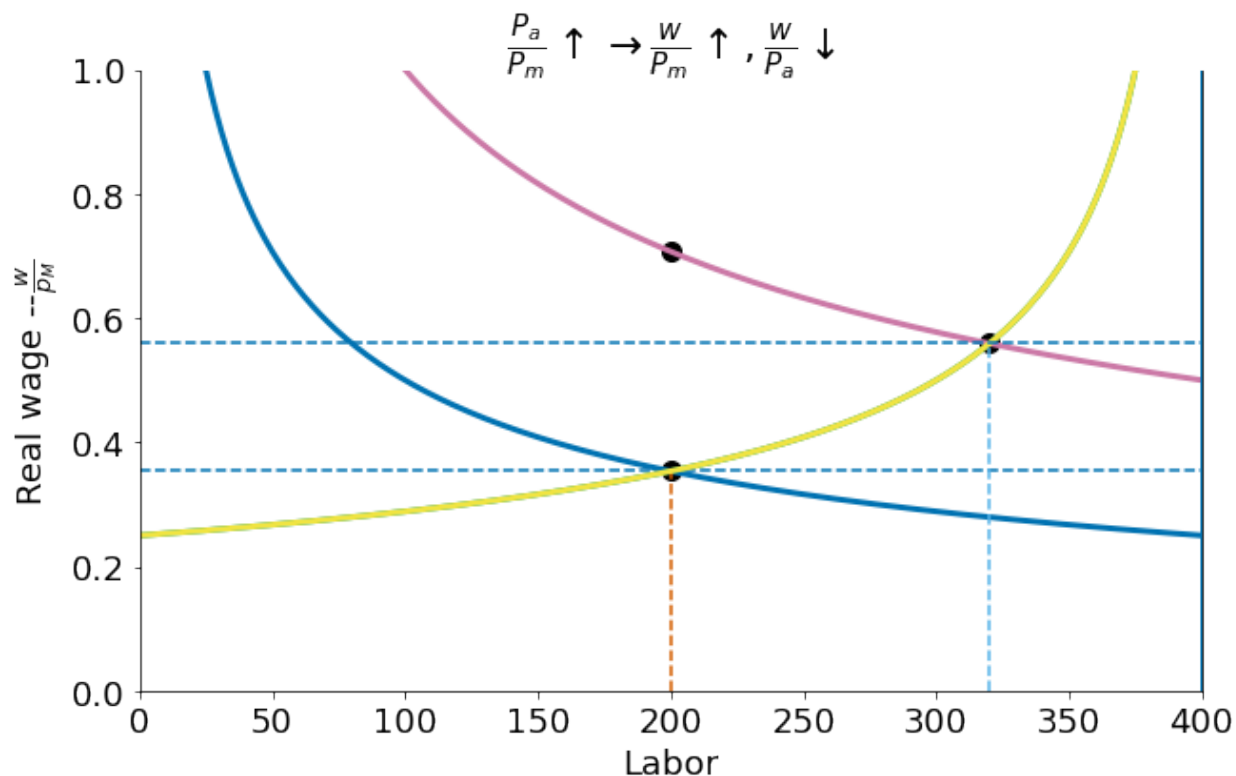
```

In [18]: def sfmplot2(p):
    sfmplot(1, show=False)
    sfmplot(p, show=False)
    plt.grid(False)
    if p == 1:
        plt.title('SF Model');
    else:
        La0, w0 = eqn(1)
        plt.scatter(La0, w0*p, s=100, color='black') #where wage would rise to without labor
        if p>1:
            plt.title(r'$\frac{P_a}{P_m}$ \uparrow \rightarrow \frac{w}{P_m}$ \uparrow, \frac{v}{P_a}$ \downarrow')
        elif p<1:
            plt.title(r'$\frac{P_a}{P_m}$ \downarrow \rightarrow \frac{w}{P_m}$ \downarrow, \frac{v}{P_a}$ \uparrow')
    plt.show();

```

In [19]: `sfmplot2(2)`





```
In [20]: interact(sfmpplot2, p=(0.1, 2, 0.1));
```

### Before and after plots

The trick here is to define a new plotting function that first plots a static plot and then allows an interaction.

```
In [21]: interact(sfmpplot2, p=(0, 2, 0.2), Lbar=fixed(400), show=fixed(True))
```

```
Out[21]: <function __main__.sfmpplot2>
```

## 1.5 Migration, urban-bias and the informal sector

### 1.5.1 The Harris-Todaro Model

This model is an adaptation of a standard two-sector open economy specific factors model (SFM) of migration.

\* The two sectors are agriculture and 'modern-manufacturing' \* The agricultural labor market is competitive. Labor is mobile between the agricultural and manufacturing sectors.

If the market of jobs in the modern manufacturing sector were also competitive this would be a standard specific factors model and migration between sectors would occur until the wage was equalized across sectors, or same thing, at the market-clearing equilibrium wage  $w^e$  the sum of labor demands from each sector equaled the economy-wide labor supply  $\bar{L}$ .

$$L_a(w^e) + L_m(w^e) = \bar{L}$$

However, for institutional/political economy reasons wages in the modern manufacturing sector will be set artificially high, for example by union activity or minimum-wage policies in that sector. This high wage will lead firms in that sector to cut back hiring but will also attract migrants to urban areas. lured by the prospect of possibly landing a high-wage job in the urban sector. Since not all migrants succeed in landing these rationed jobs however this migration will lead to the endogenous emergence of an informal urban sector in the economy and ‘urban-bias’ (a larger than efficient urban sector).

Laborers can now either stay in the rural sector to earn wage equilibrium rural wage  $w_r$  or migrate to the urban area where they may land either in (a) the informal sector where they earn a low-productivity determined wage  $w_u$  or (b) in the high-wage modern manufacturing sector where they earn the institutionally-determined wage  $w_m$ . We model assumes that only urban dwellers can apply for modern-manufacturing and that jobs are allocated among such applicants by lottery whenever those jobs are in excess demand.

A fixed constant informal sector wage can be justified by assuming that the informal sector is competitive and production takes place with a simple low-productivity linear technology. Wages in that sector are then pinned to  $w_u = a_u$  where  $a_u$  is the constant marginal product of labor in the informal sector.

Migration will now take place until the rural wage is equalized to the *expected* wage of an urban resident:

$$w_r = \frac{L_m(w_m)}{L_u + L_m(w_m)} \cdot w_m + \frac{L_u}{L_u + L_m(w_m)} \cdot w_u$$

Labor demand in each sector will depend on product prices. Without loss of generality and to simplify let’s normalize  $P_r = 1$  and now call  $p = \frac{P_r}{P_m}$  the relative price of agricultural goods.

We can then write  $L_r(w)$  as the solution to

$$p \cdot F_L(\bar{L}, L_r) = w$$

and labor demand  $L_m(w)$  as the solution to

$$G_L(\bar{K}, L_m) = w$$

Given the assumption that jobs in the high-wage manufacturing sector are allocated by fair lottery the equilibrium probability of getting such a job will be given simply by the share of the urban sector labor force in that sector.

If, without loss of generality we normalize the informal sector wage  $w_u$  to zero (we’ll change that later) the equilibrium condition becomes just:

$$w_r = \frac{L_m(w_m)}{L_u + L_m(w_m)} \cdot w_m$$

Since  $w_m$  is fixed labor use in the modern manufacturing sector will be  $L_m(w_m)$  and the market can be thought of as clearing at a rural wage  $w_r$  and a size of the urban informal sector  $L_u$  of just the right size to dissuade any further migration.

Note that this condition can be re-written more simply as follows:

$$w_m \cdot L_m = w_r \cdot (\bar{L} - L_r)$$

Since  $\bar{L}$ ,  $w_m$  and hence also  $L_m = L_m(w_m)$  are all fixed quantities this is an equation in two unknowns.

We can solve for the two unknowns  $w_r$  and  $L_r$  from a system of two equations.

The first is this last equation which is a rectangular hyperbola of the form  $x \cdot y = \kappa$ , where here  $x = \bar{L} - L_r$  and  $y = w_r$ .

The other equation is the competitive equilibrium condition

$$p \cdot F_L(\bar{K}, L_r) = w_r$$

that at a competitive optimum the rural wage  $w_r$  will be given by the marginal value product of labor in agriculture.

## Diagram analysis

Although this is a simple system of two non-linear equations in two unknowns, it's hard to get a tidy closed form solution for Cobb Douglas production functions. It is easy to see the solution graphically and solve for it numerically, however.

Suppose production in the agricultural and manufacturing sectors is carried out by identical firms in each sector each employing the following linear homogenous Cobb-Douglas technologies:

$$G(\bar{T}, L_r) = A_r \bar{T}^\alpha \cdot L_r^{1-\alpha}$$

$$F(\bar{K}, L_m) = A_m \bar{K}^\beta \cdot L_m^{1-\beta}$$

Labor demand in manufacturing as a function of  $w$ :

$$L_m(w_m) = \left[ \frac{A_m(1-\beta)\bar{K}}{w_m/P_m} \right]^{\frac{1}{\beta}}$$

and rural labor demand:

$$L_r(w_r) = \left[ \frac{A_r(1-\alpha)\bar{T}}{w_r/P_r} \right]^{\frac{1}{\alpha}}$$

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interact
from scipy.optimize import bisect, newton
%matplotlib inline

In [2]: Tbar = 200      # Fixed specific land in ag.
Kbar = 200      # Fixed specific capital in manuf
Lbar = 400      # Total number of mobile workers
LbarMax = 400    # Lbar will be on slider, max value.
A = 1
p = 1.00      # initial rel price of ag goods, p = Pa/Pm
alpha, beta = 0.75, 0.5 # labor share in ag, manuf

In [3]: def F(L, T, A=1, alpha=0.5):
    return A*(T**alpha)*(L**(1-alpha))

def G(L, K, A=1, beta=0.5):
    return A*(K**beta)*(L**(1-beta))

def mplr(L, T=Tbar, A=1, alpha=0.5):
    return (1-alpha)*F(L, T, A, alpha)/L

def mplm(L, K=Kbar, A=1, beta=0.5):
    return (1-beta)*G(L, K, A, beta)/L

def Lm(w):
    return Kbar*((p/w)*(A*(1-beta)))**(1/beta)

def expret(Lr, wm):
    return wm*Lm(wm)/(Lbar-Lr)

def expwage(Lr, wm, wu):
    return (wm*Lm(wm) + wu*(Lbar-Lm(wm)-Lr))/(Lbar-Lr)
```

The efficient competitive equilibrium is given by the point where the two labor demand curves intersect. We solve for the level of agricultural employment at which there is zero excess demand for agricultural labor. This gives an equilibrium agricultural labor demand economy-wide equilibrium wage.

```
In [4]: def effeq():
        ed = lambda L: mplr(L) - mplm(Lbar-L)
        LE = bisect(ed, 10, Lbar-10)
        return mplr(LE), LE
```

A Harris-Todaro equilibrium is one where the rural wage equals the expected urban wage. Diagrammatically the equilibrium level of rural employment is given by the intersection of the rural labor demand curve and the rectangular hyperbola running through  $(w_m, L_m(w_m))$ .

```
In [5]: def harristodaro(wm, wu):
        LM = Lm(wm)
        WE, LE = effeq()
        hteq = lambda L: mplr(L) - (wm*LM + wu*(Lbar-LM-L)) / (Lbar-L)
        LR = newton(hteq, LE)
        WR = mplr(LR)
        return WR, LR, LM, WE, LE
```

This next function plots the diagram.

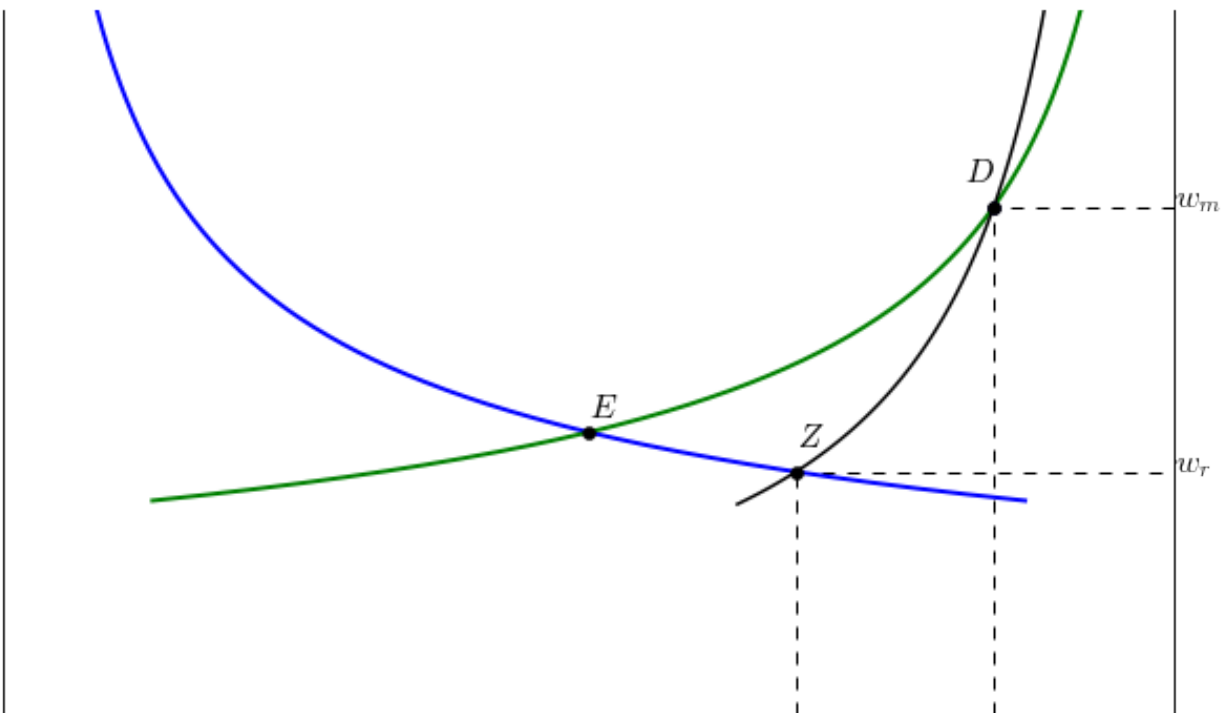
```
In [6]: def HTplot(wm, wu):
        WR, LR, LM, WE, LE = harristodaro(wm, wu)
        print('(wr, Lr), Lm, (we, le)=( {:.2f}, {:.0f}), {:.0f}, ( {:.2f}, {:.0f}), '.format(WR, LR, LM, WE, LE))
        lr = np.arange(1, Lbar)
        lup = np.arange(LR-20, Lbar-LM+20)
        fig, ax = plt.subplots(figsize=(10,6))
        ax.plot(lr[:-50], mplr(lr[:-50]), lw=2)
        ax.plot(lr[50:], mplm(Lbar-lr[50:]), lw=2)
        ax.plot(lup, expwage(lup, wm, wu), 'k', lw=1.5)
        ax.vlines(LR, 0, WR, linestyle="dashed")
        ax.vlines(Lbar-LM, 0, wm, linestyle="dashed")
        ax.hlines(wm, Lbar, Lbar-LM, linestyle="dashed")
        ax.hlines(WR, LR, Lbar, linestyle="dashed")
        ax.plot(Lbar-LM, wm, 'ob')
        ax.text(Lbar, wm, '$w_m$', fontsize=16)
        ax.text(LE, WE*1.05, '$E$', fontsize=16)
        ax.text(LR, WR*1.10, '$Z$', fontsize=16)
        ax.text(Lbar-LM-10, wm*1.05, '$D$', fontsize=16)
        ax.text(Lbar, WR, '$w_r$', fontsize=16)
        ax.plot([LE, LR, Lbar-LM], [WE, WR, wm], 'ok')
        ax.set_xlim(0, Lbar)
        ax.set_ylim(0, 1.25)
        ax.set_xlabel(r'$c_1$', fontsize=18)
        ax.set_ylabel('$c_2$', fontsize=18)
        ax.spines['top'].set_visible(False)
        ax.get_xaxis().set_visible(False)
        ax.get_yaxis().set_visible(False)
```

The high institutional wage  $w_m$  lowers demand for labor in the formal manufacturing sector relative to a competitive equilibrium. In the Harris-Todaro model it's not apriori obvious whether the high institutional wage in the formal manufacturing sector will increase or decrease the size of the urban sector relative to the efficient competitive equilibrium. Migrants have to weigh the lower probability of landing a formal sector job against the higher wage they will capture if they are lucky enough to get a job.

If we assume the informal sector (or unemployment) wage is zero, then for our Cobb-Douglas demands the following diagram suggests the policy creates an informal urban sector but overall reduces the size of the urban sector relative to the rural sector, compared to the efficient competitive equilibrium.

```
In [7]: WM, WU = 0.9, 0
        HTplot(WM, WU)

(wr, Lr), Lm, (we, le)=( 0.43, 271), 62, ( 0.50, 200,)
```



But if we make the informal sector sufficiently attractive it is possible to get ‘urban-bias’ or an excessively large urban sector relative to the efficient allocation.

For the following diagram we first make note of the efficient equilibrium wage and rural sector size:

```
In [8]: WE, LE = effeq()
        print('The efficient competitive wage is w ={:5.2f} and the rural sector is Lr ={:5.0f}'.format(WE, LE))
```

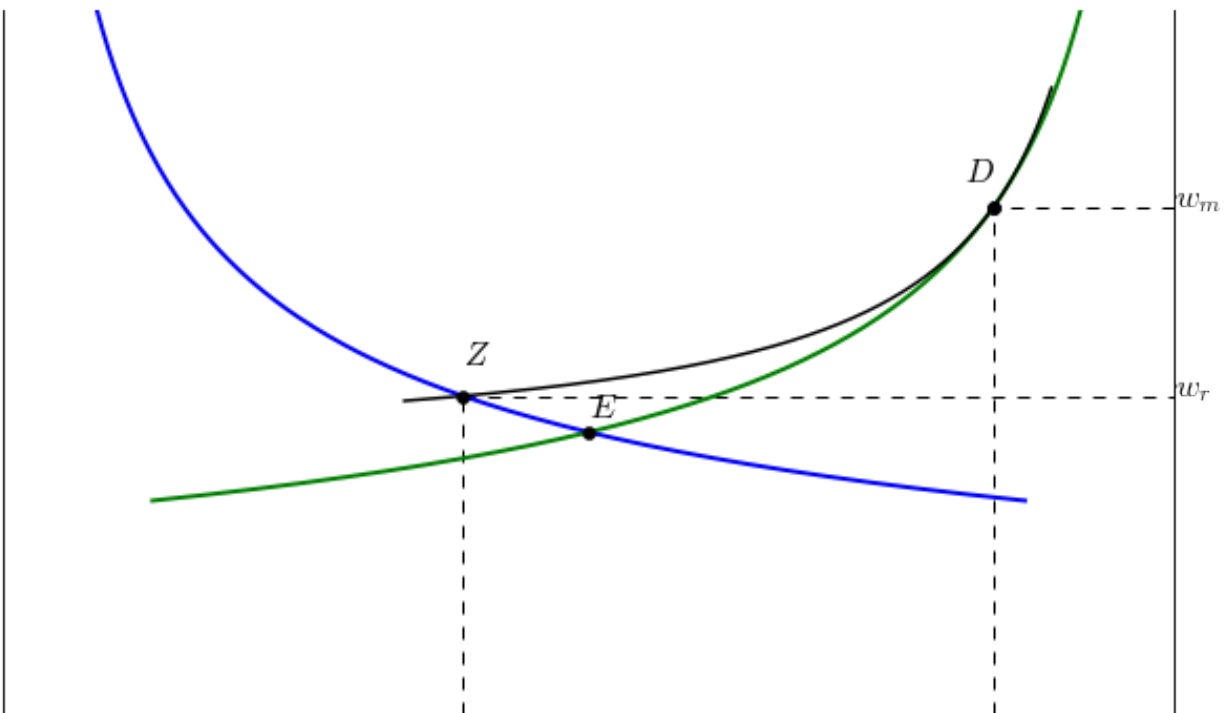
The efficient competitive wage is  $w = 0.50$  and the rural sector is  $L_r = 200$

Suppose that workers in the informal sector have access to a low-productivity occupation which pays a wage below this efficient level. That means that in an efficient allocation workers would earn more in either formal manufacturing or the rural sector and hence there would be no informal sector in equilibrium.

However the existence of this low-productivity option in the urban informal sector raises the expected return to migration in the Harris-Todaro distorted equilibrium and as the following example illustrates this may result in a smaller than efficient rural sector:

```
In [9]: WM, WU = 0.9, 0.45
        HTplot(WM, WU)

(wr, Lr), Lm, (we, le) = (0.56, 157), 62, (0.50, 200,)
```

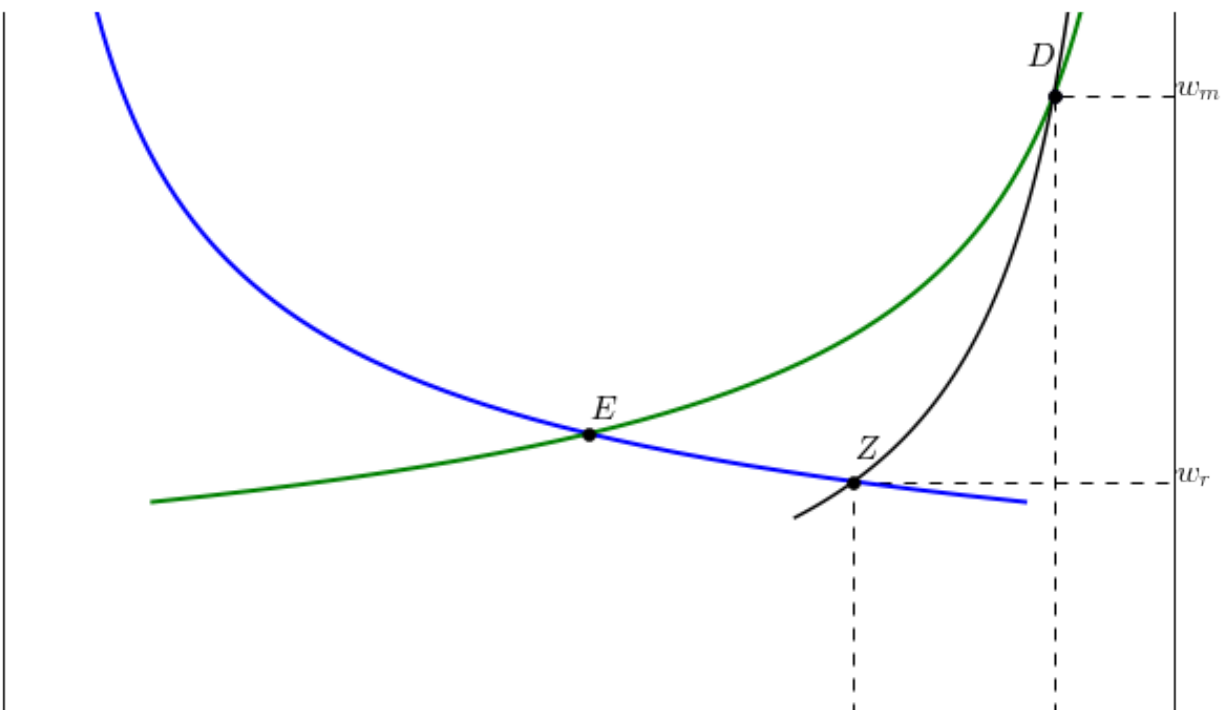


### Interactive plot

```
In [10]: interact(HTplot, wm=(WE, 3.5*WE, 0.05), wu=(0, WE, 0.05))
```

```
(wr, Lr), Lm, (we, le)=( 0.41, 290), 41, ( 0.50, 200,)
```

```
Out[10]: <function __main__.HTplot>
```



## Extensions

### Harris Todaro and Inequality

Jonathan Temple's (2005) "Growth and Wage Inequality in a dual economy" makes some simple points about wage inequality in the HT model. He shows that in the case of  $W_u = W$  the Gini coefficient can be written simply:

$$Gini = L_u \left( 2 - \frac{L_u}{u} \right)$$

where here  $L_u$  is the *proportion* of the labor force in unemployment and  $u$  (slightly redefining what we had above... or, same thing, if we normalize the total labor force to 1) and  $u$  is urban unemployment rate or the fraction of the unemployed in the urban population (i.e.  $u = \frac{L_u}{L_u + L_m}$ ). From this one can prove that inequality will unambiguously rise if one of the following statements holds if the urban unemployment rate  $u$ :

- rises, and the number of unemployed  $L_u$  goes up.
- is constant, and the number of unemployed  $L_u$  rises. Modern sector employment rises, and agricultural employment falls.
- rises, and the number of unemployed is constant. Modern sector employment falls, and agricultural employment rises

Another result is that rural growth (driven say by improved agricultural technology) leads to an unambiguous reduction in wage inequality. The effects of urban growth are ambiguous.

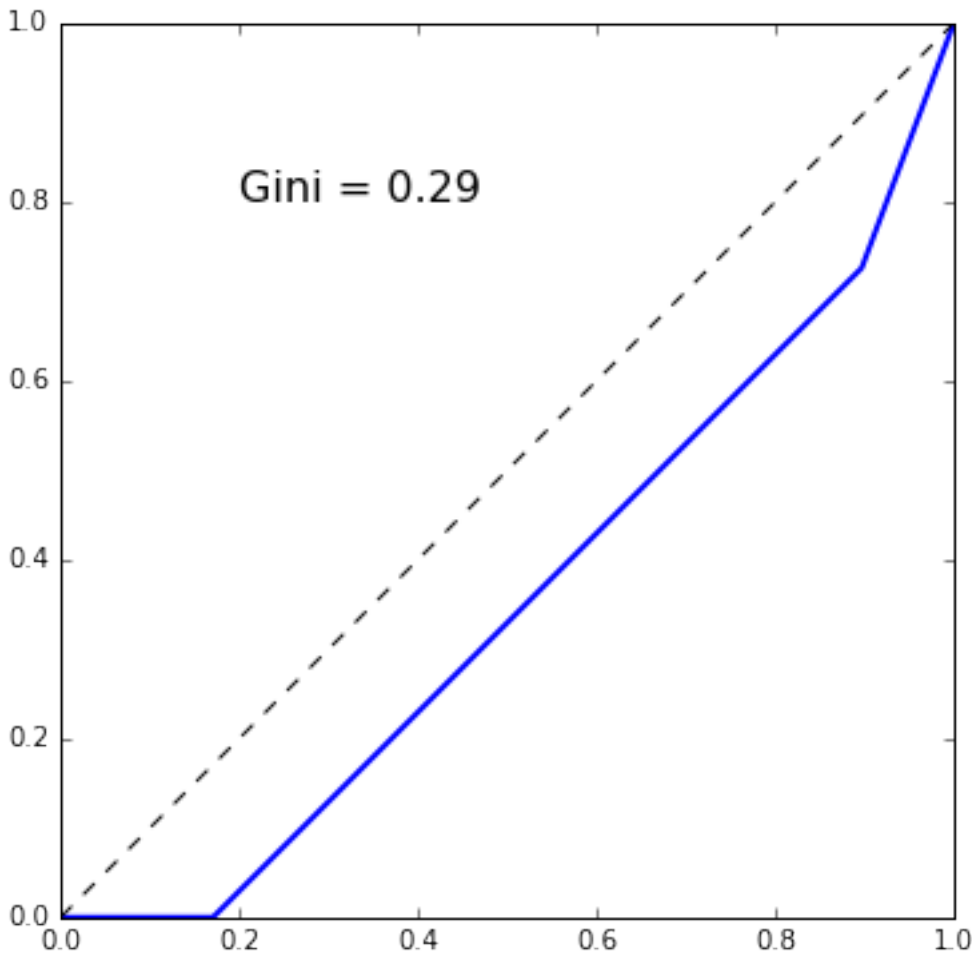
Below we plot the Lorenz curve and slightly extend Temple's analysis to the case where 'the unemployed' (or informal sector workers) earn a non-zero wage. For now we simply focus on calculating the Gini numerically.

(note/fix: GINI CALCULATION DOES NOT SEEM RIGHT)

```
In [11]: def htlorenz(wm, wu):
    WR, LR, LM, WE, LE = harristodaro(wm, wu)
    lrp = LR/Lbar
    lmp = LM/Lbar
    lup = 1 - lrp - lmp
    ytot = wu*(1-lrp-lmp) + WR*lrp + wm*lmp
    yup = wu*(1-lrp-lmp)/ytot
    yrp = WR*lrp/ytot
    ymp = wm*lmp/ytot
    A = 0.5 - (yup*((1-lup) + 0.5*lup) + (yrp-yup)*(lmp+0.5*lrp) + 0.5*lmp*(ymp))
    Gini = 2*A
    gtext = "Gini ={:5.2f}".format(Gini)
    fig, ax = plt.subplots(figsize=(6,6))
    ax.plot([0,lup,lup+lrp,1],[0,yup,yup+yrp,1], lw=2)
    ax.plot([0,1],[0,1], 'k--', lw=1)
    ax.text(0.2,0.8,gtext,fontsize=16)

In [12]: WM, WU = 0.9,0
    WR, LR, LM, WE, LE = harristodaro(WM, WU)
    interact(htlorenz, wm=(WE,3.5*WE,0.05),wu=(0,WE,0.05))

Out[12]: <function __main__.htlorenz>
```



## 1.6 Coase, Property rights and the ‘Coase Theorem’

Coase, R. H. 1960. “The Problem of Social Cost.” *The Journal of Law and Economics* 3:1–44.

Coase, Ronald H. 1937. “The Nature of the Firm.” *Economica* 4 (16):386–405.

**Note:** Most of the code to generate visuals and results below is in a *code section* at the end of this notebook. To run interactive widgets or modify content, navigate to the code section for instructions.

**Slideshow mode:** this notebook can be viewed as a slideshow by pressing Alt-R if run on a server.

### 1.6.1 Coase (1960)

#### A rancher and wheat farmer.

Both are utilizing adjacent plots of land. There is no fence separating the lands.

##### The Wheat Farmer

The wheat farm choose a production method the delivers a maximum profit of  $\Pi_W = 8$ . - to keep this simple suppose this is the farmer’s only production choice.



**The Rancher**

Chooses herd size  $x$  to maximize profits:

$$\Pi_C(x) = P \cdot F(x) - c \cdot x$$

where  $P$  is cattle price and  $c$  is the cost of feeding each animal.

To simplify we allow decimal levels (but conclusions would hardly change if restricted to integers).

First-order necessary condition for herd size  $x^*$  to max profits:

$$P \cdot F'(x^*) = c$$

**Example:**  $P_c = 4$ ,  $F(x) = \sqrt{x}$  and  $c = 1$

The FOC are  $\frac{4}{2\sqrt{x^*}} = 1$

And the rancher's privately optimal herd size:

$$x^* = 4$$

**The external cost**

No effective barrier exists between the fields so cattle sometimes strays into the wheat farmer's fields, trampling crops and reducing wheat farmer's profits.

Specifically, if rancher keeps a herd size  $x$  net profits in wheat are reduced to :

$$\Pi_W(x) = \Pi_W - d \cdot x^2$$

**The external cost**

Suppose  $d = \frac{1}{2}$

At rancher's private optimum herd size of  $x^* = 4$  the farmer's profit is reduced from 8 to zero:

$$\begin{aligned} \Pi_W(x) &= \Pi_W - d \cdot x^2 \\ &= 8 - \frac{1}{2} \cdot 4^2 \end{aligned} \quad (1.3)$$

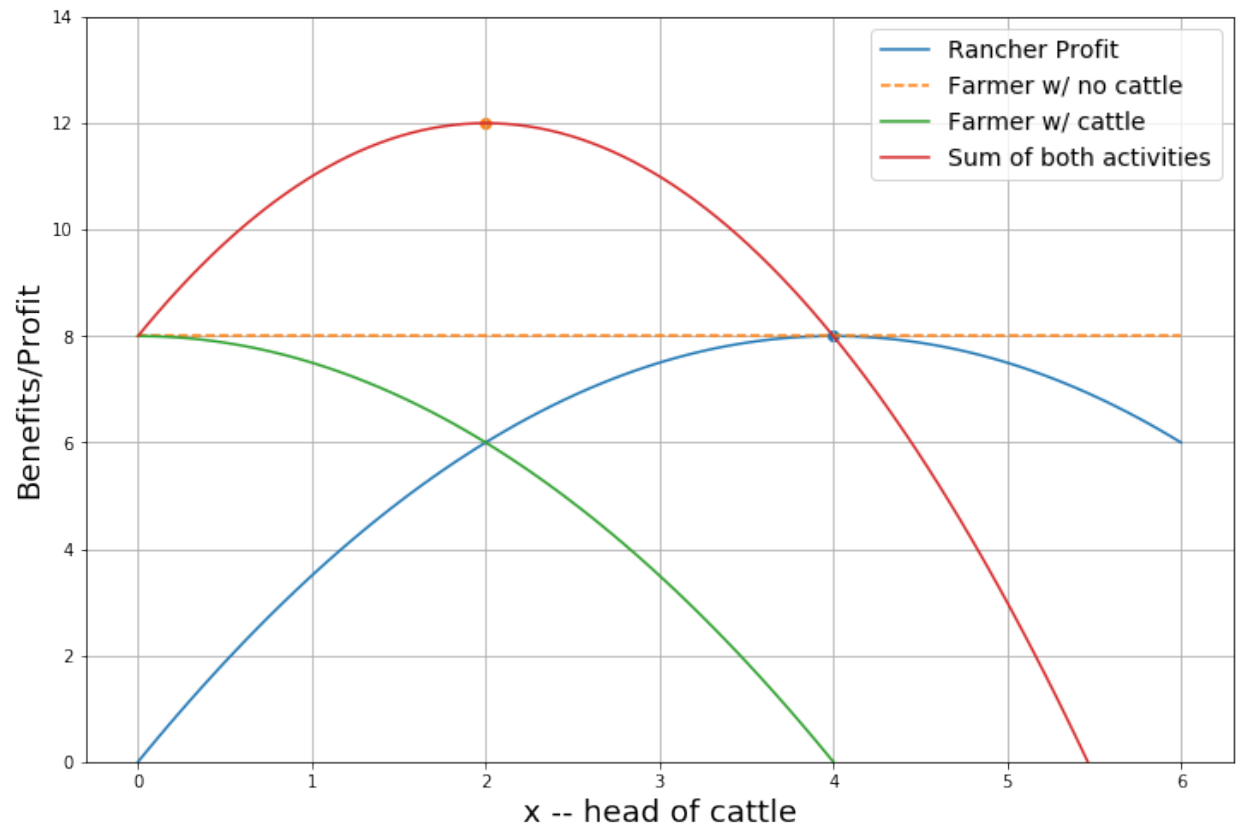
```
In [11]: Pc = 4
         Pw = 8
         c = 1/2
         d = 1/2

         CE, TE = copt(), topt()
         CE, TE
```

```
Out[11]: (4.0, 2.0)
```

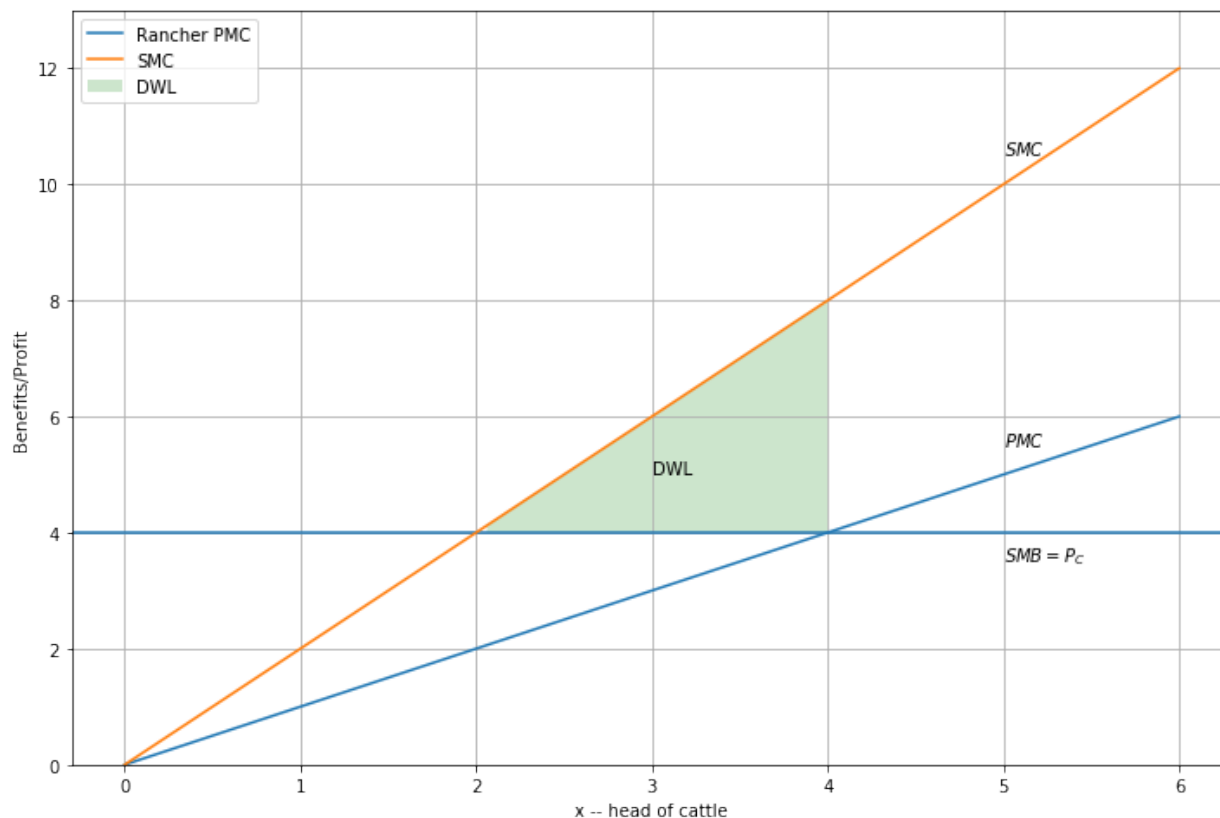
If the rancher chose his private optimum he'd earn \$8 but drive the farmer's earnings to zero.

```
In [12]: coaseplot1()
```



Private and social marginal benefits and costs can be plotted to see deadweight loss (DWL) differently:

```
In [13]: coaseplot2()
```



### 1.6.2 The assignment of property rights (liability)

**Scenario 1:** Farmer has right to enjoin cattle herding (prohibit via an injunction).

Rancher now earns \$0. Farmer \$8.

This is not efficient either.

If rancher herded just 2 would earn \$6. Could offer \$2 compensation to the wheat farmer and capture  $6-2 = \$4$ .

... or they could bargain to divide the gains to trade of \$4 in other ways.

**Scenario 2:** Rancher has right to graze with impunity.

Farmer earns \$0 if rancher herds private optimal of 4 cattle. Farmer could offer to pay \$2 to have rancher reduce herd to 2 which would leave rancher as well off and take the farmer from \$0 to \$4 ( $= 6-2$ ).

... or they could bargain to divide the gains to trade of \$4 in other ways.

#### With zero transactions costs

- **The initial assignment of property rights does not matter:** The parties bargain to an efficient outcome either way.
- However, like any scarce resource, legal rights are valuable, so **the initial allocation will affect the distribution of benefits and incomes between parties**\*
- **The emergence of property rights:** Even if there is no initial assignment of property rights, with zero transactions costs it should be in the interests of the parties to negotiate to an efficient outcome.

### 1.6.3 With positive transactions costs

- The initial distribution of property rights typically will matter.
- It's not so clear from this example but suppose that we had a situation with one wheat farmer and many ranchers. It might be difficult to get the ranchers

### 1.6.4 Coase and the development of a land market

Suppose there is an open field. In the absence of a land market whoever gets to the land first (possibly the more powerful in the village) will prepare/clear land until the marginal value product of the last unit of land is equal to the clearing cost. We contrast two situations:

- (1) Open frontier: where land is still abundant
- (2) Land Scarcity.

There will be a misallocation in (2) shown by DWL in the diagram... but also an incentive for the parties to bargain to a more efficient outcome. A well functioning land market would also deliver that outcome.

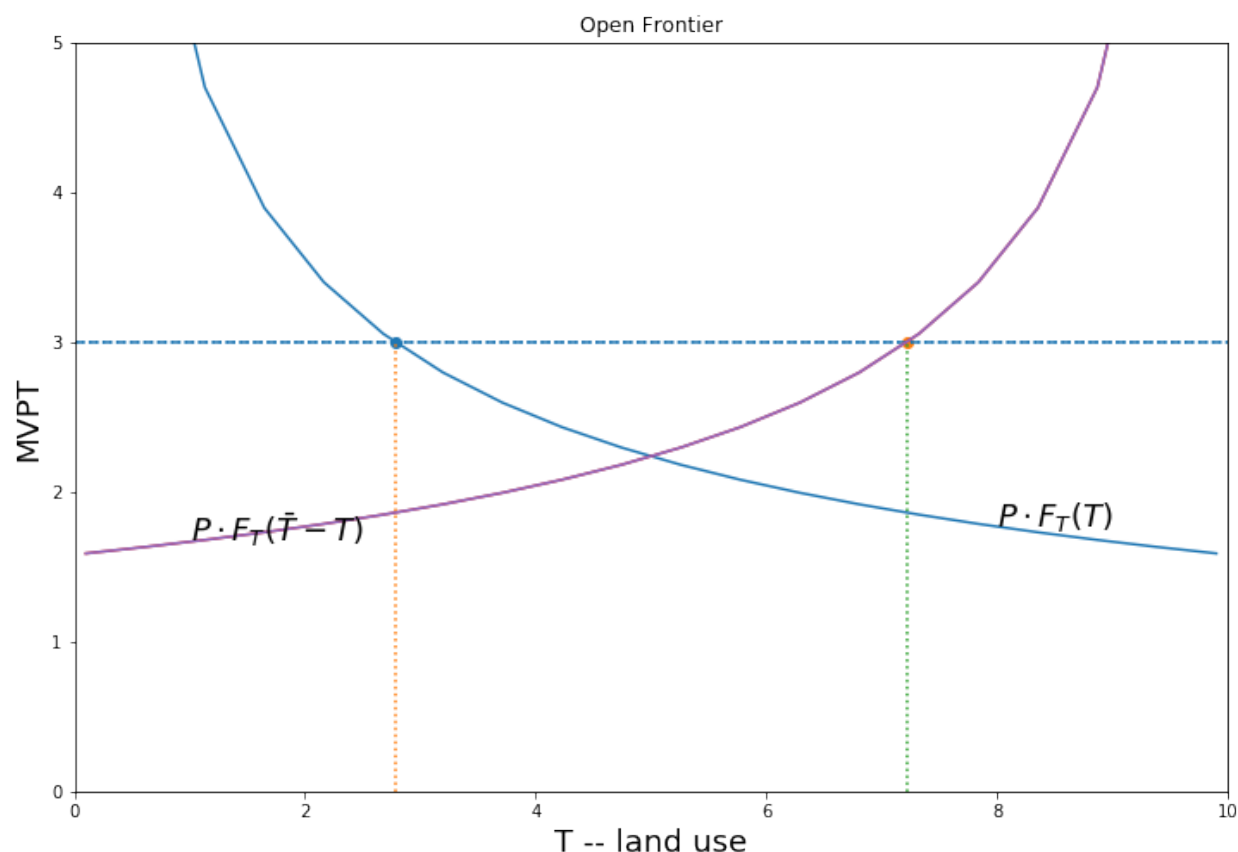
Abundant land environment

$\bar{T}$  units of land and  $N=2$  households.

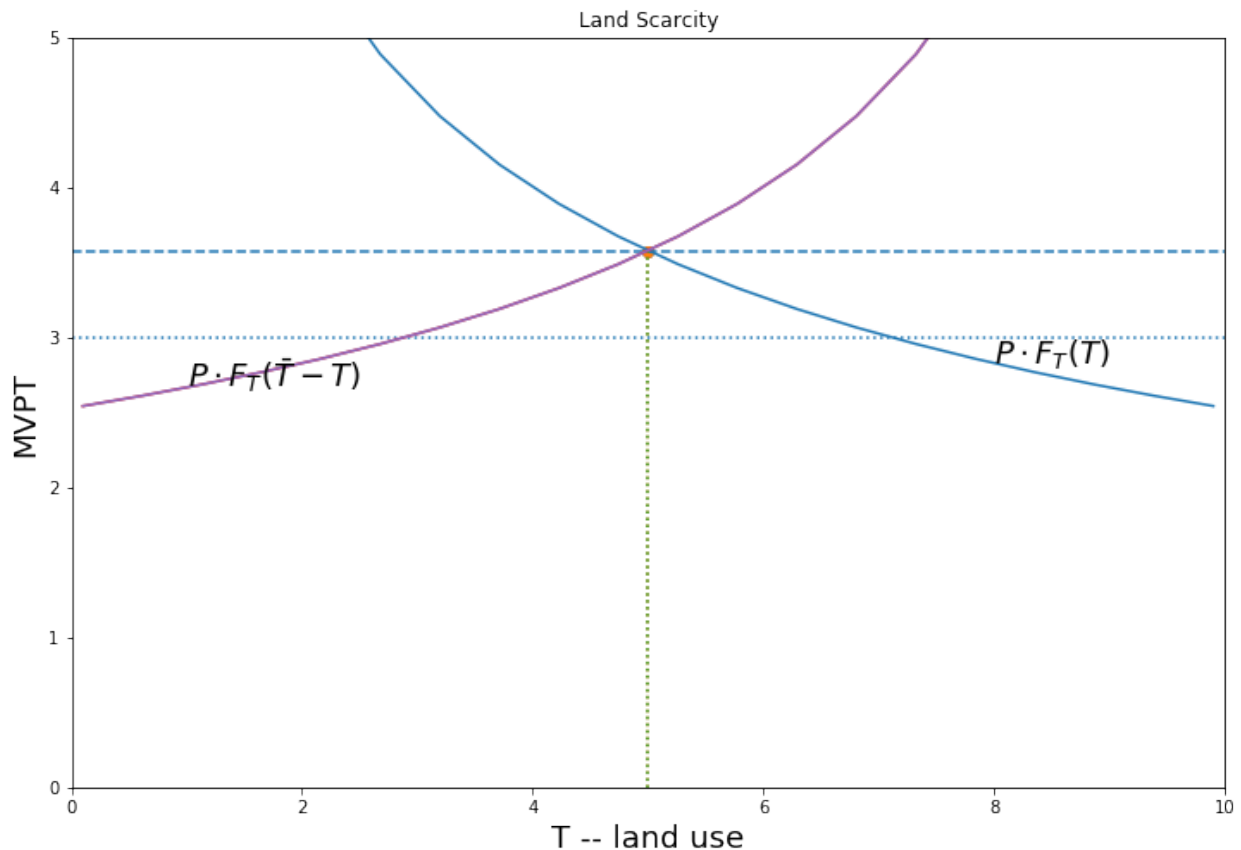
Land clearing cost  $c$ . Frontier land not yet exhausted.

Maximize profits at  $P \cdot F_T(T) = c$

In [26]: `landmarket(P=5, cl = 3, title = 'Open Frontier')`



```
In [27]: landmark(P=8, cl = 3, title = 'Land Scarcity')
```



### The ‘Coase Theorem’

Costless bargaining between the parties will lead to an efficient outcome regardless of which party is awarded the rights?

#### Coase Theorem: True, False or Tautology?

Tautology?: “if there are no costs to fixing things, then things will be fixed.”

Like the First Welfare Theorem (complete competitive markets will lead to efficient allocations, regardless of initial allocation of property rights). The Coase Theorem makes legal entitlements tradable

### 1.6.5 More useful reading of Coase result

When transactions costs the initial allocation of property rights will matter for the efficiency of the outcome.

### 1.6.6 Further notes on Coase (incomplete)

Coase can be seen as generalizing the neo-classical propositions about the exchange of goods (i.e. 1st Welfare Theorem) to the exchange of legal entitlements (Cooter, 1990). “The initial allocation of legal entitlements does not matter for an efficiency perspective so long as they can be freely exchanged. . .”

Suggests insuring efficiency of law is matter of removing impediments to free exchange of legal entitlements. ....  
define entitlements clearly and enforce private contracts for their exchange. . .

But conditions needed for efficient resource allocation. . .

Nice discussion [here](#):

Tautology: “Costless bargaining is efficient tautologically; if I assume people can agree on socially efficient bargains, then of course they will” “The fact that side payments can be agreed upon is true even when there are no property rights at all.” In the absence of property rights, a bargain *establishes* a contract between parties with novel rights that needn’t exist ex-ante.”

“The interesting case is when transaction costs make bargaining difficult. What you should take from Coase is that social efficiency can be enhanced by institutions (including the firm!) which allow socially efficient bargains to be reached by removing restrictive transaction costs, and particularly that the assignment of property rights to different parties can either help or hinder those institutions.”

Transactions cost: time and effort to carry out a transaction.. any resources needed to negotiate and enforce contracts. . .

Coase: initial allocation of legal entitlements does not matter from an efficiency perspective so long as transaction costs of exchange are nil. . .

Like frictionless plane in Physics. . . a logical construction rather than something encountered in real life..

Legal procedure to ‘lubricate’ exchange rather than allocate legal entitlement efficiently in the first place. . .

As with ordinary goods the gains from legal

### The Political Coase Theorem

Acemoglu, Daron. 2003. “Why Not a Political Coase Theorem? Social Conflict, Commitment, and Politics.” *Journal of Comparative Economics* 31 (4):620–652.

### 1.6.7 Incomplete contracts

- Hard to think of all contingencies
- Hard to negotiate all contingencies
- Hard to write contracts to cover all contingencies

Incomplete contracts - silent about parties’ obligations in some states or state these only coarsely or ambiguously

- Incomplete contracts will be revised and renegotiated as future unfolds. . . This implies
  - ex-post costs (my fail to reach agreement..).
  - ex-ante costs

Relationship-specific investments.. Party may be reluctant to invest because fears expropriation by the other party at recontracting stage.. - hold up: after tenant has made investment it is sunk, landlord may hike rent to match higher value of property (entirely due to tenant investment).. Expecting this tenant may not invest..

“Ownership – or power – is distributed among the parties to maximise their investment incentives. Hart and Moore show that complementarities between the assets and the parties have important implications. If the assets are so complementary that they are productive only when used together, they should have a single owner. Separating such complementary assets does not give power to anybody, while when the assets have a single owner, the owner has power and improved incentives.”

## Code Section **Note:** To re-create or modify any content go to the

‘Cell’ menu above run all code cells below by choosing ‘Run All Below’. Then ‘Run all Above’ to recreate all output above (or go to the top and step through each code cell manually).

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve
%matplotlib inline
```

Default parameter values:

```
In [2]: Pc = 4
Pw = 8
c = 1/2
d = 1/2
```

```
In [3]: def F(x,P=Pc,c=c):
    '''Cattle Profit'''
    return P*x - c*x**2

def AG(x, P=Pw):
    '''Wheat farm profit before crop damage'''
    return P*(x**0) # to return an array of len(x)
```

```
def AGD(x,P=Pw,d=d):
    return AG(x,P) - d*x**2
```

```
In [4]: def copt(P=Pc,c=c):
    '''rancher private optimum'''
    return P/(2*c)
```

```
def topt(P=Pc,c=c, d=d):
    '''Social efficient optimum'''
    return P/(2*(c+d))
```

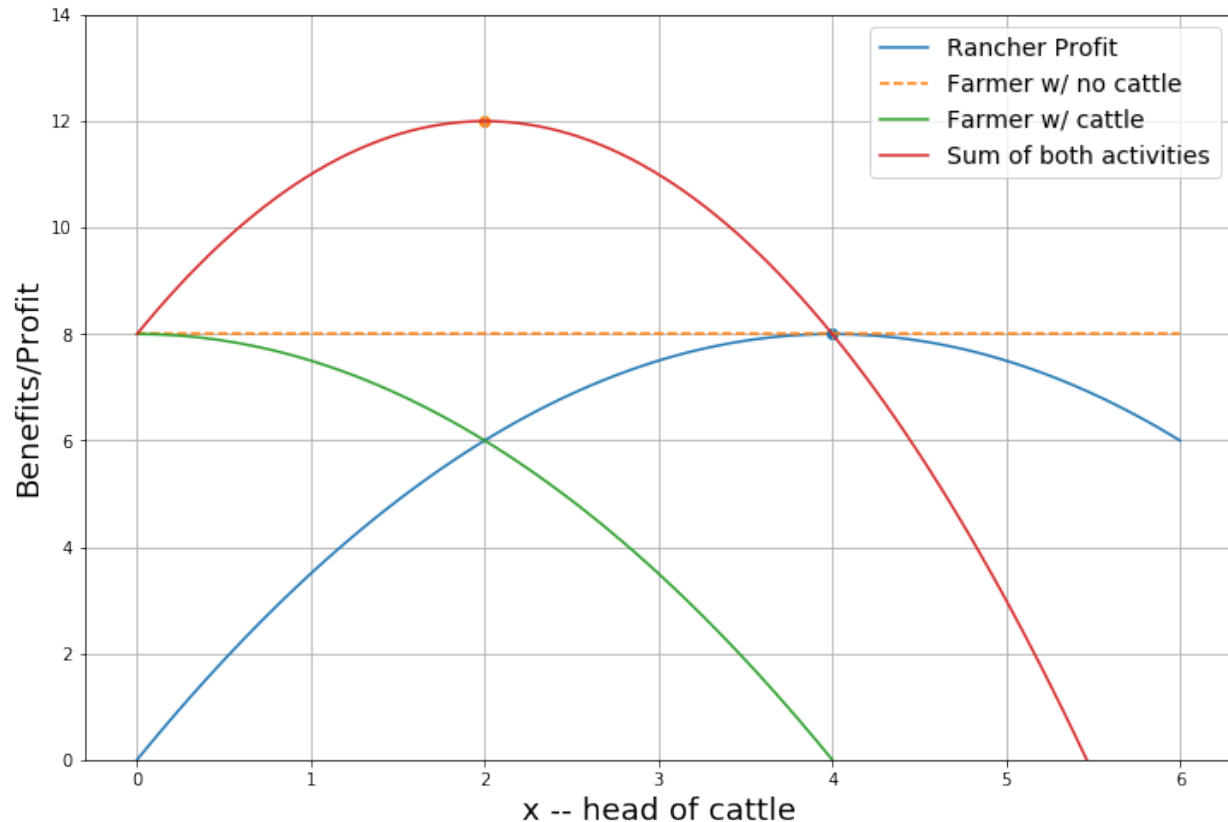
```
In [5]: CE, TE = copt(),topt()
CE, TE
```

```
Out[5]: (4.0, 2.0)
```

```
In [6]: xx = np.linspace(0,6,100)
```

```
In [7]: def coaseplot1():
    fig = plt.subplots(figsize=(12,8))
    plt.plot(xx, F(xx), label = 'Rancher Profit' )
    plt.plot(xx, AG(xx), '--', label = 'Farmer w/ no cattle' )
    plt.plot(xx, AGD(xx), label = 'Farmer w/ cattle')
    plt.plot(xx, F(xx) + AGD(xx),label='Sum of both activities')
    plt.scatter(copt(),F(copt()))
    plt.scatter(topt(),F(topt()) + AGD(topt()))
    plt.grid()
    plt.ylim(0,14)
    plt.xlabel('x -- head of cattle', fontsize=18)
    plt.ylabel('Benefits/Profit', fontsize=18)
    plt.legend(fontsize=14);
```

```
In [8]: coaseplot1()
```



Let's plot a standard 'external cost' diagram

```
In [9]: def MC(x, c=1/2):
        '''Cattle MC'''
        return 2*c*x

        def excost(x, d=1/2):
            return 2*d*x

In [10]: def coaseplot2(Pw=Pw, Pc=Pc):
        fig = plt.subplots(figsize=(12,8))
        plt.axhline(Pc);
        plt.plot(xx, MC(xx), label = 'Rancher PMC')
        plt.plot(xx, MC(xx)+excost(xx), label = 'SMC')
        plt.fill_between(xx, MC(xx)+excost(xx), Pc*xx**0, where=((MC(xx) <= Pc*xx**0) & (xx>2)),
                        facecolor='green', alpha=0.2, label='DWL')

        plt.text(3,5, 'DWL')
        plt.text(5,3.5, r'$SMB = P_C$')
        plt.text(5,5.5, r'$PMC$')
        plt.text(5,10.5, r'$SMC$')
        #plt.scatter(topt(), G(topt()) + AGD(topt()))
        plt.grid()
        plt.ylim(0,13)
        plt.xlabel('x -- head of cattle')
        plt.ylabel('Benefits/Profit')
        plt.legend();

In [14]: A=1

In [15]: def F(T, A=A):
```



```

        return A*np.sqrt(T)
In [16]: def MVPT(P,T,A=A):
        return A*P/T**(1/2)

        def LD(P,r,A=A):
            return (P*A/r)**2
In [17]: A=1
        Tbar = 10 # Total land endowment
        P = 5.5 # Price of output
        cl = 3 # cost of clearing land

```

Land demand for each farmer is given by  $P \cdot F_T(T_i) = r$ . So for this production  $P \frac{1}{\sqrt{T_i}} = r$  or  $P \frac{1}{\sqrt{T_i}} = cl$  so we can write

$$T_i^*(r) = (P/r)^2$$

If there is an open frontier the sum of demands falls short of total land supply and the marginal cost of land is the cost of clearing  $r = c_l$ . ‘Land scarcity’ results when there is an equilibrium price of land and  $r > c_l$  where  $r$  is found from

$$\sum T_i^*(r) = \bar{T}$$

```

In [18]: def req(P,cl, Tb=Tbar, N=2, A=A):
        '''equilibrium rental rate'''
        def landemand(r):
            return N*(A*P/r)**2 - Tb
        return fsolve(landemand, 1)[0]
In [19]: P, cl, req(P,cl)
Out[19]: (5.5, 3, 2.4596747752497685)
In [20]: LD(P, req(P,cl))*2, Tbar
Out[20]: (10.000000000000002, 10)
In [21]: def mopt(P,cl,A=A):
        '''Optimum land use for each i at the P*MPT = max(cl,r)'''
        r = req(P,cl)
        ru = max(cl, r)
        return (A*P/ru)**2

```

Farmer A will demand

```

In [22]: mopt(P,cl), MVPT(P, mopt(P,cl) )
Out[22]: (3.3611111111111107, 3.0)
In [23]: #plt.style.use('bmh')

def landmarket(P, cl, title, A=A):
    t = np.linspace(0.1,Tbar-0.1, 2*Tbar)
    fig = plt.subplots(figsize=(12,8))
    x0 = mopt(P,cl,A=A)
    plt.ylim(0,5)
    #plt.axhline(cl,linestyle=':')
    plt.axhline(max(cl,req(P,cl,A=A)),linestyle='--')
    plt.axhline(cl,linestyle=':')
    plt.plot(t,MVPT(P,t))
    plt.text(8, MVPT(P,8),r'$P \cdot F_T(T)$', fontsize=18)
    plt.text(1, MVPT(P,Tbar-1),r'$P \cdot F_T(\bar{T} - T)$', fontsize=18)
    plt.xlabel('T -- land use', fontsize=18)
    plt.ylabel('MVPT', fontsize=18)

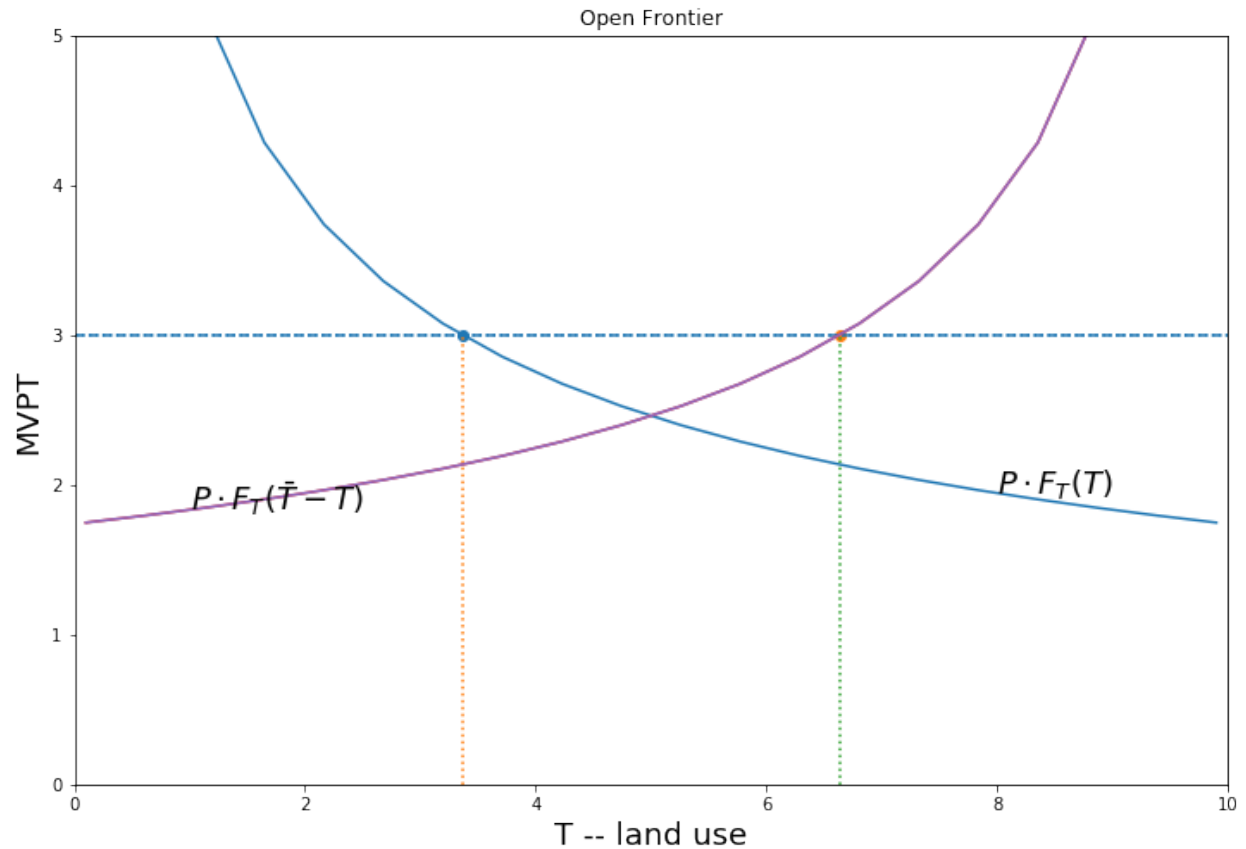
```

```

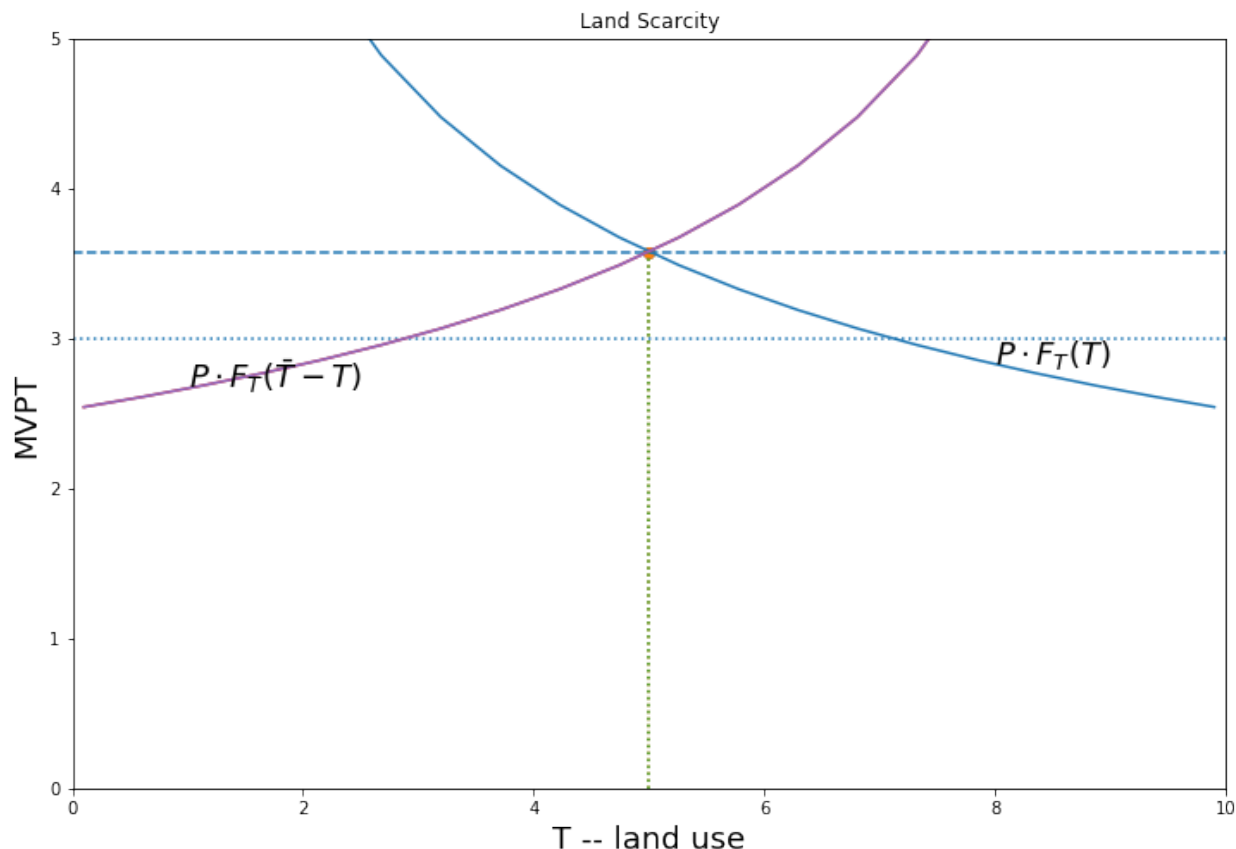
plt.scatter(x0, MVPT(P,x0))
plt.scatter(Tbar-mopt(P,c1),MVPT(P,x0))
plt.plot([x0,x0],[0,MVPT(P,x0)],':')
plt.plot([Tbar-x0,Tbar-x0],[0,MVPT(P,x0)],':')
plt.plot(t,MVPT(P,Tbar - t))
plt.plot(t,MVPT(P,Tbar-t))
plt.title(title)
plt.xlim(0,Tbar);

```

In [24]: `landmarket(P=5.5, c1 = 3, title = 'Open Frontier')`



In [25]: `landmarket(P=8, c1 = 3, title = 'Land Scarcity')`



## 1.7 Farm Household Models

Most economics textbooks typically present the decisions of firms in separate chapters from the decisions of households. In this artificial but useful dichotomy the firm organizes production and hires factors of production while households own the factors of production (and ownership shares in firms). Households earn incomes from supplying these factors to firms and use that income to purchase products from the firms.

Many households in developing countries however may operate small farms, garden plots, stores or other types of businesses, and may also at the same time sell or hire labor in the labor market. In fact a large literature suggests that one defining characteristic of developing countries labor markets is the very large fraction of its labor force that are self-employed ‘own account’ workers and the many who run small businesses alongside wage labor jobs (Gollin, 2008).

For this reason it will be useful to build models that allow households to both utilize labor and other factors as potential producers who may also choose to sell or hire these factors on factor markets. Here we offer stylized representations of the so called ‘farm household model’ or ‘agricultural household model’ (Singh, Squire, Strauss, 1986).

In these models the household acts both as a consumption unit, maximizing utility over consumption and ‘leisure’ choices and as a production unit, deciding how to allocate factors of production to the household farm or business.

A key question in this literature is whether the household’s production decisions (such as its choice of labor and other inputs and the scale of production) are *separable* or not from its preferences and endowments (e.g. consumer preferences, household demographic composition and ownership of land and other resources). For example, will a rural farm household with more working age adults run a larger farm compared to a neighboring household that is otherwise similar and has access to the same technology but has fewer working-age adults? If the household’s decisions are non-separable then the answer might be yes: the larger household uses of its larger labor force to run a larger farm.

When households are embedded in well functioning competitive markets however we tend to expect the household's decisions to become separable:

- **Household as a profit-maximizing firm:** acting as a profit-maximizing firm the household first optimally chooses labor and other input allocations to extract maximum profit income from the farm.
- **Household as a consumption unit:** the household makes optimal choices over consumption and leisure subject to its income budget which includes income from selling its labor endowment and maximized profits from the household as a firm.

In the simple example above of two otherwise identical households the separable farm household model would predict that both households run farms of similar size and use wage labor markets to equalize land-labor ratios and hence marginal products of labor across farms. The larger household would be a net seller of labor on the market compared to the smaller household.

Without the separation property the common approach of many studies to separately estimate consumer demand from output supply functions is no longer valid.

Another way to state the separation hypothesis is that if they have access to markets the marginal production decisions of farms (and firms more generally) should not depend on their owners' ownership of tradable factors (except in so far as it might raise their overall income) or their preferences in consumption. When markets are complete then production decisions will be separable, the price mechanism will equalize the marginal products of factors across uses, and the initial distribution of tradable factors should not matter for allocative efficiency (the first and second welfare theorems). Much of modern micro-development since at least the mid 1960s however is concerned with how transactions costs, asymmetric information, conflict and costly enforcement can lead to market frictions and imperfections that make production decisions non-separable, which then means that the initial distribution of property rights (over land and other tradable factors) may in fact well matter for determining the patterns of production organization and its efficiency in general equilibrium. A good example of such analysis is Eswaran and Kotwal (1986) paper on "Access to Capital and Agrarian production organization," which explores how the combination of transaction costs in labor hiring and in access to capital can lead to a theory of endogenous class structure or occupational choice which can change dramatically depending on the initial distribution of property rights.

### 1.7.1 The Chayanovian Household

We'll start with a non-separable model, inspired by the work of the Russian agronomist Alexander Chayanov whose early 20th century ideas and writings on "the peasant economy" became widely influential in anthropology, economics, and other fields. Chayanov played a role in the Soviet agrarian reforms but his focus on the importance of the household economy led him to be (presciently) skeptical of the appropriateness and efficiency of large-scale Soviet farms. He was arrested, sent to a labor camp and later executed.

The following model is not anywhere as rich as Chayanov's description of the peasant household economy but instead a stripped down version close in spirit to Amartya Sen's (1966) adaptation.

Farm household  $i$  has land and labor endowment  $(\bar{T}_i, \bar{L}_i)$ . We can assume that it can buy and sell the agricultural product at a market price  $p$  but in practice we will model the household as self-sufficient and cut off from the market for land and labor. The household (which we treat as a single decision-making unit for now) allocates labor maximizes utility over consumption and leisure

$$\max_{c_i, l_i} U(c_i, l_i)$$

Although we haven't included it above, we think of the utility function  $U(c_i, l_i; A)$  as depending on 'preference shifters'  $A$  which might include such things as the household's demographic composition or things that affect it's preference for leisure over consumption).

The household maximizes this utility subject to the constraints that consumption  $c_i$  not exceed output and that the sum of hours in production  $L_i$  plus hours in leisure  $l_i$  not exceed the household's labor endowment  $\bar{L}_i$

$$c_i \leq F(\bar{T}_i, L_i)$$

$$L_i + l_i \leq \bar{L}_i$$

Substituting the constraints into the objective allows one to redefine the problem as one of choosing the right allocation of labor to production:

$$\max_{L_i} U(F(\bar{T}_i, L_i), \bar{L}_i - L_i)$$

From the first order necessary conditions we obtain

$$U_c \cdot F_L = U_l$$

which states that the farm household should allocated labor to farm production up to the point where the marginal utility benefit of additional consumption  $U_c \cdot F_L$  equals the opportunity cost of leisure.

Since the household sets the marginal product of labor (or shadow price of labor) equal to the marginal rate of substitution between leisure and consumption  $F_L = U_l/U_c$  and the latter is clearly affected by ‘preference shifters’  $A$  and also by the household’s land endowment. The shadow price of labor will hence differ across households that are otherwise similar (say in their access to production technology and farming skill). For example households with a larger endowment of labor will run larger farms.

It will be useful to draw things in leisure-consumption space. Leisure is measured on the horizontal from left to right which then means that labor  $L$  allocated to production can be measured from right to left with an origin starting at the endowment point  $\bar{L}_i$

To fix ideas with a concrete example assume farm household  $i$  has Cobb-Douglas preferences over consumption and leisure:

$$U(c, l) = c^\beta l^{1-\beta}$$

and its production technology is a simple constant returns to scale Cobb-Douglas production function of the form:

$$F(\bar{T}_i, L_i) = \bar{T}_i^\alpha L_i^{1-\alpha}$$

The marginal product of labor  $F_L$  which the literature frequently refers to as the shadow price of labor will be given by:

$$F_L(\bar{T}_i, L_i) = (1 - \alpha) \left[ \frac{\bar{T}_i}{L_i} \right]^\alpha$$

The first order necessary condition can therefore be solved for and for these Cobb-Douglas forms we get a rather simple and tidy solution to the optimal choice of leisure

$$l^* = \frac{a \cdot \bar{L}}{1 + a}$$

where  $a = \frac{1-\beta}{\beta \cdot (1-\alpha)}$

## 1.7.2 Graphical analysis

(Note: formulas and plots work for default parameters but not checked for other values.)

```
In [1]: %matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
from ipywidgets import interact
```

```
In [2]: ALPHA = 0.5
        BETA = 0.7
        TBAR = 100
        LBAR = 100

        def F(T,L,alpha=ALPHA):
            return (T**alpha)*(L**(1-alpha))

        def FL(T,L,alpha=ALPHA):
            """Shadow price of labor"""
            return (1-alpha)*F(T,L,alpha=ALPHA)/L

        def FT(T,L,alpha=ALPHA):
            """Shadow price of labor"""
            return alpha*F(T,L,alpha=ALPHA)/T

        def U(c, l, beta=BETA):
            return (c**beta)*(l**(1-beta))

        def indif(l, ubar, beta=BETA):
            return (ubar/(1**(1-beta)) )**(1/beta)

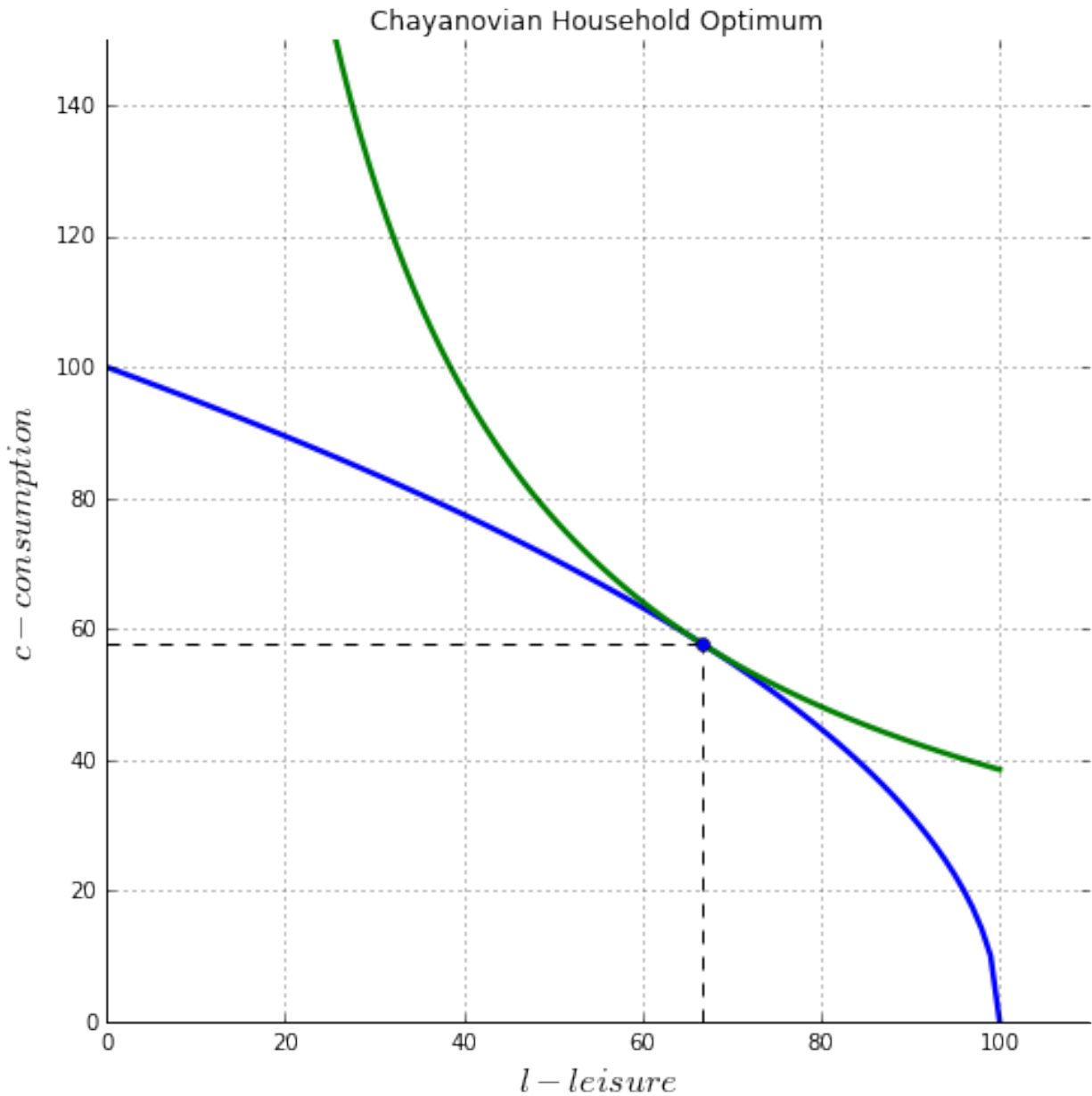
        def leisure(Lbar,alpha=ALPHA, beta=BETA):
            a = (1-alpha)*beta/(1-beta)
            return Lbar/(1+a)

        def HH(Tbar,Lbar,alpha=ALPHA, beta=BETA):
            """Household optimum leisure, consumption and utility"""
            a = (1-alpha)*beta/(1-beta)
            leisure = Lbar/(1+a)
            output = F(Tbar,Lbar-leisure, alpha)
            utility = U(output, leisure, beta)
            return leisure, output, utility

In [3]: def chayanov(Tbar,Lbar,alpha=ALPHA, beta=BETA):
        leis = np.linspace(0.1,Lbar,num=100)
        q = F(Tbar,Lbar-leis,alpha)
        l_opt, Q, U = HH(Tbar, Lbar, alpha, beta)
        print("Leisure, Consumption, Utility =({:5.2f},{:5.2f},{:5.2f})"
              .format(l_opt, Q, U))
        print("shadow price labor:{:5.2f}".format(FL(Tbar,Lbar-l_opt,beta)))
        c = indif(leis,U,beta)
        fig, ax = plt.subplots(figsize=(8,8))
        ax.plot(leis, q, lw=2.5)
        ax.plot(leis, c, lw=2.5)
        ax.plot(l_opt,Q,'ob')
        ax.vlines(l_opt,0,Q, linestyles="dashed")
        ax.hlines(Q,0,l_opt, linestyles="dashed")
        ax.set_xlim(0, 110)
        ax.set_ylim(0, 150)
        ax.set_xlabel(r'$l - leisure$', fontsize=16)
        ax.set_ylabel(r'$c - consumption$', fontsize=16)
        ax.spines['right'].set_visible(False)
        ax.spines['top'].set_visible(False)
        ax.grid()
        ax.set_title("Chayanovian Household Optimum")
        plt.show()

In [4]: chayanov(TBAR,LBAR,0.5,0.5)
```

Leisure, Consumption, Utility = (66.67, 57.74, 62.04)  
 shadow price labor: 0.87



### 1.7.3 Inverse farm size productivity relationship

In this non-separable household with no market for land or labor, each household farms as much land as it owns. We find the well-known inverse farm size-productivity relationship: output per hectare is higher on smaller farms. Households with larger farms enjoy higher marginal products of labor – a higher shadow price of labor.

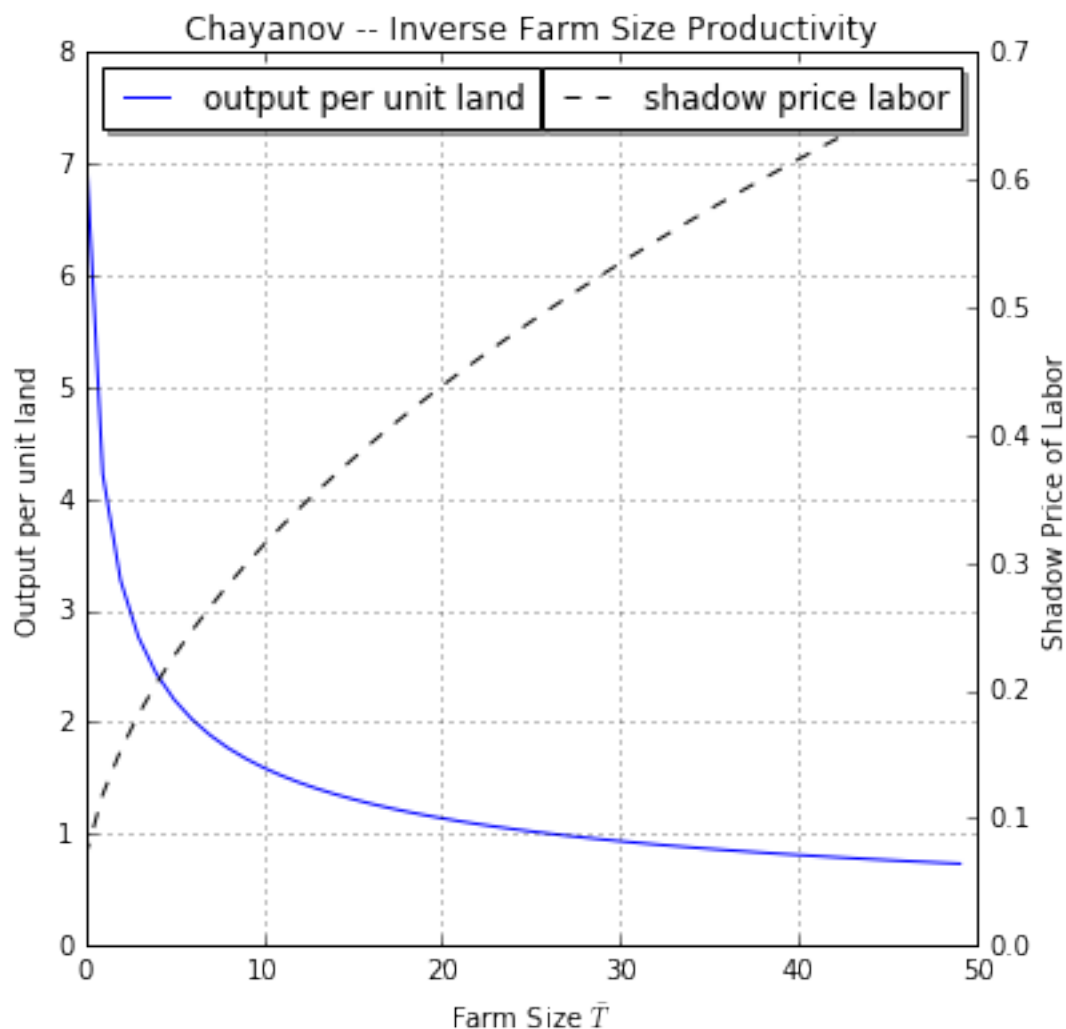
These two relationships can be seen below on this two-axis plot.

```
In [5]: Tb = np.linspace(1, LBAR)
        le, q, _ = HH(Tb, LBAR)
        fig, ax1 = plt.subplots(figsize=(6, 6))
```

```

ax1.plot(q/Tb,label='output per unit land')
ax1.set_title("Chayanov -- Inverse Farm Size Productivity")
ax1.set_xlabel('Farm Size '+r'$\bar{T}$')
ax1.set_ylabel('Output per unit land')
ax1.grid()
ax2 = ax1.twinx()
ax2.plot(FL(Tb,LBAR-le),'k--',label='shadow price labor')
ax2.set_ylabel('Shadow Price of Labor')
legend = ax1.legend(loc='upper left', shadow=True)
legend = ax2.legend(loc='upper right', shadow=True)
plt.show()

```



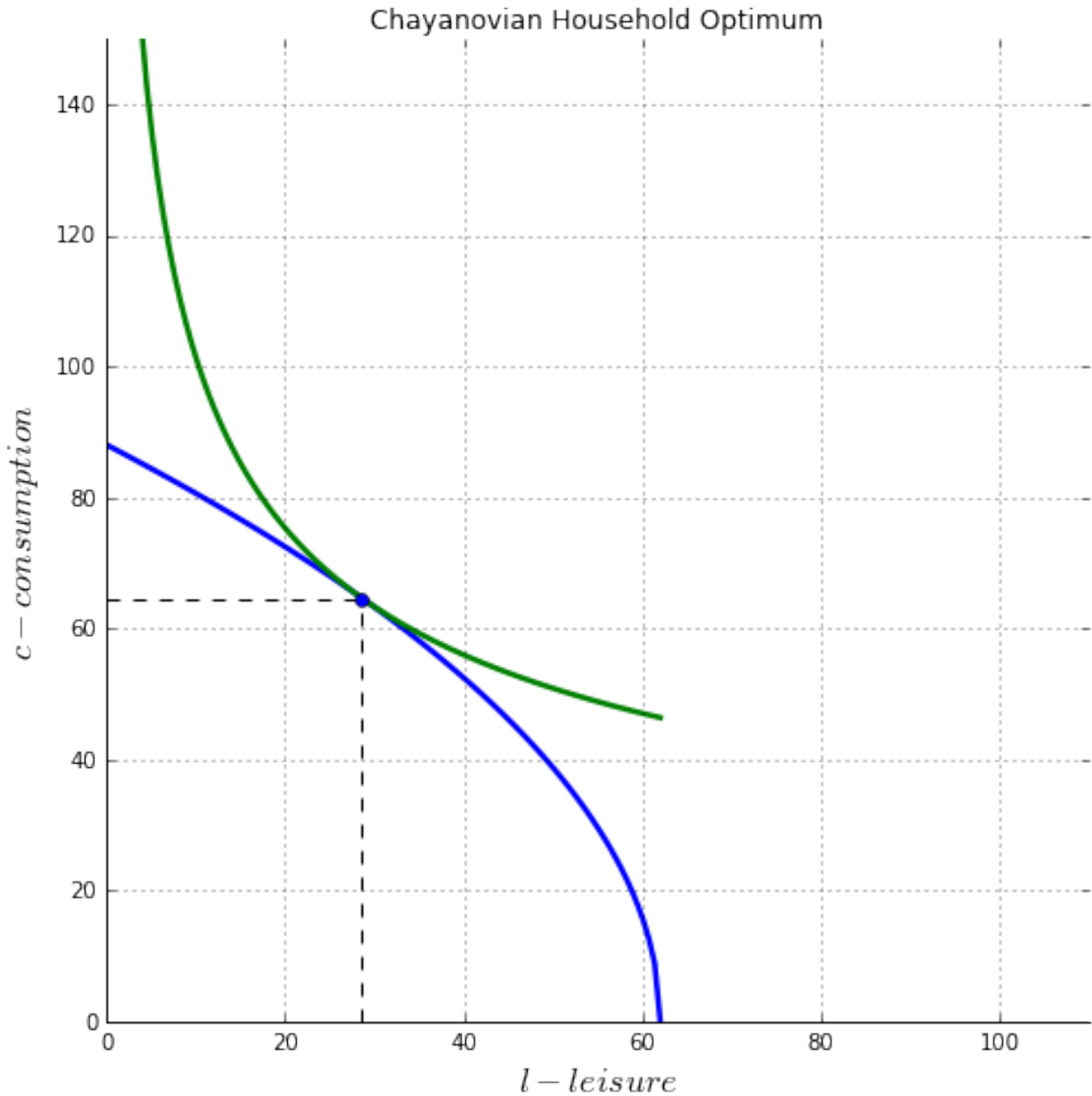
If you are running this notebook in interactive mode you can play with the sliders:

```

In [6]: interact(chayanov,Tbar=(50,200,1),Lbar=(24,100,1),alpha=(0.1,0.9,0.1),beta=(0.1,0.9,0.1))
Leisure, Consumption, Utility =(28.62,64.60,50.60)
shadow price labor: 0.58

```





Out[6]: <function \_\_main\_\_.chayanov>

### The Separable Farm Household

Consider now the case of the household that participates in markets. It can buy and sell the consumption good at market price  $p$  and buy or sell labor at market wage  $w$ . Hired labor and own household labor are assumed to be perfect substitutes.

To keep the analysis a little bit simpler and fit everything on a single graph we'll assume that the market for land leases remains closed. However it is easy to show that for a linear homogenous production function this will not matter for allocative efficiency – it will be enough to have a working competitive labor market for the marginal product of land and labor to equalize across farms.

When the farm household is a price-taker on markets like this, farm production decisions become 'separable' from

farm consumption decisions. The farm household can make its optimal decisions as a profit-maximizing farm and then separately choose between consumption and leisure subject to an income constraint.

So the household's problem can be boiled down to:

$$\max_{c,l} U(c,l)$$

$$s.t.$$

$$p \cdot c \leq pF(\bar{T}_i, L^*) + w \cdot (\bar{L}_i - l)$$

This last constraint states that what the household spends on consumption cannot exceed the value of farm sales at a profit maximizing optimum plus wage income from labor sold to the market. This last constraint can also be rewritten as:

$$p \cdot c + w \cdot l \leq \Pi(T_i, w, p) + w \cdot \bar{L}_i$$

where  $\Pi(T_i, w, p) = p \cdot F(T_i, L_i^*) - w \cdot L_i^*$  is the maximized value of farm profits and  $L_i^*$  is the optimal quantity of labor that achieves that maximum.

In other words the farm household can be thought of as maximizing profits by choosing an optimal labor input into the farm  $L_i^*$  which will be satisfied with own and/or hired labor. The household then maximizes utility from consumption and leisure subject to the constraint that it not spend more on consumption and leisure as its income which is made up of farm profits (a return to land) and the market value of its labor endowment.

In this last description the household is thought of as buying leisure at its market wage opportunity cost.

### The efficient competitive farm

As a production enterprise the farm acts to maximize farm profits  $pF(\bar{T}_i, L_i) - w \cdot L_i$ . The first order necessary condition for an interior optimum is: The separation property is immediately apparent from this first order condition, as this equation alone is enough to determine the farm's optimum labor demand which depends only on the production technology and the market wage, and not at all on household preferences. While the quantity of labor demanded here does depend on the households' ownership of land, with a functioning labor market the number of workers per unit land will be the same across all farms and hence also independent of household land endowment.

For the Cobb-Douglas production function, optimal labor demand can then be solved to be:

$$L_i^* = \bar{T}_i \cdot \left[ \frac{p(1 - \alpha)}{w} \right]^{\frac{1}{\alpha}}$$

Substituting this value into the profit function we can find an expression for farm profits  $\Pi(T_i, w, p)$  as a function of the market product price and wage and the farm's land holding. If we had allowed for hiring on both land and labor markets we would have found zero profits with this linear homogenous technology so in this context where there is no working land market, profits are just like a competitively determined rent to non-traded land (equal in value to what the same household would have earned on its land endowment had the land market been open).

Note how the optimal allocation of labor to the farm is independent of the size of the household  $\bar{L}_i$  and household preferences. If we had allowed for a competitive land market the labor allocation would also have been independent of the household's

Household demand for consumption and leisure is very simple given this Cobb-Douglas utility:

$$c^*(w) = \beta \cdot I$$

$$l^*(w) = (1 - \beta) \cdot \frac{I}{w}$$

where household income  $I$  is given by the sum of farm profits and the market value of the factor endowment:

$$I = \Pi(w, \bar{T}) + w \cdot \bar{L}$$

In what follows we shall normalize the product price  $p$  to unity. This is without loss of generality as all that matters for real allocations is the relative price of labor (or leisure) relative to output, or the real wage  $\frac{w}{p}$ . So in all the expressions that follow when we see  $w$  it should be thought of as  $\frac{w}{p}$

```
In [7]: def farm_optimum(Tbar, w, alpha=ALPHA, beta=BETA):
        """returns optimal labor demand and profits"""
        LD = Tbar * ((1-alpha)/w)**(1/alpha)
        profit = F(Tbar, LD) - w*LD
        return LD, profit

        def HH_optimum(Tbar, Lbar, w, alpha=ALPHA, beta=BETA):
            """returns optimal consumption, leisure and utility.
               Simple Cobb-Douglas choices from calculated income """
            _, profits = farm_optimum(Tbar, w, alpha)
            income = profits + w*Lbar
            consumption = beta * income
            leisure = (1-beta) * income/w
            utility = U(consumption, leisure, beta)
            return consumption, leisure, utility
```

Let's assume the market real wage starts at unity:

```
In [8]: W = 1

In [9]: def plot_production(Tbar, Lbar, w):
        lei = np.linspace(1, Lbar)
        q = F(Tbar, Lbar-lei)
        ax.plot(lei, q, lw=2.5)
        ax.spines['right'].set_visible(False)
        ax.spines['top'].set_visible(False)

In [10]: def plot_farmconsumption(Tbar, Lbar, w, alpha=ALPHA, beta=BETA):
        lei = np.linspace(1, Lbar)
        LD, profits = farm_optimum(Tbar, w)
        q_opt = F(Tbar, LD)
        yline = profits + w*Lbar - w*lei
        c_opt, l_opt, u_opt = HH_optimum(Tbar, Lbar, w)

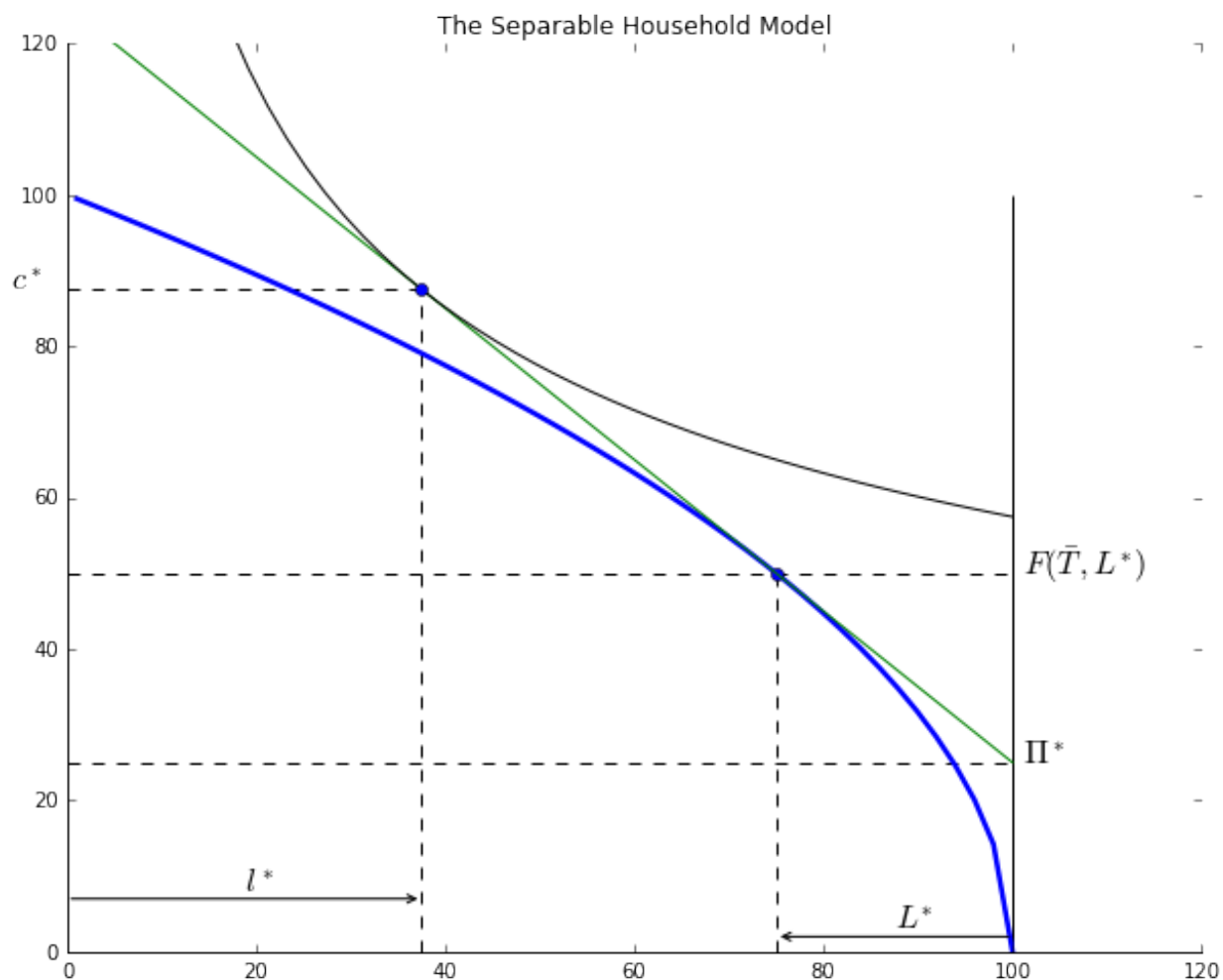
        ax.plot(Lbar-LD, q_opt, 'ob')
        ax.plot(lei, yline)
        ax.plot(lei, indif(lei, u_opt, beta), 'k')
        ax.plot(l_opt, c_opt, 'ob')
        ax.vlines(l_opt, 0, c_opt, linestyle="dashed")
        ax.hlines(c_opt, 0, l_opt, linestyle="dashed")
        ax.vlines(Lbar - LD, 0, q_opt, linestyle="dashed")
        ax.hlines(profits, 0, Lbar, linestyle="dashed")
        ax.vlines(Lbar, 0, F(Tbar, Lbar))
        ax.hlines(q_opt, 0, Lbar, linestyle="dashed")
        ax.text(Lbar+1, profits, r'$\Pi^{*}$', fontsize=16)
        ax.text(Lbar+1, q_opt, r'$F(\bar{T}, \bar{L}^{*})$', fontsize=16)
        ax.text(-6, c_opt, r'$c^{*}$', fontsize=16)

        ax.annotate('', (Lbar-LD, 2), (Lbar, 2), arrowprops={'arrowstyle': '->'})
        ax.text((2*Lbar-LD)/2, 3, r'$\bar{L}^{*}$', fontsize=16)

        ax.text(l_opt/2, 8, r'$l^{*}$', fontsize=16)
        ax.annotate('', (0, 7), (l_opt, 7), arrowprops={'arrowstyle': '<-'})
```

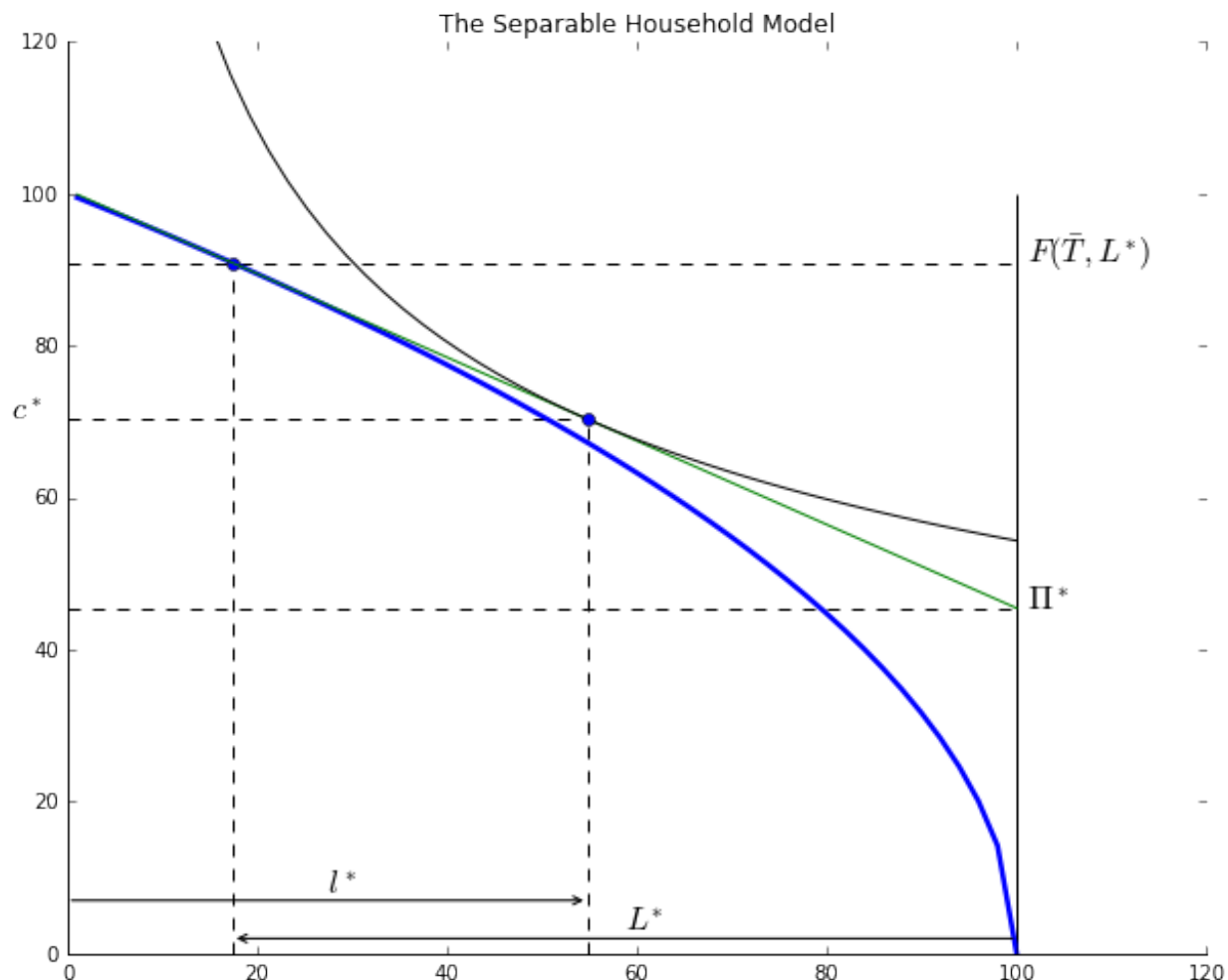
Here is an example of a household that works and their own farm *and* sells labor to the market:

```
In [11]: fig, ax = plt.subplots(figsize=(10,8))
         plot_production(TBAR,LBAR,W)
         plot_farmconsumption(TBAR, LBAR, W)
         ax.set_title("The Separable Household Model")
         ax.set_xlim(0,LBAR+20)
         ax.set_ylim(0,F(TBAR,LBAR)+20)
         plt.show()
```



Here is the same household when facing a much lower wage (55% of the wage above). They will want to expand output on their own farm and cut back on labor sold to the market (expand leisure). This household will on net hire workers to operate their farm.

```
In [12]: fig, ax = plt.subplots(figsize=(10,8))
         plot_production(TBAR,LBAR, W*0.55)
         plot_farmconsumption(TBAR, LBAR, W*0.55)
         ax.set_title("The Separable Household Model")
         ax.set_xlim(0,LBAR+20)
         ax.set_ylim(0,F(TBAR,LBAR)+20)
         plt.show()
```



### 1.7.4 No Inverse farm size productivity relationship

Since every farm takes the market wage as given the marginal product of labor will be equalized across farms. The shadow price of labor will equal the market wage on all farms, regardless of their land size. The land labor ratio and hence also the marginal product of land will also equalize across farms so output per unit land will also remain constant.

The next section describes some empirical tests of the separation hypothesis as well as models with ‘selective-separability’ where missing markets and/or transactions costs end up determining that, depending on their initial ownership of assets, some households may face different factor shadow prices depending on whether they are buyers or sellers or non-participants on factor markets.

This last example should drive home the point that when the farm household’s optimization problem is separable the farm’s optimal choice of inputs will be independent of both household characteristics *and* its endowment of *tradable* factors such as land and labor.

In this last example when we shut down the labor market and introduce a transaction wedge on the market for land we create a non-separable region of autarkic or non-transacting households (shaded above). A framework such as this would generate data that displays the classic inverse farm-size productivity relationship. Farms that lease in land are smaller and have higher output per hectare than larger farms that lease land out and within the autarky zone farm size is increasing in the size of the land endowment and the shadow price (and hence output per unit land) falling.

## 1.8 Testing for Separation in Household models

There are a few related approaches to testing for separation. The early studies were ‘global’ tests that aimed at accepting or rejecting the null hypothesis of a separable household model for the entire sample of households. Some later studies argue that a more typical situation is one where only ‘local’ subsets of households may be constrained and therefore caught in a ‘non-separable’ regime and that any econometric estimation strategy should be sensitive to this fact.

### 1.8.1 Global separation tests:

One approach attempts to test the implication of the separation hypothesis that labor demand should be independent of household preference shifters  $A$  (i.e. demographic characteristics of the household). Basically this approach regresses labor demand on prices, fixed non-traded inputs, and a measure of  $A$ . The null hypothesis is that the model is separable and the coefficient on  $A$  is zero. Therefore an insignificant estimated coefficient is interpreted as a failure to reject the null hypothesis of separation. There are econometric issues such as simultaneity bias to worry about of course. This is the approach of papers such as Lopez (1984), Benjamin (1992) and many others.

Another approach builds from the observation that under separation the shadow price of labor or the marginal product of labor on the farm – call it  $w^*$  – should equal the market wage  $w$ . Since the shadow wage is not directly observed it must be calculated off an estimated production function. If estimates of  $w^*$  have been estimated across farms the separation test boils down to a test of whether  $\beta_0 = 0$  and  $\beta_1 = 1$  in the regression  $w^* = \beta_0 + \beta_1 w$ . Issues of endogeneity and other econometric concerns must be addressed. This is the approach of Jacoby (1993), Skoufias (1994) and others.

Kien Le (2010) proposes a semi-parametric method that combines elements of both of the above approaches but uses less data. his approach boils down to regressing the log of the measured value of output per worker on each farm against the market wage and a few controls.

### 1.8.2 Selective-separability

A small but important literature attempts to model farm household behavior on markets with transactions costs. The basic idea is that transactions costs such as the cost of getting to work can establish a wedge between the cost of hiring and selling labor. Suppose for example that farms that hire have to pay workers a wage of  $w$  per hour but workers take home only  $w(1 - \tau)$  of the wage due to transaction costs. If a would-be worker has the option of operating their own farm it should be clear that they would work sell their labor to the market as long as the shadow price (marginal product) of labor on their own farm falls below  $w(1 - \tau)$ , they will neither hire nor sell labor to the market if the shadow price of labor is above  $w(1 - \tau)$  but below  $w$  and they will hire labor if the shadow price is above  $w$ . In a simple model where other factors of production (say land or farming skill) are missing then households may be placed into each of these three regimes (hiring out, self-sufficient, hiring in) depending on the initial distribution of these non-traded assets across households. The extent of allocative inefficiency (misallocation compared to the same economy without transaction costs) will depend on both the size of the transaction costs and the initial distribution of non-traded assets. In the simple model I’ve just traced out households that operate farms and also hire out labor will in general be producing using more labor-intensive methods compare to households that hire-in labor since the latter face a higher wage.

de Janvry et al (1991) highlight some of the many ‘puzzles’ and behaviors that can be explained by adding transaction costs to an otherwise neoclassical model. Sadoulet et al (1996) appear to be one of the earliest, if not the earliest paper to use the term ‘selective separability’ to refer to this fact that households may be selectively in one regime or another, depending on their asset holdings.

## Carter and Yao (2002): Local versus Global Separability

Carter and Yao's (2002) paper "Local versus Global Separability in Agricultural Household Models: The factor price equalization effect of land transfer rights," offers an example of local versus global separability tests. They consider a model where transaction costs establish a wedge between the rental rate tenants must pay to hire-in land and the rental rate that landlords receive for hiring-out land. In a relatively simple model where the labor market is closed (which they argue fits their Chinese panel dataset from 1988 and 1993) this situation will generate a three regime equilibrium: (1) households with an endowment ratio of land relative to labor below some threshold will be tenants and hire-in land but these households are price-takers and the regime is separable in that all households in this group employ a common high labor intensity and low shadow wage. (2) households with an endowment ratio above a different higher threshold will rent-out land and are also in a separable regime but with a lower labor-land intensity and higher shadow wage. Finally households in the non-separable autarky regime (3) are those with endowment ratios between the two thresholds; these households do not hire in or hire out land but instead employ the land they have which means that for households in this group the shadow wage rises with the endowment ratio.

As the authors point out, referring to earlier 'global' tests:

...neither separability, nor non-separability exist globally across the full endowment continuum ...the reduced form global separability tests found in the econometrics literature (which do not distinguish between rental regimes) would be mis-specified and subject to a degree of bias that depends on the distribution of households across the three regimes.

They employ a switching regression approach where the model of labor intensity to be estimated depends on the discrete rental regime a household is classified into. The authors employ a simulated MLE method to estimate the system with panel data. They contrast their estimates with what would be found on the same dataset with one of the global tests of separability, arguing that the results from a global test are biased and misleading.

Carter and Yao's (2002) paper "Local versus Global Separability in Agricultural Household Models: The Factor Price Equalization Effect of Land Transfer Rights"

were analyzing households in rural China in the late 80s and 90s, a period of transition where land markets were being slowly liberalized. The authors describe the situation as one where it was reasonable to assume no market for wage labor and a limited market for land. They model this last idea as a wedge between the rental rate that is received by households that lease out land  $r^O = r(1 - c^O(M))$  and the amount that households had to pay to rent land in  $r^I = r(1 + c^I(M))$ , where  $r^O \leq r \leq r^I$ .

To see this clearly, suppose households (HH) are otherwise identical to each other in all respects – including the size of their labor force, call it  $\bar{L}_e$  – and only differ in terms of their endowment of land  $T_i$ . Households can then be classified as belonging to one of three behavioral regimes, depending the relative size of their land endowment.

, two where household behavior can be characterized as separable, and one w.

1. **HH that rent-in land,  $T_i \in (0, T^I]$ :** Households with a low endowment of land will lease in land at the high rent-in market rate  $r^I$  to operate a farm of size  $T^I$ .
2. **HH in autarky,  $T_i \in (T^I, T^O]$ :** Households with a land endowment within these two thresholds will operate their own land at shadow rate  $r^0 < r^* < r^I$ .
3. **HH that rent-out land,  $T_i \in [T^O, \bar{T}]$ :** Households with a high endowment of land will lease out land at the low rent-out market rate  $r^O$  and operate a farm of size  $T^O$ .

Leaving aside the issue of how the equilibrium rental rate is determined (discussed in other sections) suppose  $r^I$  and  $r^O$  are given. Then Households in the first regime operate farms of size maximize farm profits

$$p \cdot F_T(T^I, \bar{L}_e) = r^I$$

Households in the third regime operate farms of size maximize farm profits

$$p \cdot F_T(T^O, \bar{L}_e) = r^O$$

And farms with a land endowment of  $T_i \in (T_I, T^O]$  will operate a farm of size  $T_i$  and the shadow rental rate of land on their farm will be

$$r^*(T_i) = p \cdot F_T(T_i, \bar{L}_e)$$

Below is a plot for our simple Cobb-Douglass example and reasonable parameter values.

```
In [1]: %matplotlib inline
import matplotlib.pyplot as plt
import numpy as np

In [2]: ALPHA = 0.5
BETA = 0.7
TBAR = 100
LBAR = 100

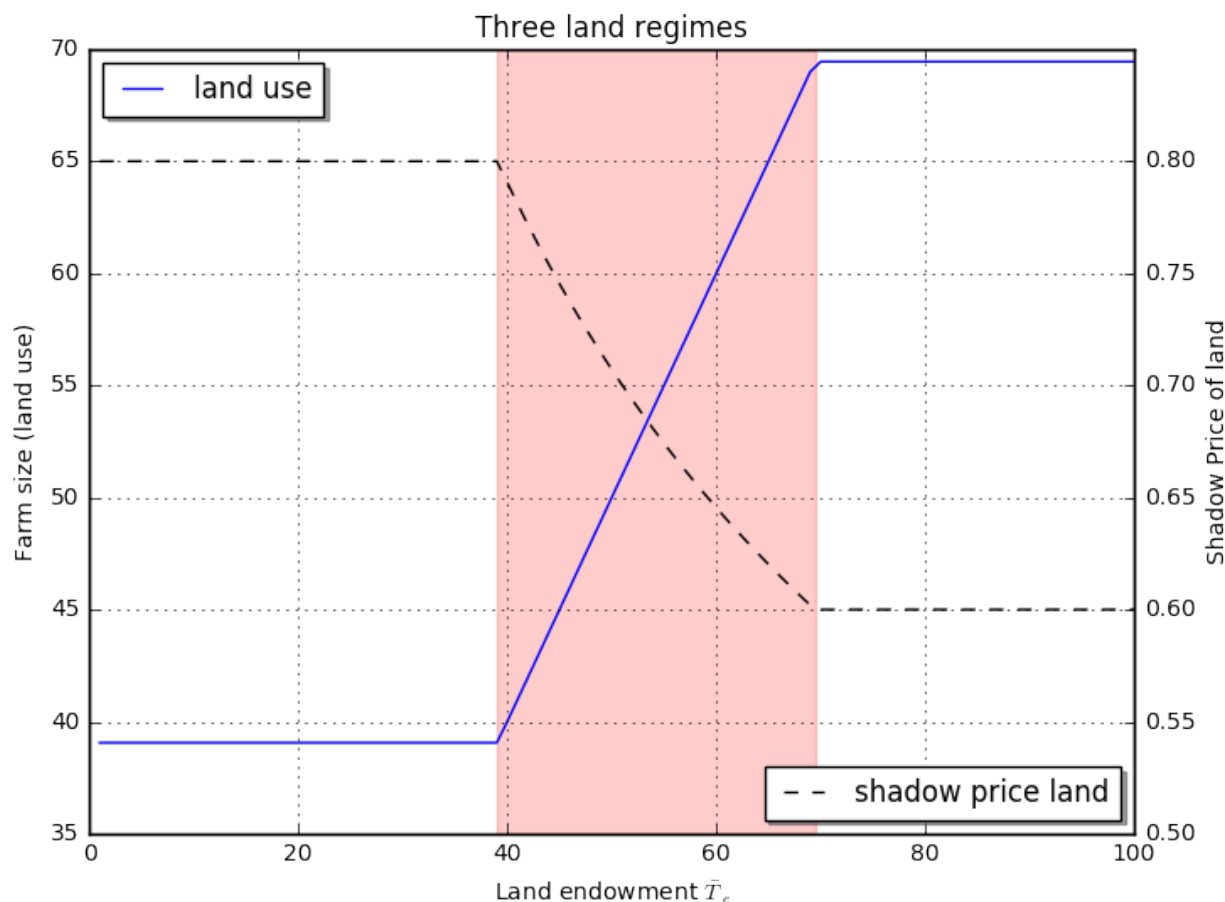
def F(T,L,alpha=ALPHA):
    return (T**alpha)*(L**(1-alpha))

def FT(T,L,alpha=ALPHA):
    """Shadow price of labor"""
    return alpha*F(T,L,alpha=ALPHA)/T

In [3]: def yao_carter(Ti, rO, rI, alpha=ALPHA):
    """returns optimal land use and shadow rental price of land"""
    r = FT(Ti, LBAR, alpha)
    Tout = LBAR * (alpha/rO)**(1/(1-alpha))
    Tin = LBAR * (alpha/rI)**(1/(1-alpha))
    TD = (r < rO)*Tout + (r > rI)* Tin + ((r>=rO) & (r<=rI))*Ti
    rs = (r < rO)*rO + (r > rI)* rI + ((r>=rO) & (r<=rI)) * r
    fig, ax1 = plt.subplots(figsize=(8,6))
    ax1.plot(Ti,TD,label='land use')
    ax1.set_title("Three land regimes")
    ax1.set_xlabel('Land endowment '+r'$\bar{T}_e$')
    ax1.set_ylabel('Farm size (land use)')
    ax1.grid()
    ax2 = ax1.twinx()
    ax2.plot(Ti, rs, 'k--',label='shadow price land')
    ax2.set_ylabel('Shadow Price of land')
    ax2.set_ylim(0.5,0.85)
    ax1.axvspan(Tin, Tout, alpha=0.2, color='red')
    legend = ax1.legend(loc='upper left', shadow=True)
    legend = ax2.legend(loc='lower right', shadow=True)
    plt.show()

In [4]: Ti = np.linspace(1,TBAR,num=100)
RI = 0.8
RO = 0.6
yao_carter(Ti, RO, RI, alpha=ALPHA)
```





## 1.9 Equilibrium Size Distribution of Farms

Like many of these notebooks this one was written quickly.

### 1.9.1 Indeterminacy of size distribution with constant returns to scale technology

In an earlier analysis we described the optimal consumption and production allocations of a *farm household* that took product and factor prices as given. We argued that if the production function  $F(T, L)$  was linear homogenous in land and labor than efficiency in allocation could be achieved when both land and labor markets were competitive and even when one of the two factor markets is shutdown. That's because by the definition of what it means to be linear homogenous we have:

$$F(\tau T, \tau L) = \tau F(T, L)$$

which means that

$$P_A F_L(\tau T, \tau L) = P_A F_L(T, L) = w$$

so any farm that operates using the same land-labor ratio will have the same marginal value product, so for any given market wage and rental the optimal scale of production cannot be pinned down, all we can determine is the optimal land-to-labor ratio. The size distribution of farms is indeterminate (but also irrelevant since there is an infinite number

of efficient ways to distribute the efficient output among farms that all have access to the same technology – and every farm makes zero profits).

If in a scenario like this we shut down the land market then the size distribution of farms becomes determined simply by however much land each household has in its endowment. Efficiency in production allocation will still be achieved however if there is a competitive labor market as households hire in or hire out labor to bring the land to labor ratio to a level that efficiently equalizes marginal value products across farms.

### 1.9.2 Non-traded skills, diseconomies of scale and determinate size distribution

Suppose that we instead had a production function  $\hat{F}(S, T, L)$  which is linear homogenous in its three arguments  $S$ ,  $T$ , and  $L$  where  $S$  refers to a *non-traded* farming skill or ability,  $T$  is tradable land and  $L$  is tradable labor. Consider for example Cobb-Douglas function of the form:

$$\hat{F}(S, T, L) = S^{1-\gamma} [T^\alpha L^{1-\alpha}]^\gamma$$

where  $\gamma$  and  $\alpha$  are both numbers greater than 0 and less than or equal to 1. This production function is clearly linear homogenous in its three arguments. Suppose now however that farming skill  $S$  is non-traded and each household had an endowment of exactly  $S = 1$  units of farming skill. The conditional production

$$F(T, L) = \hat{F}(S = 1, T, L)$$

$$F(T, L) = [T^\alpha L^{1-\alpha}]^\gamma$$

is clearly linear homogenous of degree  $\gamma$  and if  $\gamma$  is strictly less than one then it's a production function subject to diseconomies of scale. In contrast to a production function subject to constant returns to scale where the marginal cost and average cost of production to a price taking firm are constant, when production is subject to diseconomies of scale the firm's marginal cost and average cost curves will both be upward sloping: as the firm doubles its inputs output less than doubles so average (and marginal) cost increases with output. This will then determine an optimal scale to the firm (where marginal cost equals the market price of output).

Note also that if there are two firms that are otherwise identical but where one firm has a larger endowment of the non-traded farming skill input, then it will be optimal for this more skilled farmer to operate at a larger scale. This can be seen very simply by noting that the marginal product of labor and capital are both augmented by farming skill.

This then means that if there is some initial distribution of farming skill  $S$  across households then the equilibrium size distribution of operational farm sizes will be in some way proportional to that distribution – farm households with higher farming skills will operate larger farms that use both more land and labor. Since the production function is homogenous in land and labor it's also homothetic and though different farms will operate at different scale they will still all operate using the same land-labor ratio in an efficient competitive allocation.

Lucas (1978) calls a model very similar to this a 'span of control model'. The basic idea is that because farm management/supervision ability cannot be hired on the market due perhaps to moral hazard considerations. As a farmer attempts to use their fixed farming skill to supervise a larger and larger farm they face diseconomies of scale ( $F_{ST} < 0$  and  $F_{SL} < 0$ ) or rising costs. This will mean that a household that starts with a large endowment of land but only medium farming ability would find it optimal to operate a farm up to a certain scale and then lease out remaining land to the market (because the shadow return to employing land on a yet larger farm would fall below the land rental rate they can get from the market).

Lucas' (1978) model complicates things by assuming that the household has to choose between allocating its time endowment between being a pure farm manager/supervisor or being a laborer. This then leads to a partition of households: those above some threshold level of farming skill become full time farm operators who hire in labor and possibly land all those below this threshold do not operate farms and derive income only from selling labor and possibly land to the market.

It turns out that simplifying Lucas' model yields a richer model of the farm economy. In the model that follows every household has an endowment of farming skill, household labor and land, all of which will be supplied inelastically.

Farming skill is independent of labor use, so the household is not forced to choose between farm supervision and work in the labor market, they can do both.

### Competitive Factor Markets with no distortions

Let's first study the benchmark cases where though there is no market for the non-traded farming skill, the other two factors land and labor can be costlessly hired in or out on competitive markets, resulting in competitive efficient equilibria.

### Equal distribution of farming skill across households

Consider the simplest version of the model. Every household has access to the same Cobb Douglas production technology, has  $S = 1$  units of farming skill and there is some distribution of the total endowment of land  $\bar{T}$  and labor  $\bar{L}$  across households. So tradable endowments  $(\bar{T}_i, \bar{L}_i)$  follow some distribution  $\Gamma$ .

To fix ideas suppose that there is an integer number of farm households  $\bar{L}$ . Since every farm household has the same technology and farming skill and face the same equilibrium market factor prices  $w$  and  $v$  they'll choose the same optimal factor demands call them  $(T^D(w, r), L^D(w, r))$ . Equilibrium in the factor markets requires:

$$\bar{L}L^D(w, r) = \bar{L}$$

$$\bar{L}T^D(w, r) = \bar{T}$$

it follows that the optimal farm size will employ  $L^* = 1$  worker,  $T^* = \tau^* = \frac{\bar{T}}{\bar{L}}$  units of land and will produce  $F(1, \tau^*, 1)$  units of output. Equilibrium factor prices are also then simply determined as  $w^* = F_L(1, \tau^*, 1)$  and  $r^* = F_T(1, \tau^*, 1)$ . If the household has a land endowment  $\bar{T}_i$  larger than (less than) this optimal farm scale  $\tau^*$  the household leases out (leases in) land and likewise any household has a labor endowment  $\bar{L}_i$  larger than (less than) this optimal farm scale  $L^*$  hires out (hires in) labor.

### Unequal distribution of farming skill across households

This simple model is easily adapted to the situation where households have access to the same technology but differ in their initial level of farm management/supervision skill. Conceptually farmers with larger non-traded farming skill will operate proportionately larger farms but the land-to-labor ratio is equalized across farms. It's quite easy to demonstrate that if the initial distribution of skills is distributed as a log-normal then so too will the distribution of optimal farm sizes. Once again the farm household will lease out (lease in) land and hire out (hire in) labor depending on whether their initial holding of the traded land endowment exceeds (falls short of) the optimal land size for their farming skill and whether their initial holding of the traded labor endowment exceeds (falls short of) the optimal labor demand for their farming skill.

If we add in a fixed cost to operating a farm then farmers of very low skill will find it unprofitable to operate a farm and will become pure laborers and lease out all land.

Depending on the initial allocation of non-traded skills and tradable inputs households the model delivers an endogenous fourfold classification of the types of engagement in this farm labor economy (the labels I use here are similar to Eswaran and Kotwal's (1986) paper): \* **pure laborers**: low skill households do not operate farms. \* **laborer-cultivators**: low-medium skill households operate small farms and also seek outside employment \* **capitalist**: higher skill households operate larger farms with hired labor.

In the economy described thus far, farm household decisions are separable and the initial distribution of *tradable* land and labor does not matter for efficiency in production.

Note that because our production function was linear homogenous in the three inputs we could shut down one of the markets – the market for farming skill – and still achieve efficiency in production.

We can think of the Eswaran and Kotwal (1986) model in these terms. They describe a model with land, labor and labor supervision. We've shut down the market for supervisory labor whereas they tell a more complicated story about moral hazard which at the end of the day introduces a similar type of diminishing returns to supervision. That market imperfection by itself is not enough to matter for efficiency in allocation but they then go ahead and introduce a second distortion: they impose the requirement that farms must pay for their land and labor hiring in advance of the crop but then make a farm household's access to capital proportional to its initial holding of land (they also impose a fixed cost of operation similar to that described above).

The working capital constraint means that some farms will not be able to achieve their optimum scale of operation and the initial distribution of land will now determine both efficiency in allocation and the endogenous agrarian structure: the proportion of different types of farm operators in the economy.

Let's see a simple simulation of this model after a brief note on the code we will use to solve for equilibria.

### A note on python implementation and object-oriented programming

Most of the code to run the model below is contained in a separate module `geqfarm.py` outside of this notebook that will be imported as any other library.

It's often said that in python 'everything is an object.' Objects have attributes and methods. 'Attributes' can be thought of as data or variables that describe the object and 'methods' can be thought of as functions that operate on object.

Take a very simple example. When in python you declare a string variable like so:

```
In [29]: mystring = 'economics'
```

python treats `mystring` as an instance of a string object. One then has access to a long list of attributes and methods associated with this object. In a jupyter notebook if you type the variable name `mystring` followed by a period and then hit the tab key you will see a list of available attributes and methods. Here are a few:

```
In [30]: # return the string capitalized
mystring.upper()
```

```
Out[30]: 'ECONOMICS'
```

```
In [31]: # count the number of occurrences of the letter 'o'
mystring.count('o')
```

```
Out[31]: 2
```

```
In [32]: # tell me if the string ends with the letter 'M'
mystring.endswith('M')
```

```
Out[32]: False
```

### Class statements to create new objects

Let's import the `geqfarm.py` library:

```
In [33]: import numpy as np
```

```
In [34]: from geqfarm import *
```

If you take a look at the code you will see how I have used class statements to create a new prototype 'Economy' object. An object of type Economy has attributes such as the number of households in the economy, parameters of the production function, and arrays that summarize the initial distribution of skill, land and labor across households. Once this class of object is defined one can make assignments such as the following:

```
In [35]: myeconomy = Economy(20)
```

This creates `myeconomy` as an instance of an Economy object. Several attributes are set to default values. We can easily find out what these are. For instance this is an economy with a production function with the  $\gamma$  parameter which measures the extent of homogeneity or diseconomies of scale. To find out what value it's set to we just type:

```
In [36]: myeconomy.GAMMA
```

```
Out[36]: 0.8
```

And we can easily change it to another value:

```
In [37]: myeconomy.GAMMA = 0.9
```

I've written a method to get a convenient summary of all important parameters:

```
In [38]: myeconomy.print_params()
```

```
ALPHA : 0.5 GAMMA : 0.9 H : 0.0 LAMBDA : 0.05
LBAR : 100 N : 20 TBAR : 100 s : [ 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

For example, the number of households is  $N = 20$ , total land endowment and labor force are both set to 100 and  $s$  is an array of skills across each of those 20 households. For the moment every household has the same skill level.

Let's work with a different simpler economy of just  $N = 5$  identical households. We'll call this new economy just E.

```
In [39]: N = 5
         E = Economy(N)
         E.ALPHA = 0.5
         E.GAMMA = 0.90
```

I've written a simple object method which calculates a competitive equilibrium allocation for the economy. I've set it up to take as required inputs a vector of economywide endowments and an array summarizing the skill distribution. It returns a 'named-tuple' with the vector of equilibrium factor prices, and the optimal land and labor allocations on each farm.

```
In [40]: eqnE = E.smallhold_eq([E.TBAR, E.LBAR], E.s)
```

The competitive equilibrium land rent and labor wage that clears this market are given by:

```
In [41]: eqnE.w
Out[41]: array([ 0.3335,  0.3335])
```

The optimal allocation of land and labor to each of the five farm skill groups is given by:

```
In [42]: eqnE.X
Out[42]: array([[ 20.,  20.,  20.,  20.,  20.],
               [ 20.,  20.,  20.,  20.,  20.]])
```

Since every one of the 5 farm household has the same skill and there are 100 units of land and labor the optimal allocation is for every farm to operate with 20 units of land and 20 units of labor.

Suppose however we now increase the skill of only the very first household, household zero.

```
In [43]: E.s[0] = 1.2
         E.s
Out[43]: array([ 1.2,  1. ,  1. ,  1. ,  1. ])
```

Let's recalculate the competitive equilibrium:

```
In [44]: eqnE = E.smallhold_eq([E.TBAR, E.LBAR], E.s)
```

Since there is more skill in this economy we would expect real wages and rents to rise, as indeed they do:

```
In [45]: eqnE.w
Out[45]: array([ 0.3581,  0.3581])
```

Since farm household zero now has a considerably higher level of skill compare to other households it's optimal scale of operations increases and, since it bids on the market for the same factors driving up the costs to other farms, the optimal scale of operations of the other farms shrinks.

```
In [46]: eqnE.X
Out[46]: array([[ 60.7525,   9.8119,   9.8119,   9.8119,   9.8119],
                [ 60.7525,   9.8119,   9.8119,   9.8119,   9.8119]])
```

## Equilibria with credit market distortions

We can easily modify to model above to get a simplified version of the model used in Eswaran and Kotwal’s 1996 Economic Journal paper “Access to Capital and Agrarian Production Organization.”

The model is as above except that now we add in a farm-specific working capital constraint, or the requirement that farms must be able to hire their inputs up-front prior to sale of their crop.

### 1.9.3 Latifundia Economics: factor market-power distortions

```
In [47]: %load_ext autoreload
        %autoreload 2
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

```
In [48]: import numpy as np
        from matplotlib import pyplot as plt
        plt.rcParams["figure.figsize"] = (10, 8)

        np.set_printoptions(precision=4)
        %matplotlib inline
```

What follows is a simple description of my (2007) ‘Latifundia Economics’ paper, which I used to try to explain the Latifundia-Minifundia complex of bimodal distribution of farm sizes that existed for long periods of time in many parts of historical Latin America and other parts of the world.

To understand the main arguments, consider the simplest case of a single landlord (or a cartel of landlords) surrounded by a fringe of small landowning or landless agricultural households. If the landlord owns a large fraction of the land endowment a standard partial-equilibrium analysis of non-price discriminating monopoly suggests the landlord would drive up the rental price of land by withholding land from the lease market. In a general equilibrium setting however there is another effect: by restricting other farmers’ access to land landlords also lower the marginal product of labor on those farms. This increases the supply of labor to landlord estates at any given wage increasing landlords’ potential income from monopsony rents. This can lead to equilibria where landlords increase the size of their production estates scale well above efficient scale in a competitive economy. A Latifundia-Minifundia type economy can emerge in which landlords operate large estates employing overly land-intensive production techniques while a large mass of farmers operate inefficiently and labor-intensive small parcels of land and sell labor to the landlord estate(s).

Let’s start with a simplified statement of the problem with just two types of households – landlords and ‘peasants’ and a very simple discrete distribution of farming skills. We’ll later extend this to a much more general setting.

The economy has  $\bar{T}$  units of cultivable land and  $\bar{L}$  households with one unit of labor each. The economy-wide land to labor ratio is therefore  $\bar{t} = \bar{T}/\bar{L}$ .

Household are indexed  $i = 1.. \bar{L}$  and each household has a non-traded farming skill level  $s_i$  drawn from a known distribution  $Z$ . There is also an initial distribution of property rights over land. We’ll make more flexible assumptions later but for now suffice to say that there is a group of “landlord” households (fraction  $\lambda$  of the total) who together own fraction  $\theta$  of the land endowment. In some scenarios below they will collude to coordinate their fa

As the  $\lambda\bar{L}$  landlord households own  $\theta\bar{T}$  units of land, peasant households own the remaining  $(1 - \theta)\bar{T}$  units. The average non-landlord household therefore owns  $\frac{(1-\theta)\bar{t}}{(1-\lambda)}$  units and, for the moment, all peasant households have the same initial land endowment. Under these assumptions it can be shown that the land Gini coefficient is exactly  $[\theta - \lambda]$ .

A single tradable good such as corn is produced and consumed in the economy at a unity price fixed by trade with the world market. Households maximize utility from consumption subject to household income from farm production plus net factor sales.

All peasant households have access to the same production technology represented by a standard concave production function  $\hat{F}(T, L, s)$  assumed for now to be linearly homogenous in its three arguments: land  $T$ , labor  $L$ , and a third factor which we label  $s$ . This last factor is assumed to be a non-traded factor that captures farming skill or labor supervision ability.

In the illustrative simulations below we assume a Cobb-Douglas form

$$\hat{F}(T, L, s) = s^{1-\gamma} \cdot [T^\alpha L^{1-\alpha}]^\gamma$$

## Latifundia Economics (factor market power distorted equilibria)

The following contrived example helps to starkly highlight the mechanisms behind the factor-market power distorted equilibrium.

TO start simple assume that there just  $N=5$  farmers each with a skill level normalized to 1.

```
In [49]: N = 5
```

Now create an economy and change a few parameters from their default. The  $\gamma$  parameter which measures the degree of homogeneity in production is purposefully set very high.. We are very close to assuming constant returns to scale (but setting it just below 1 is needed to make sure the size-distribution remains determinate).

```
In [50]: E = Economy(N)      # an instance takes N length as parameter
         s = np.ones(N)      # skill distribution in the economy
         E.ALPHA = 0.5
         E.GAMMA = 0.90
```

The Economy has default resource endowment:

```
In [51]: E.TBAR, E.LBAR
```

```
Out[51]: (100, 100)
```

So as expected the efficient (competitive) resource allocation has every farmer operating a farm of equal unit size. The `smallhold_eq` method gives the competitive (efficient) allocation for a subeconomy with  $XE = (TE, LE)$  endowments and skill distribution  $s$ . It returns a named tuple with equilibrium factor prices and  $X=[T, L]$  allocations.

If  $XE$  and  $s$  are those of the entire economy then we've found the economywide competitive allocation.

```
In [52]: Xc = E.smallhold_eq([E.TBAR, E.LBAR], s)
         Xc
```

```
Out[52]: result(w=array([ 0.3335,  0.3335]), X=array([[ 20.,  20.,  20.,  20.,  20.],
              [ 20.,  20.,  20.,  20.,  20.])))
```

```
In [53]: (Xrc, Xr, wc, wr) = scene_print(E, 10, detail=True)
```

Running 10 scenarios...

Assumed Parameters

=====

```
ALPHA  : 0.5 GAMMA  : 0.9 H      : 0.0 LAMBDA : 0.2
LBAR    : 100 N     : 5 TBAR   : 100 s      : [ 1.  1.  1.  1.  1.]
```

```
Efficient:[ Trc, Lrc]      [rc,wc]      w/r      F( )      [r*Tr]  [w*Lr]
```

```
=====
          [ 20.00, 20.00]  [0.33,0.33]  1.00  | 14.82   6.67   6.67
```

```
Theta  [ Tr, Lr ]      [rM,wM]      w/r  | F( )  [T_hire]  [T_sale]  [L_hire]
```

```
=====
0.00 [ 17.56, 20.69] [ 0.33, 0.34] 1.04 | 14.19      5.74      0.00      7.03
0.10 [ 18.57, 20.17] [ 0.33, 0.34] 1.02 | 14.39      6.13      3.30      6.79
0.20 [ 20.00, 20.00] [ 0.33, 0.33] 1.00 | 14.82      6.67      6.67      6.67
0.30 [ 21.95, 20.19] [ 0.34, 0.33] 0.98 | 15.52      7.41     10.13      6.67
0.40 [ 24.58, 20.78] [ 0.34, 0.33] 0.95 | 16.55      8.43     13.72      6.79
0.50 [ 28.15, 21.88] [ 0.35, 0.32] 0.92 | 18.00      9.85     17.50      7.04
0.60 [ 33.08, 23.67] [ 0.36, 0.32] 0.88 | 20.05     11.92     21.61      7.47
0.70 [ 40.17, 26.55] [ 0.38, 0.31] 0.81 | 23.05     15.13     26.36      8.14
0.80 [ 51.31, 31.70] [ 0.41, 0.29] 0.71 | 27.87     20.94     32.65      9.22
0.90 [ 74.78, 46.29] [ 0.53, 0.25] 0.47 | 39.14     39.33     47.34     11.43
0.99 [ 99.90, 94.35] [  4, 0.07] 0.02 | 61.44    401.03    397.42      6.65
=====
```

Thus far we've said nothing of the ownership of land or labor. Let's assume every household has one unit of labor but that the 'landlord' class (which WLOG we index to be the last skill group  $s[-1]$ ) owns fraction  $\theta$  of the land. Assuming a uniform distribution of households across skills every skill group has  $L_{\text{bar}}/N$  households, and so there are that many landlords who act as a single cartel.

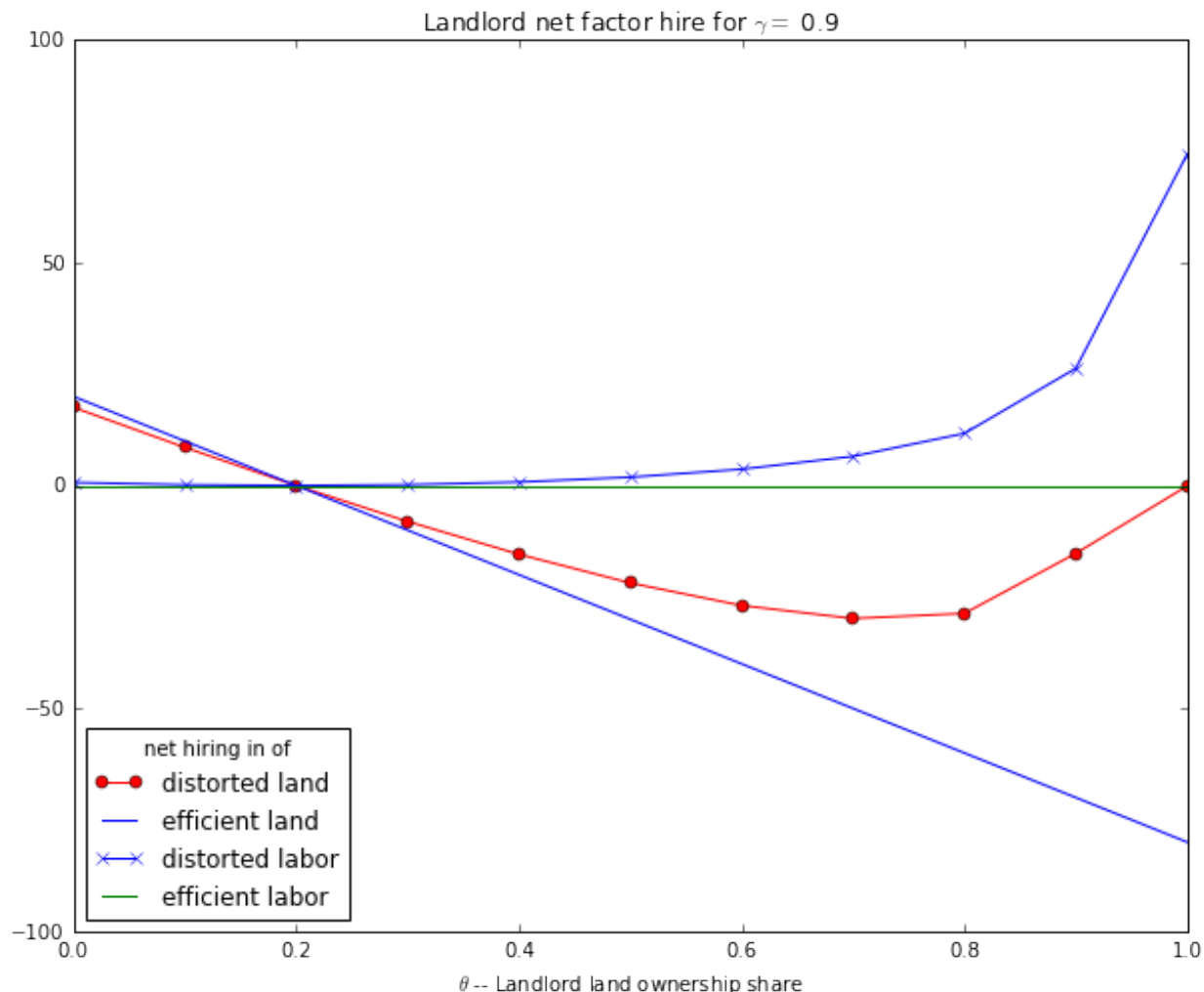
The following code is useful for printing and plotting out equilibria as a function of initial landlords' land ownership share.

We pass our economy instance to the scenarios function and it solves for competitive and market-power equilibria (at different levels of  $\theta$ ) and prints out the results.

Let's plot the results.. This is the classic diagram from Latifundia Economics..

```
In [54]: factor_plot(E,Xrc,Xr)
```





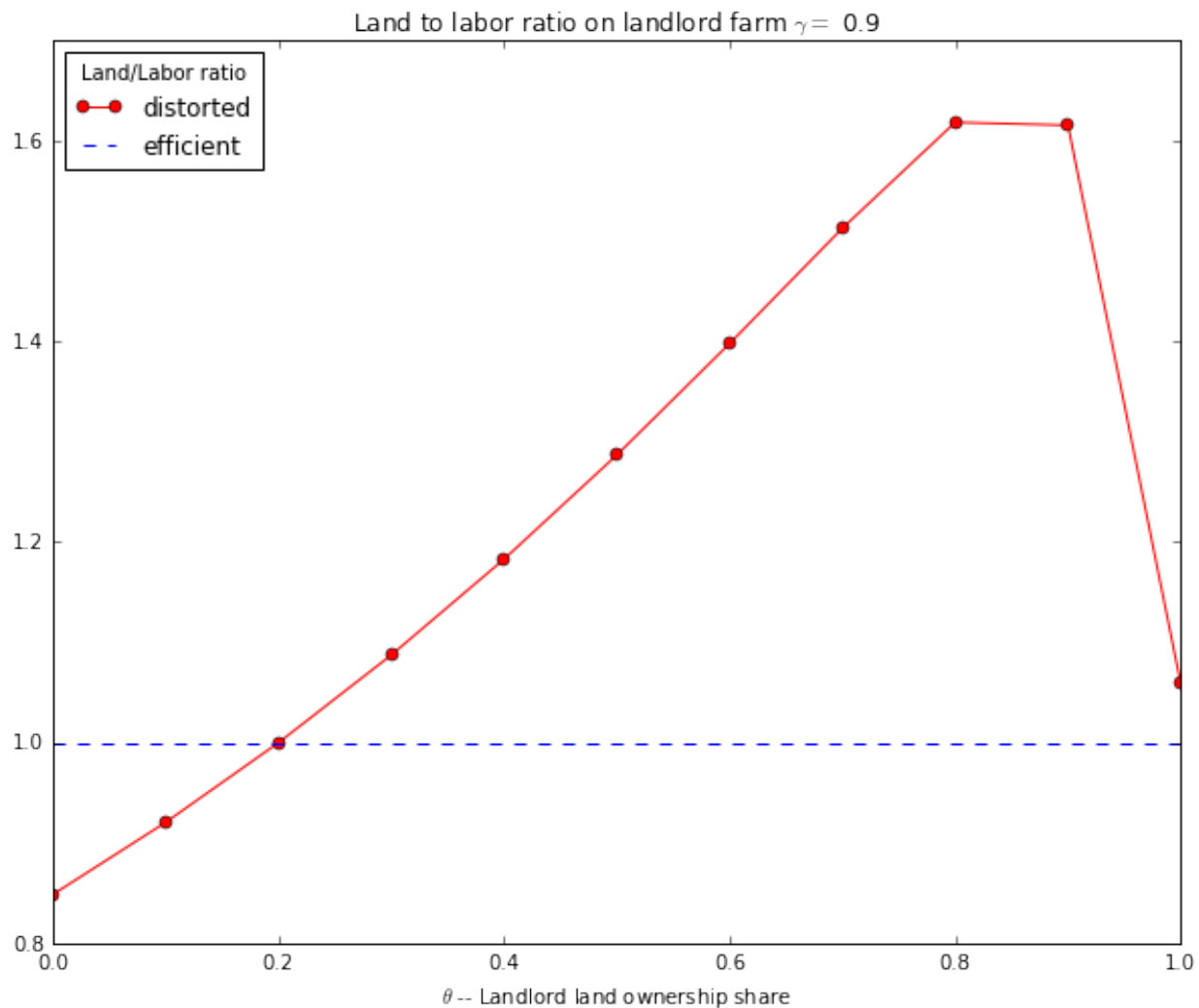
In the example above the ‘landlord’ farmer was in every way the same as the other farmers, the only difference being he had more land ownership (fraction  $\theta$  of the total). He had the same skill parameter as every other farmer. In an efficient equilibrium his operational farm size should therefore be the same size as every other farmer. The plot above shows how monopoly power (which rises with  $\theta$  allows the monopolist to distort the economy – he withholds land from the lease market to drive up the land rental rate and, since this deprives the ‘fringe’ of farmers of land, lowers the marginal product of labor on each smallholder farm, increasing the smallholder labor supply to the market which pushes down the labor wage. Hence we see how at higher levels of  $\theta$  the landlord expands the size of his estate and establish monopsony power wages.

A key force keeping the landlord from becoming too large is the fact that there are diseconomies of scale in production. The landlord is expanding the scale of his operation (raising the land to labor ratio on his farm in this example) earn more via distorted factor prices, but he balances off the increase in extraction from distorted wages against the cost of operating an inefficiently large farm (i.e. the cost of being too big).

Now let’s see the effect of making the landlord just a little bit more ‘skilled’ than the others. This lowers the cost of being big. But note that it also makes him bigger at lower  $\theta$  and makes what I call the ‘size monopsony or ‘Feenstra’ effect matter more..

So let’s raise the landlord farmer’s productivity 10% relative to the rest of the farmers.

```
In [55]: TLratio_plot(E,Xrc,Xr)
```



```
In [56]: E.s[-1]=1.10
```

Let's recalculate the new equilibria under the different scenarios.

```
In [57]: (Xrc,Xr,wc,wr) = scene_print(E,10,detail=True)
```

Running 10 scenarios...

Assumed Parameters

=====

```
ALPHA : 0.5 GAMMA : 0.9 H : 0.0 LAMBDA : 0.2
LBAR : 100 N : 5 TBAR : 100 s : [ 1. 1. 1. 1. 1.1]
```

```
Efficient:[ Trc, Lrc]      [rc,wc]      w/r      F( )      [r*Tr] [w*Lr]
=====
          [ 39.34, 39.34] [ 0.34,0.34]  1.00 | 29.97  13.49  13.49
```

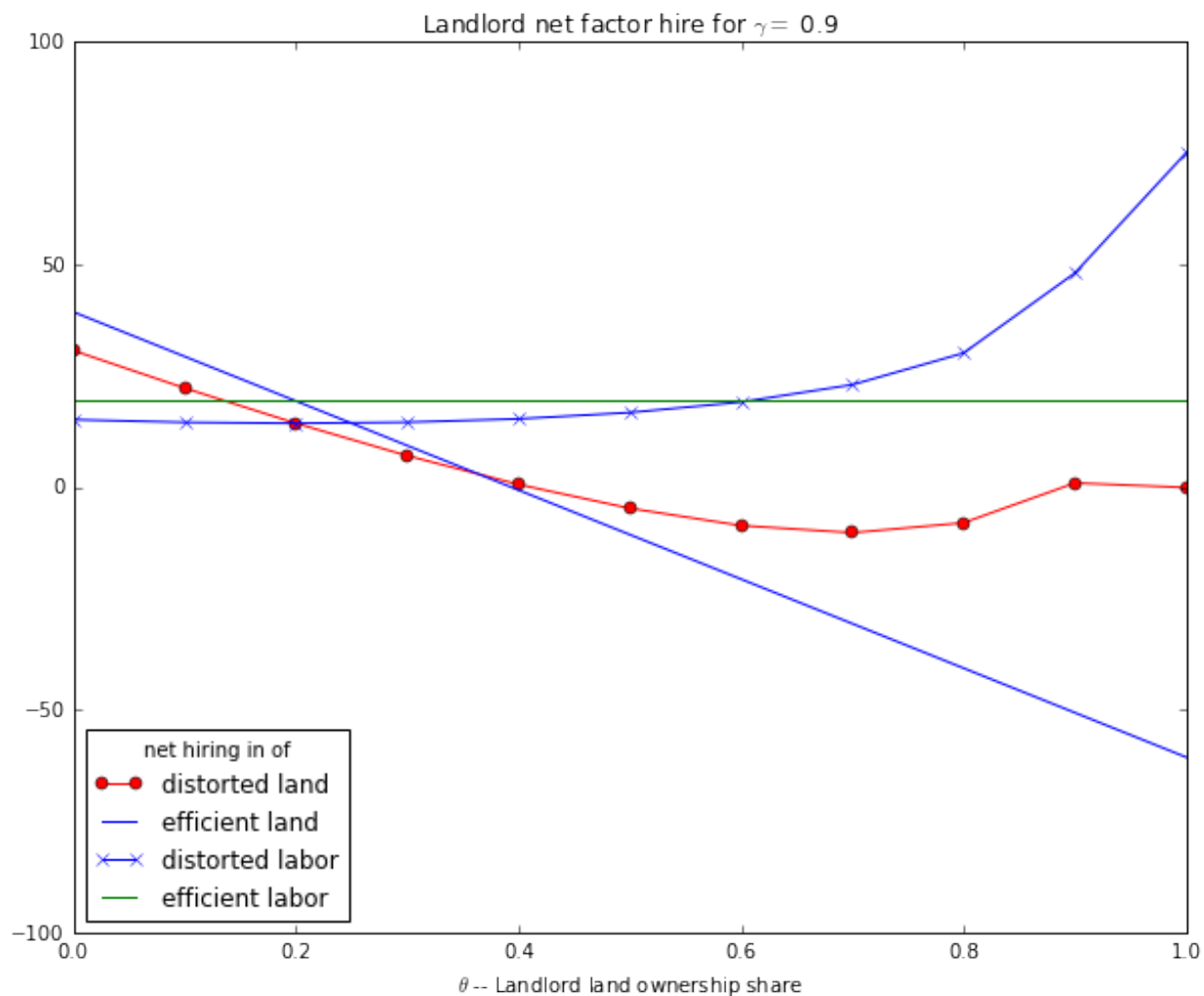
```
Theta [ Tr, Lr ]      [rM,wM]      w/r | F( )      [T_hire] [T_sale] [L_hire]
=====
0.00 [ 30.67, 35.14] [ 0.33, 0.35]  1.07 | 25.47  10.07  0.00  12.33
0.10 [ 32.24, 34.54] [ 0.33, 0.35]  1.04 | 25.84  10.76  3.34  11.94
0.20 [ 34.32, 34.32] [ 0.34, 0.34]  1.00 | 26.51  11.68  6.80  11.68
0.30 [ 37.05, 34.56] [ 0.35, 0.33]  0.96 | 27.52  12.88  10.43  11.55
```

```

0.40 [ 40.60, 35.31] [ 0.36, 0.33] 0.92 | 28.96 14.50 14.28 11.58
0.50 [ 45.23, 36.74] [ 0.37, 0.32] 0.87 | 30.95 16.72 18.48 11.76
0.60 [ 51.36, 39.10] [ 0.39, 0.31] 0.80 | 33.70 19.92 23.27 12.11
0.70 [ 59.75, 42.98] [ 0.42, 0.29] 0.71 | 37.65 24.97 29.25 12.68
0.80 [ 71.98, 50.11] [ 0.48, 0.27] 0.56 | 43.86 34.56 38.42 13.51
0.90 [ 90.93, 68.06] [ 0.73, 0.21] 0.28 | 55.93 66.42 65.74 14.12
0.99 [ 99.92, 95.08] [ 4.1, 0.07] 0.02 | 67.82 411.11 407.35 6.74
=====

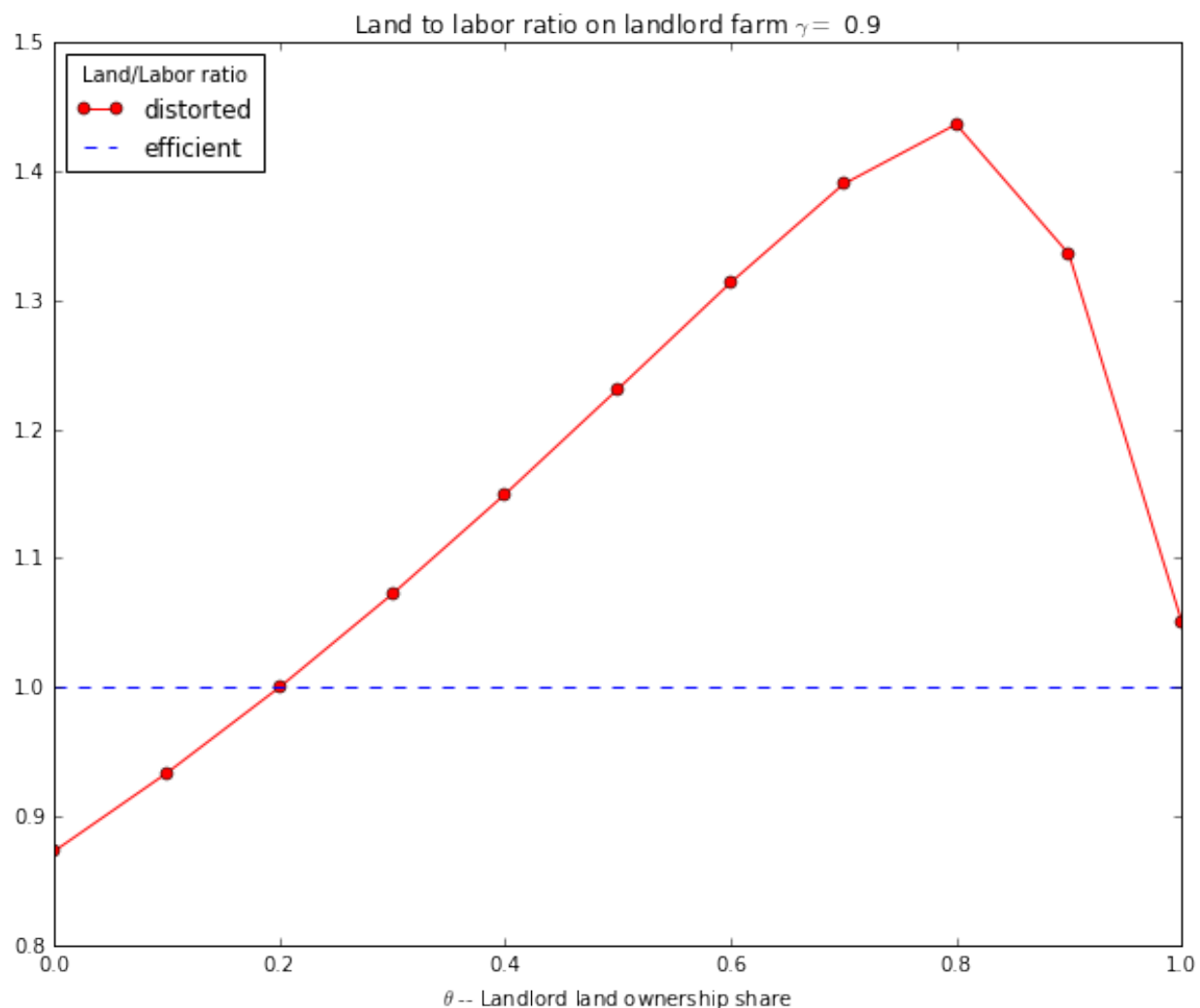
```

```
In [58]: factor_plot(E,Xrc,Xr)
```



Given that he is more skilled than before the landlord's efficient scale of production has increased. This lowers the cost of being big. Interestingly at low  $\theta$  this leads the landlord to hire less land and labor ...

```
In [59]: TLratio_plot(E,Xrc,Xr)
```



## 1.10 Causes and consequences of insecure Property Rights to land

Insecure property rights to land can affect investment decisions and the nature and efficiency of agrarian production organization by affecting the market for tenancies and land sales.

The relationship between property rights insecurity and investment is not as obvious as it might appear at first. While it might seem obvious that property rights insecurity (e.g. the possibility of expropriation) could only reduce incentives to invest and trade this is less obvious once you consider that informal investments might be undertaken to strengthen property rights claims in environments where they might be contested. Think of the homesteader or the informal slum dweller who builds a solid foundation for their home, fences what they consider their property, and builds other 'defences' in the hope that this will exclude others and dissuade a would-be-evicter.

There's a rich literature on this topic, including the readings by Besley (1995), de Soto's *Mystery of Capital*, and *The Other Path* books, the survey by Conning and Deb (2008) and many many others. In time I'll try to find time to fill out this introduction, in the meantime the rest of the notebook focuses on a short synthesis of Conning and Robinson (2007).

### 1.10.1 Conning & Robinson (2007): Property Rights and the Political Organization of Agriculture

This is a simplified version of Conning and Robinson's 2007 paper in the Journal of Development Economics, a "general equilibrium model where the organization of agriculture and the political equilibrium determining the security of property rights are jointly determined.

In its present form this notebook focuses primarily on the economic side of the model which demonstrates how insecure property rights to land (e.g. the risk of expropriation) might affect the equilibrium level of land leasing and equilibrium factor prices. The dull details of the probabilistic voting model that determines the equilibrium political risk of expropriation are left for another time.

The model is another in the family of general equilibrium models of the size distribution of farms that we've already analyzed. In all cases we have an economy with an agricultural technology that is linear homogenous in farming skill  $S$ , land  $T$  and labor  $L$ . We showed that even though there was no market for farm management skills (so  $S$  is a non-traded asset) efficient resource allocations could be achieved if we had competitive land and labor markets that allowed households to achieve their efficient scale. This model is similar to Lucas' 1978 size distribution of business firms and would explain the size distribution of farms as determined by an underlying distribution of farming skills across households. In this model efficiency of allocation is independent of the initial distribution of factors  $S, T$ , and  $L$  in the population.

If, in addition to the imperfection in the market for non-traded farming skills, we add a distortion to either the market for land or labor, efficiency can no longer be achieved.

Eswaran and Kotwal 1996 paper on "Access to Capital and Production Organization" can be viewed as a variant on this type of benchmark model. They introduce two simultaneous distortions. They require that supervisory labor be used at increasing cost to supervise any hired labor. This is loosely analogous to our shutting down the market for farming skill (or supervision) ability. The second constraint is a distortion in the market for working capital which determines that households with low initial wealth (principally ownership of land) will be constrained in the market for working capital (in our benchmark model we implicitly assumed working capital could always be financed). The interaction of these two types of distortion determines that allocations cannot be efficient and also the shape of the equilibrium agrarian 'class structure' or occupational choice hierarchy. Even though in effect all farms have the same farming skill, households with less ownership of land are more likely to be constrained in access to working capital and hence have to drop out of production entirely (to become 'pure laborers') or may be a constrained 'labor-cultivator' who runs a small farm and then sells labor on the market. In addition households may be constrained or unconstrained 'capitalist' farms that hire other workers.

Conning and Robinson's paper sets itself up against a similar benchmark model, but in a two period world. In a world with complete security of property rights land and labor markets will allocate land efficiently across farms in agriculture and labor efficiently across farms in agriculture and manufacturing activities in urban areas. The size distribution of farms in agriculture will be efficient and completely determined by the initial distribution of non-traded farming skills. Land tenancy markets will be efficient and active. Take the case of all households having the same farming skill. If landlords have relatively large amounts of land they will lease it out to tenants to bring the distribution of operational farm sizes in line with the distribution of farming skills.

Starting with this benchmark the model asks what would happen if we introduce property rights insecurity in the form of the possibility of a 'land to the tiller' land or tenancy reform.

#### The Model with secure property rights

To simplify greatly assume there are just two population groups: landlords and peasant households. They have access to the following production technologies:

*Peasants:*  $F(T, L)$  *Landlords:*  $G(T, L) = A_l F(T, L)$

When  $A_l = 1$  landlord and peasant production technologies are identical, when  $A_l > 1$  landlords are more skilled farmers and we associate that with higher productivity of labor and capital. For the moment assume  $A_l = 1$ .

Landlord and peasant households are each endowed with 1 unit of labor. There are  $\bar{L}$  households overall of which proportion  $n_p$  are peasant households and proportion  $n_l$  are landlord households. For example if  $\bar{L} = 100$  and  $n_p = 0.90$  there would be 90 peasant households and 10 landlord households.

We will assume there is an urban sector where production takes place using production function  $H(L)$  which is subject to decreasing returns. In equilibrium  $L_u$  units of labor will leave agriculture to work in the city.

The only difference between landlord and peasant households then is their ownership of land. Landlords as a group own fraction  $\theta$  of the overall land endowment. That means that each landlord household owns

$$\frac{\theta}{n_l} \bar{t}$$

units of land where  $\bar{t} = \frac{\bar{T}}{\bar{L}}$

Since by definition landlords own more land then peasants  $\theta > n_l$

In a world of secure property rights the first period allocations will be identical to the second period allocations, and in any given period efficient allocation will be characterized by:

$$w = F_L(T_p, L_p) = G_L(T_l, L_l) = H(L_u)$$

$$r = F_T(T_p, L_p) = G_T(T_l, L_l)$$

In the special case where landlord and peasant farming skill is identical efficient production allocations will be identical on all  $\bar{L}$  farms and equal to

$$T^e = \frac{\bar{T}}{\bar{L}}$$

$$L^e = \frac{\bar{L} - L_u^e}{\bar{L}}$$

where  $L_u^e$  is pinned down by  $F_L(T^e, L^e) = w = H_L(L_u^e)$ .

It's easy to show the efficient share of cultivated land under tenancy should be

$$\tau_e = \theta - n_l$$

Intuitively, the higher the concentration of land  $\theta$  the more land needs to be under tenancy to equalize farm sizes.

## The Model with insecure property rights

We now extend the setting to allow the initial production period to be followed by a political contest that will decide the likelihood of future property rights reforms that may affect ownership claims in a final production period.

Households will choose factors of production as before except that there is now the threat that a tenancy reform may occur with (possibly zero) probability  $\alpha$ .

If a reform takes place all sitting tenants obtain protection from eviction on fraction  $(1-\kappa)$  of the land they leased in the pre-reform period and they will only have to pay a new capped rental rate  $\bar{v}$  for those leases in the following period, where  $\bar{v}$  will generally be set at or below the post-reform market equilibrium rental rate  $v^e$

One interpretation of  $\kappa$  is that the reform beneficiary might have to pay monetary expenses  $(v^e - \bar{v})$  per unit land transferred to cover such things as property registration paperwork, new taxes, excess loan financing costs, or other transaction or setup costs. The value of  $\kappa$  will be a key parameter establishing the size of the gap between the value of property rights lost by the landlord and the value of benefits transferred to the tenant or squatter.

In a post-reform competitive equilibrium this transaction cost will be reflected in land rent and sale prices and how its burden is shared between landlord and tenant it would not matter if we had instead imposed these transaction costs directly on the landlord.

To simplify matters it is assumed that no further alteration of property rights can take place after the reform. This implies that in the post-reform production period efficient resource allocation  $(T_e, L_e, L_{ue})$  will be achieved at market factor prices  $v^e = F_T(T_e, L_e)$  and  $w^e = F_L(T_e, L_e)$ , except of course that reform beneficiaries now earn additional incomes generated by the transfer of property rights.

We can model agrarian reforms of different type and depth by varying the three parameters  $v$ ,  $\alpha$  and  $\kappa$ . For example, one may think of  $v = 0$  as expropriation without compensation to the landlord while  $0 < v < v^e$  could be thought of as a tenancy reform with a rent ceiling, or as a land reform with partial compensation. From a landlord's perspective a property rights reform means facing the prospect of being forced to cede the  $(t/n^l - T_l)$  units of land they put under lease in the pre-reform phase at below-market rental rate  $v$  in the post-reform period.

Anticipating this the landlord will now choose  $L_l$  and  $T_l$  (and hence also how much land to lease out) in a preemptive manner, considering the risk that tenants may become squatters or agrarian reform beneficiaries.

(Each individual landlord and peasant is assumed too small to internalize how their own production decisions might affect the subsequent political equilibrium and takes  $\alpha$  as given.)

Taking their conjecture of  $\alpha$  as given, each landlord now chooses pre-reform factor inputs  $T_l$  and  $L_l$  to maximize the expected discounted value of farm profits plus factor sales taking into account that property rights over land leased out will be challenged in the event of reform:

$$\begin{aligned} V^l = & [^l(T_l, L_l) + w + vt/n^l] \\ & + \alpha [^l(T_e, L_e) + w + v^e t/n^l - (v^e - v)[t/n^l - T_l]] \\ & + (1-\alpha) [^l(T_e, L_e) + w^e + v^e t/n^l] \end{aligned}$$

Stated more compactly:

$$\begin{aligned} V^l = & [^l(T_l, L_l) + w + vt/n^l] + [^l(T_e, L_e) + w + v^e t/n^l] \\ & - \alpha(v^e - v)(t/n^l - T_l) \end{aligned}$$

The first two bracketed terms measure expected returns over the two periods under secure property rights (albeit with distortions in the first period) while the last term captures the expected loss of income from having to possibly cede property rights to a squatter or reform beneficiary in the second period.

The peasant household will similarly choose  $T_p$  and  $L_p$  to maximize earnings from farm profits plus factor sales taking into account the prospect (summarized by  $\alpha$ ) that they may acquire squatter rights over any land that they leased in the first period. We assume that in a post-reform period a tenant has to pay only the regulated rate  $v$  but can lease that land back out onto the market, to earn a windfall rent of  $(v^e - v)$  per unit land. The tenant's discounted expected payoff is therefore:

$$\begin{aligned} V^p = & [^p(T_p, L_p) + w + v(1-\alpha)t/n^p] + [^p(T_e, L_e) + w^e + v^e(1-\alpha)t/n^p] \\ & + (1-\alpha)(v^e - v)(T_p - (1-\alpha)t/n^p) \end{aligned}$$

The second line captures the expected windfall gain the peasant/tenant from a transfer of rights toward the tenant.

The first-order conditions for a competitive equilibrium with respect to first period land input choices are now given by:

$$G_T(T_l, L_l) = F_T(T_p, L_p) - (v^e - v)$$

$$G_L(T_l, L_l) = F_L(T_p, L_p) = H_L(L_u)$$

**REMARK** Even with a positive threat of property rights reform  $\alpha > 0$ , equilibrium allocations will remain efficient as long as  $\kappa = 0$

At first blush this might seem slightly surprising. When there are no transaction costs involved in transferring land under a reform market allocations will be efficient even though there may be a positive probability (or even a certainty) of a property rights reforms.

Yes a reform will compel a landlord to surrender property rights to tenant-cum-squatters in the following period but this possibility is fully anticipated and causes no distortion because we have in effect left open a market for “squatter rights”. Tenants understand that they may capture a windfall rent from the reform and in effect pay the going market rate for the right to be a sitting tenant/squatter.

Now this might sound somewhat incredible but in fact if you read *The Other Path: the invisible revolution in the third world* the book that made Hernando de Soto world famous and that describes in great detail the organization of land squatting invasions in Lima Peru you will find de Soto dedicating several pages to describing precisely such arrangements. He argues in effect that many land-invasions take place with the explicit cooperation of owners who often in fact demand payments in advance for allowing the squatters into their properties ahead of staged invasions. This happens in particular in situations where land is not zoned for residential occupancy.

But even more striking evidence of this happening comes from work of Jeon and Kim (2000) who analyze what happened in the land market in Korea following when the Japanese were forced to suddenly and completely abandon the Korean peninsula and Korea came under the US military administration in August 1945. Tenancy under the Japanese colonial administration 1919-45 had been a widespread phenomenon as by one measure over 56 percent of farmer households were tenants and 58 percent of farmland was under tenancy in 1939. Although tenant protests demanding lower rents were not uncommon, the Japanese colonial military presence had strictly enforced landlord’s property rights. With the Japanese military suddenly gone the eventuality of land reform became a near certainty. The interesting thing is that the authors document that in the period just prior to this anticipated reform 60 percent of landlords – mostly the larger ones – sold their land to tenants via the market at reduced prices before 1950. More than twice as much land was sold by landlords in anticipation of the reform than was eventually transferred directly via the land reform process.

Most land reforms are however messy and somewhat uncertain affairs. A more realistic assumption is that the transfer of property rights will not be costless and without leakage, and therefore that  $\kappa > 0$ . When this is the case:

**Proposition:** When  $\kappa > 0$ , and as long as non-traded input  $S$  remains an essential input, then the expectation of reform ( $\alpha > 0$ ) leads landlords to defensively *suppress* tenancy, or  $T_l < T^e$ . Landlord farms will become larger and more land-intensive, and peasant farms smaller and more labor-intensive than the first best efficient scale. Peasant off-farm labor supply and rural to urban migration will increase.

Anything that increases the size of the wedge between the effective rental rate the tenant pays and the landlord can expect to recover  $\alpha\kappa(v^e - \bar{v})$  will strengthen tenancy suppression. So a higher probability of reform  $\alpha$  a higher leakage rate  $\kappa$  or a lower cap  $\bar{v}$  on the rental rate in the post-reform period all contribute to a landlord trying to protect a larger fraction of his land from tenancy-would-be-reform-beneficiaries in the first period. It can be shown that in equilibrium this withholding of land makes landlord farms bigger, peasant farms smaller and leads peasant off-farm labor supply to increase which would drive down equilibrium wages.

## 1.11 Consumer Choice and Intertemporal Choice

We setup and solve a very simple generic one-period consumer choice problem over two goods. We later specialize to the case of intertemporal trade over two periods and choice over lotteries. The consumer is assumed to have time-consistent preferences. Later, in a separate notebook, we look at a three period model where consumers have time-inconsistent (quasi-hyperbolic) preferences.

The code to generate the static and interactive figures is at the end of this notebook (and should be run first to make this interactive).



### 1.11.1 Choice over two goods

A consumer chooses a consumption bundle to maximize Cobb-Douglas utility

$$U(c_1, c_2) = c_1^\alpha c_2^\beta$$

subject to the budget constraint

$$p_1 c_1 + p_2 c_2 \leq I$$

To plot the budget constraint we rearrange to get:

$$c_2 = \frac{I}{p_2} - \frac{p_1}{p_2} c_1$$

Likewise, to draw the indifference curve defined by  $u(c_1, c_2) = \bar{u}$  we solve for  $c_2$  to get:

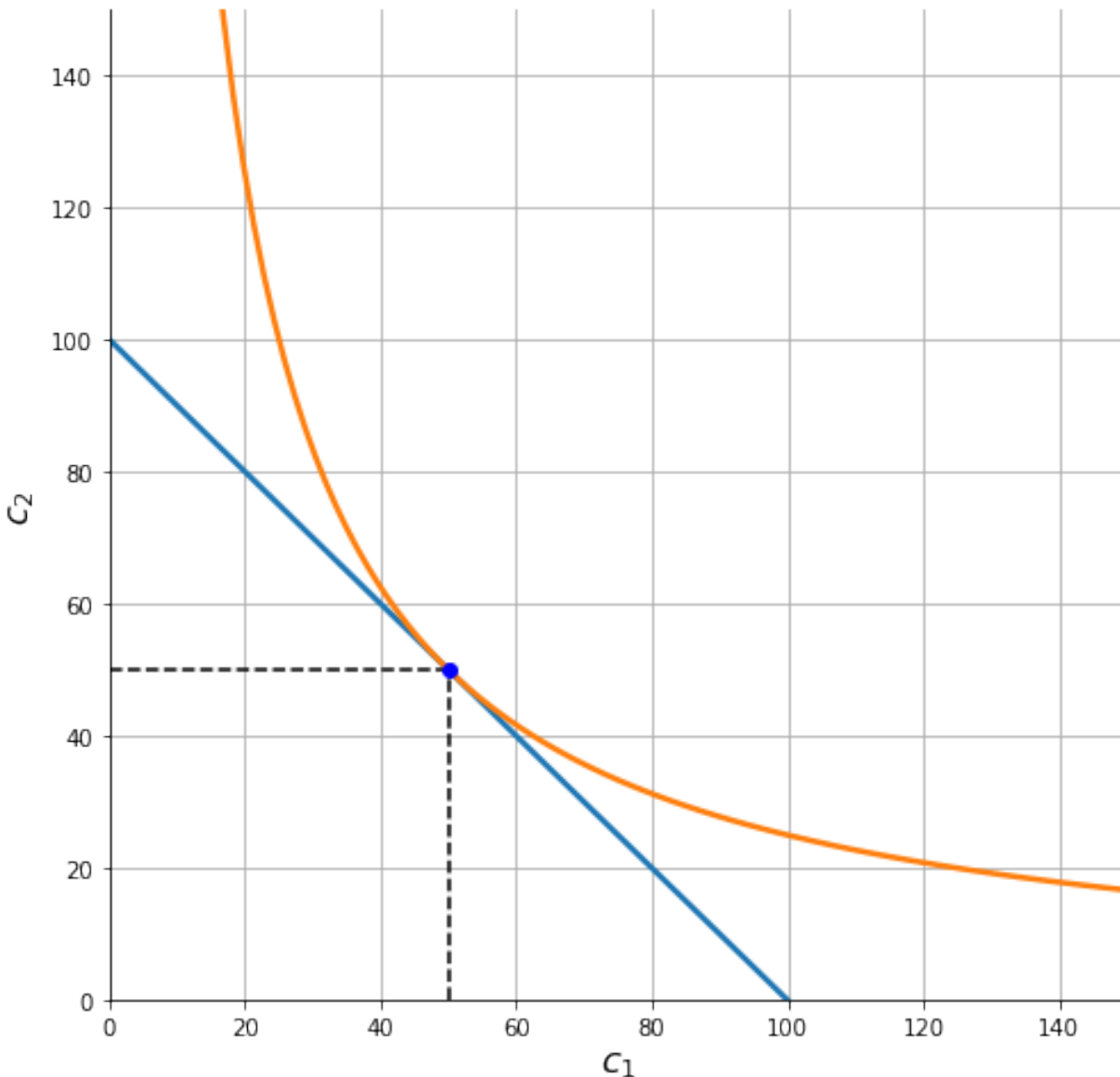
$$c_2 = \left( \frac{\bar{u}}{c_1^\alpha} \right)^{\frac{1}{\beta}}$$

For Cobb-Douglas utility the marginal rate of substitution (MRS) between  $c_2$  and  $c_1$  is:

$$MRS = \frac{U_1}{U_2} = \frac{\alpha}{\beta} \frac{c_2}{c_1}$$

where  $U_1 = \frac{\partial U}{\partial c_1}$  and  $U_2 = \frac{\partial U}{\partial c_2}$

```
In [17]: consume_plot()
```



### 1.11.2 The consumer's optimum

$$L(c_1, c_2) = U(c_1, c_2) + \lambda(I - p_1c_1 - p_2c_2)$$

Differentiate with respect to  $c_1$  and  $c_2$  and  $\lambda$  to get:

$$U_1 = \lambda p_1$$

$$U_2 = \lambda p_2$$

$$I = p_1c_1 + p_2c_2$$

Dividing the first equation by the second we get the familiar necessary tangency condition for an interior optimum:

$$MRS = \frac{U_1}{U_2} = \frac{p_1}{p_2}$$

Using our earlier expression for the MRS of a Cobb-Douglas indifference curve, substituting this into the budget constraint and rearranging then allows us to solve for the **Marshallian demand functions**:

$$c_1(p_1, p_2, I) = \frac{\alpha}{\alpha + \beta} \frac{I}{p_1}$$

$$c_2(p_1, p_2, I) = \frac{\beta}{\alpha + \beta} \frac{I}{p_2}$$

Interactive plot with sliders (visible if running on a notebook server):

```
In [20]: interact(consume_plot, p1=(pmin, pmax, 0.1), p2=(pmin, pmax, 0.1), I=(Imin, Imax, 10), alpha=(0.05, 0.95),
interactive(children=(FloatSlider(value=1.0, description='p1', max=4.0, min=1.0), FloatSlider(value=1.0, description='p2', max=4.0, min=1.0),
```

### 1.11.3 The expenditure function

The indirect utility function:

$$v(p_1, p_2, I) = u(c_1(p_1, p_2, I), c_2(p_1, p_2, I))$$

$$c_1(p_1, p_2, I) = \alpha \frac{I}{p_1}$$

$$c_2(p_1, p_2, I) = (1 - \alpha) \frac{I}{p_2}$$

$$v(p_1, p_2, I) = I \cdot \alpha^\alpha (1 - \alpha)^{1-\alpha} \cdot \left[ \frac{p_2}{p_1} \right]^\alpha \frac{1}{p_2}$$

$$E(p_1, p_2, \bar{u}) = \frac{\bar{u}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \cdot \left[ \frac{p_2}{p_1} \right]^\alpha \frac{1}{p_2}}$$

To minimize expenditure needed to achieve level of utility  $\bar{u}$  we solve:

$$\min_{c_1, c_2} p_1 c_1 + p_2 c_2$$

s.t.

$$u(c_1, c_2) = \bar{u}$$

The first order conditions are identical to those we got for the utility maximization problem, and hence we have the same tangency

$$\frac{U_1}{U_2} = \frac{\alpha}{1 - \alpha} \frac{c_2}{c_1} = \frac{p_1}{p_2}$$

From this we can solve:

$$c_2 = \frac{p_1}{p_2} \frac{1 - \alpha}{\alpha} c_1$$

Now substitute this into the constraint to get:

$$u\left(c_1, \frac{p_1}{p_2} \frac{1 - \alpha}{\alpha} c_1\right) = \bar{u}$$

$$u\left(c_1, \frac{p_1}{p_2} \frac{1 - \alpha}{\alpha} c_1\right) = \bar{u}$$

### 1.11.4 Intertemporal Consumption choices

We now look at the special case of intertemporal consumption, or consumption of the same good (say corn) over two periods. As modeled below the consumer's income is now given by the market value of an endowment bundle  $(y_1, y_2)$ .

The variables  $c_1$  and  $c_2$  now refer to consumption of (say corn) in period 1 and period 2.

As is typical of intertemporal maximization problems we will use a time-additive utility function. The consumer who has access to a competitive financial market (they can borrow or save at interest rate  $r$ ) maximizes:

$$U(c_1, c_2) = u(c_1) + \delta u(c_2)$$

subject to the intertemporal budget constraint:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

This is just like an ordinary utility maximization problem with prices  $p_1 = 1$  and  $p_2 = \frac{1}{1+r}$ . Think of it this way, the price of corn is \$1 per unit in each period, but \$1 in period 1 can be placed into savings that will grow to  $(1+r)$  dollars in period 2. That means that from the standpoint of period 1 owning one unit of corn (or one dollar worth of corn) in period 2 is the equivalent of owning  $\frac{1}{1+r}$  units of corn today (because placed in savings that amount of period 1 corn would grow to  $\frac{1+r}{1+r} = 1$  units of corn in period 2).

The first order necessary condition for an interior optimum is:

$$u'(c_1^*) = \delta u'(c_2^*)$$

Let's adopt the widely used Constant-Relative Risk Aversion (CRRA) felicity function of the form:

$$u(c_t) = \begin{cases} \frac{c_t^{1-\rho}}{1-\rho}, & \text{if } \rho > 0 \text{ and } \rho \neq 1 \\ \ln(c) & \text{if } \rho = 1 \end{cases} \quad (1.5)$$

if  $\rho = 1$

The first order condition then becomes simply

$$c_1^{*- \rho} = \delta(1+r)c_2^{*- \rho}$$

or

$$c_2^* = [\delta(1+r)]^{\frac{1}{\rho}} c_1^*$$

From the binding budget constraint we also have

$$c_2^* = E[y](1+r) - c_1^*(1+r)$$

where  $E[y] = y_1 + \frac{y_2}{1+r}$

Solving for  $c_1^*$  (from the FOC and this binding budget):

$$c_1^* = \frac{E[y]}{1 + \frac{[\delta(1+r)]^{\frac{1}{\rho}}}{1+r}}$$

If we simplify to the simple case where  $\delta = \frac{1}{1+r}$ , where the consumer discounts future consumption at the same rate as the market interest rate then at an optimum the consumer will keep their consumption flat at  $c_2^* = c_1^*$ . If we specialize further and assume that  $r = 0$  then the consumer will set  $c_2^* = c_1^* = \frac{y_1 + y_2}{2}$

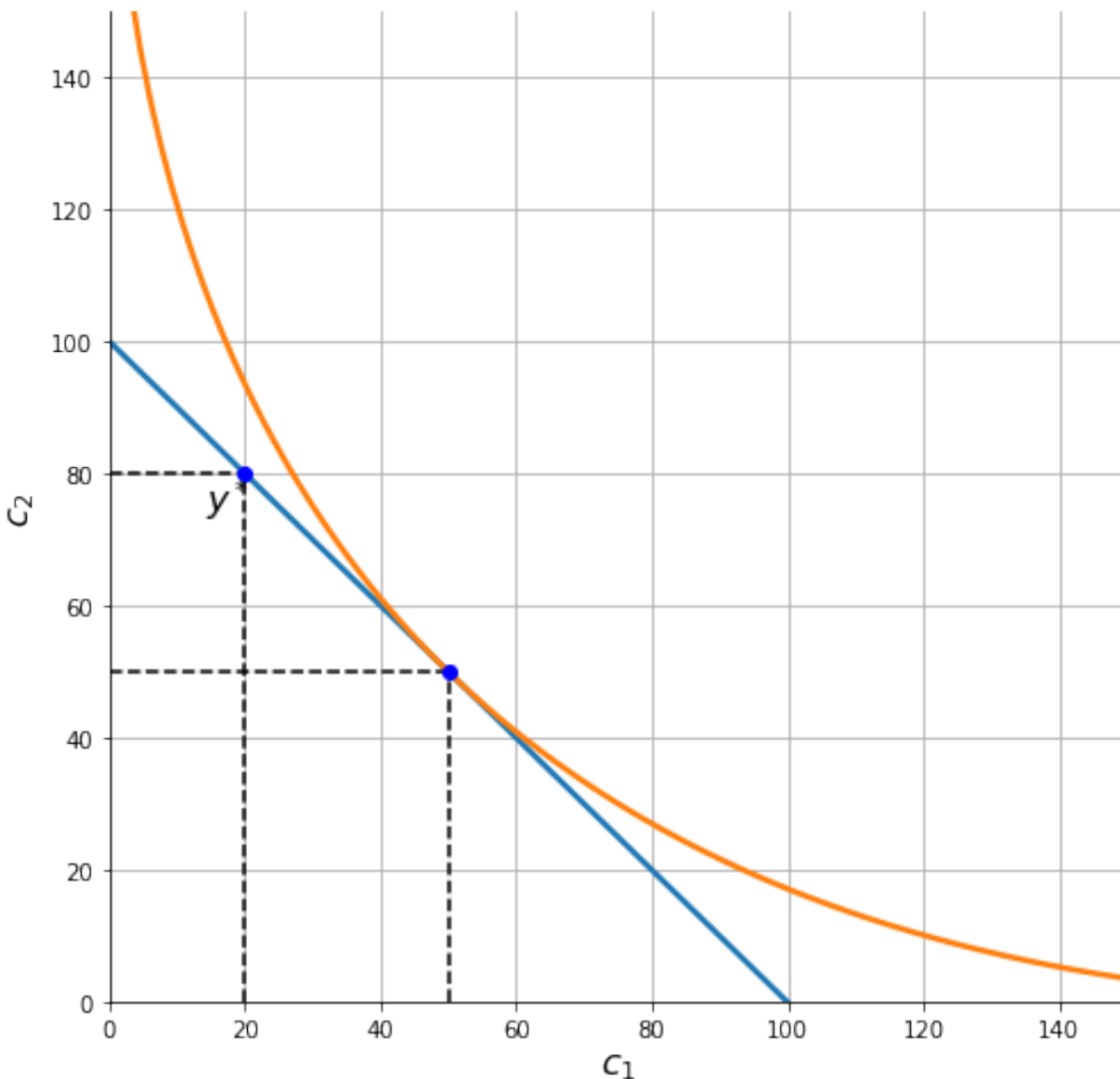
### 1.11.5 Saving and borrowing visualized

Let us visualize the situation. The consumer has CRRA preferences as described above (summarized by parameters  $\delta$  and  $\rho$  and starts with an income endowment  $(y_1, y_2)$ . The market cost of funds is  $r$ .

#### A savings case

In the diagram below the consumer is seen to be saving, i.e.  $s_1^* = y_1 - c_1^* > 0$ .

```
In [23]: consume_plot2(r, delta, rho, y1, y2)
```



Interactive plot with sliders (visible if running on a notebook server):

```
In [21]: interact(consume_plot2, r=(rmin,rmax,0.1), rho=fixed(rho), delta=(0.5,1,0.1), y1=(10,100,1),  
interactive(children=(FloatSlider(value=0.0, description='r', max=1.0), FloatSlider(value=0.7, descr  
In [12]: c1e, c2e, uebar = find_opt2(r, rho, delta, y1, y2)
```

In this particular case the consumer consumes:

```
In [13]: c1e, c2e  
Out[13]: (50.0, 50.0)
```

Her endowment is

```
In [14]: y1,y2  
Out[14]: (80, 20)
```

And she therefore saves

```
In [15]: y1-c1e
```

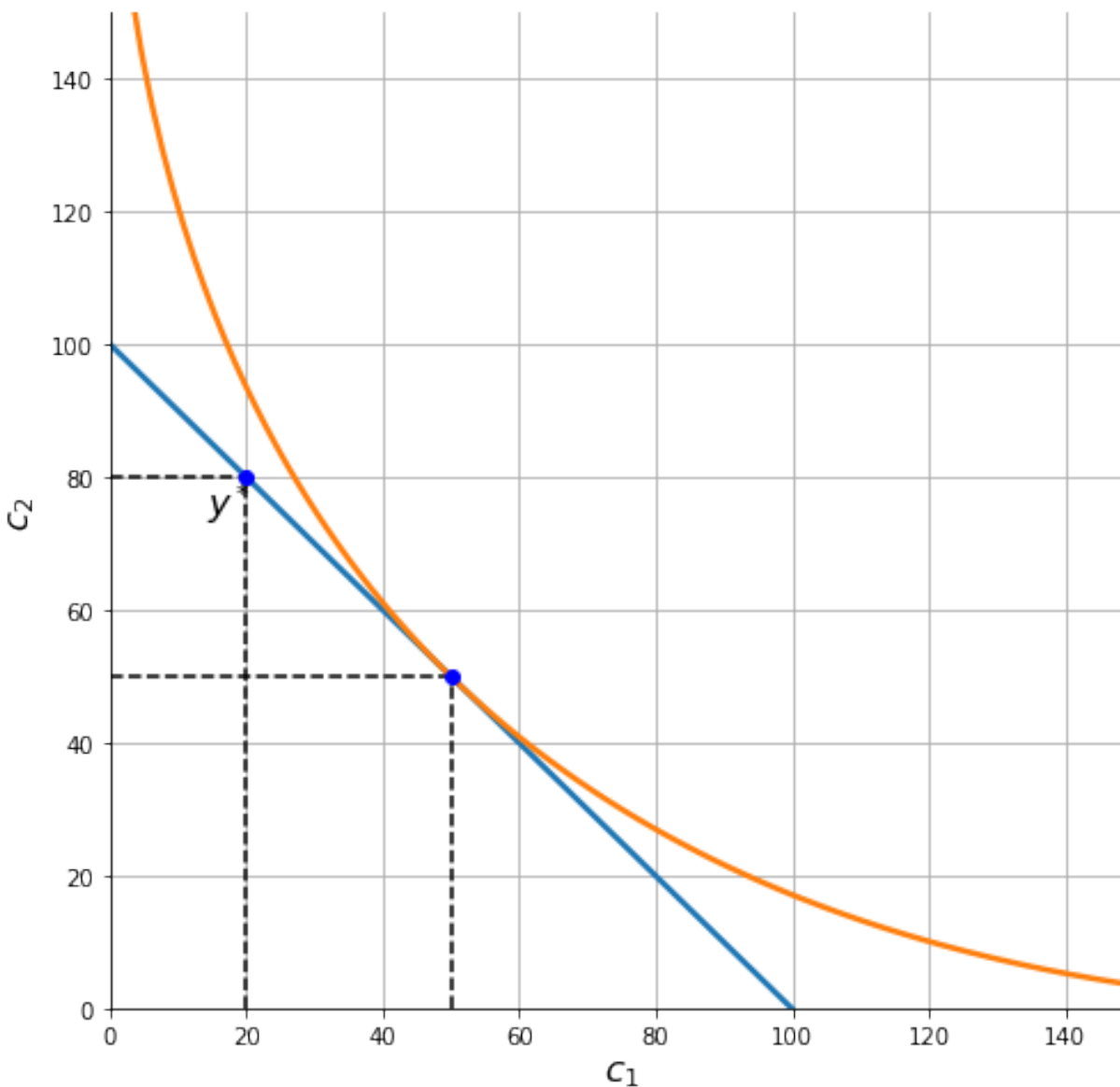
```
Out[15]: 30.0
```

in period 1.

### A borrowing case

In the diagram below the consumer is seen to be borrowing, i.e.  $s_1^* = y_1 - c_1^* < 0$ .

```
In [16]: y1,y2 = 20,80
         consume_plot2(r, delta, rho, y1, y2)
```



## Code section Run the code below first to re-generate the figures and interactions above.

```
In [1]: %matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interact, fixed
```

### Code for simple consumer choice

```
In [2]: def U(c1, c2, alpha):
        return (c1**alpha)*(c2**(1-alpha))

        def budgetc(c1, p1, p2, I):
            return (I/p2)-(p1/p2)*c1

        def indif(c1, ubar, alpha):
            return (ubar/(c1**alpha))**(1/(1-alpha))

In [3]: def find_opt(p1,p2,I,alpha):
        c1 = alpha * I/p1
        c2 = (1-alpha)*I/p2
        u = U(c1,c2,alpha)
        return c1, c2, u
```

### Parameters for default plot

```
In [4]: alpha = 0.5
        p1, p2 = 1, 1
        I = 100

        pmin, pmax = 1, 4
        Imin, Imax = 10, 200
        cmax = (3/4)*Imax/pmin

In [5]: def consume_plot(p1=p1, p2=p2, I=I, alpha=alpha):

        c1 = np.linspace(0.1,cmax,num=100)
        cle, c2e, uebar = find_opt(p1, p2 ,I, alpha)
        idfc = indif(c1, uebar, alpha)
        budg = budgetc(c1, p1, p2, I)

        fig, ax = plt.subplots(figsize=(8,8))
        ax.plot(c1, budg, lw=2.5)
        ax.plot(c1, idfc, lw=2.5)
        ax.vlines(cle,0,c2e, linestyles="dashed")
        ax.hlines(c2e,0,c1e, linestyles="dashed")
        ax.plot(c1e,c2e,'ob')
        ax.set_xlim(0, cmax)
        ax.set_ylim(0, cmax)
        ax.set_xlabel(r'$c_1$', fontsize=16)
        ax.set_ylabel('$c_2$', fontsize=16)
        ax.spines['right'].set_visible(False)
        ax.spines['top'].set_visible(False)
        ax.grid()
        plt.show()
```

### Code for intertemporal Choice model

```
In [6]: def u(c, rho):
        return (1/rho)* c**(1-rho)
```



```

def U2(c1, c2, rho, delta):
    return u(c1, rho) + delta*u(c2, rho)

def budget2(c1, r, y1, y2):
    Ey = y1 + y2/(1+r)
    return Ey*(1+r) - c1*(1+r)

def indif2(c1, ubar, rho, delta):
    return ((1-rho)/delta)*(ubar - u(c1, rho))**(1/(1-rho))

In [7]: def find_opt2(r, rho, delta, y1, y2):
    Ey = y1 + y2/(1+r)
    A = (delta*(1+r))**(1/rho)
    c1 = Ey/(1+A/(1+r))
    c2 = c1*A
    u = U2(c1, c2, rho, delta)
    return c1, c2, u

```

Parameters for default plot

```

In [8]: rho = 0.5
        delta = 1
        r = 0
        y1, y2 = 80, 20

        rmin, rmax = 0, 1
        cmax = 150

In [9]: def consume_plot2(r, delta, rho, y1, y2):

    c1 = np.linspace(0.1, cmax, num=100)
    cle, c2e, uebar = find_opt2(r, rho, delta, y1, y2)
    idfc = indif2(c1, uebar, rho, delta)
    budg = budget2(c1, r, y1, y2)

    fig, ax = plt.subplots(figsize=(8,8))
    ax.plot(c1, budg, lw=2.5)
    ax.plot(c1, idfc, lw=2.5)
    ax.vlines(cle, 0, c2e, linestyle="dashed")
    ax.hlines(c2e, 0, cle, linestyle="dashed")
    ax.plot(cle, c2e, 'ob')
    ax.vlines(y1, 0, y2, linestyle="dashed")
    ax.hlines(y2, 0, y1, linestyle="dashed")
    ax.plot(y1, y2, 'ob')
    ax.text(y1-6, y2-6, r'$y^*$', fontsize=16)
    ax.set_xlim(0, cmax)
    ax.set_ylim(0, cmax)
    ax.set_xlabel(r'$c_1$', fontsize=16)
    ax.set_ylabel(r'$c_2$', fontsize=16)
    ax.spines['right'].set_visible(False)
    ax.spines['top'].set_visible(False)
    ax.grid()
    plt.show()

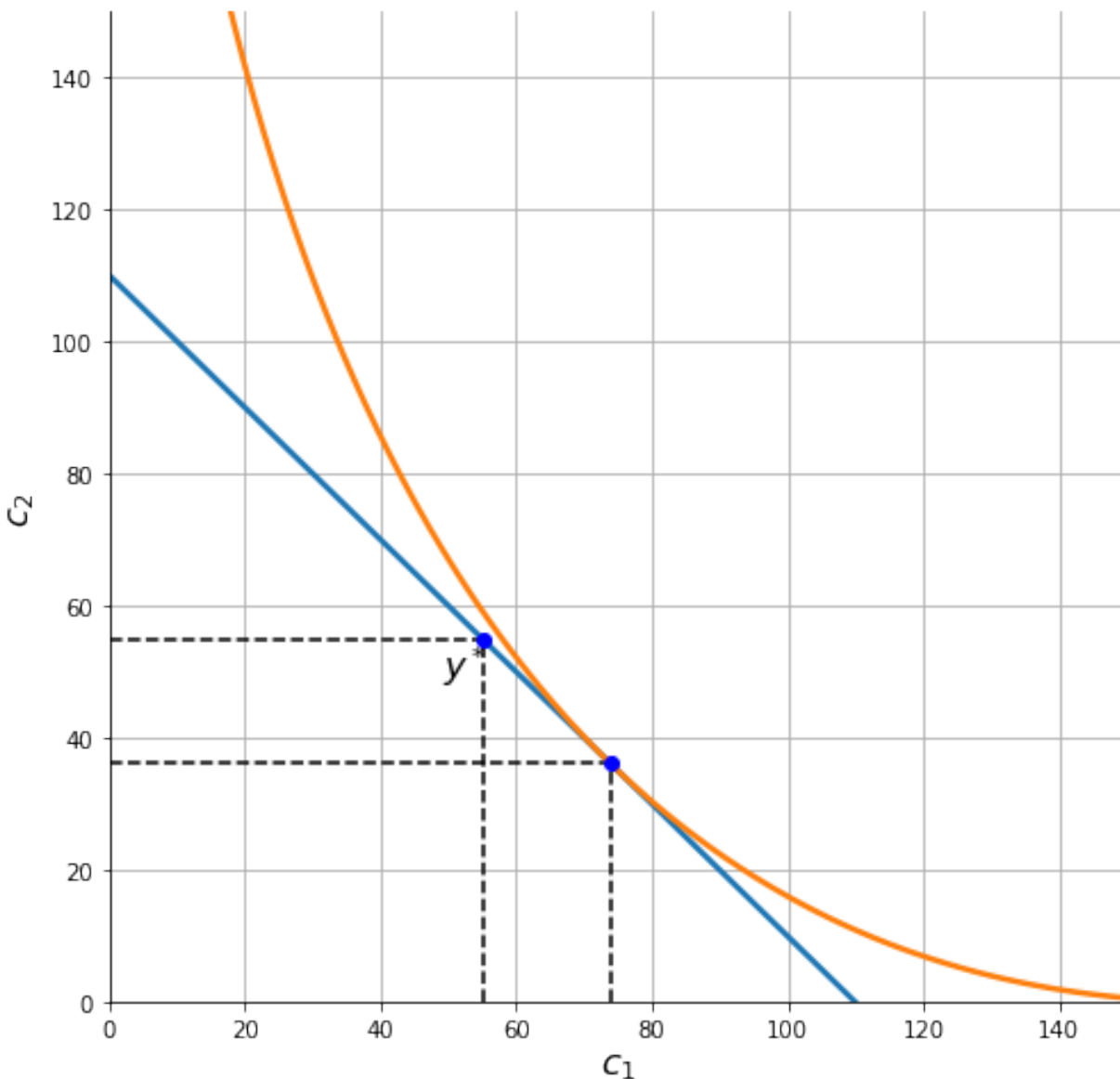
```

## Interactive plot

```

In [19]: interact(consume_plot2, r=(rmin, rmax, 0.1), rho=fixed(rho), delta=(0.5, 1, 0.1), y1=(10, 100, 1),

```



In [ ]:

## 1.12 Quasi-hyperbolic discounting and commitment savings

### 1.12.1 The exponential discounter

Consider a simple extension of the standard intertemporal optimization problem (seen in [an earlier notebook](#) from two to three periods.

Our *exponential* or time-consistent discounter wishes use own-savings strategies and/or the services of a competitive financial service sector to exchange existing endowment income stream  $(y_0, y_1, y_2)$  for a more optimal consumption stream.

The consumer optimization problem is to choose  $c_0, c_1, c_2$  to solve:

$$\begin{aligned} \max_{c_0, c_1, c_2} \quad & u(c_0) + \delta u(c_1) + \delta^2 u(c_2) \\ \text{s.t.} \quad & c_0 + \frac{c_1}{1+r} + \frac{c_2}{1+r} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{1+r} \end{aligned}$$

where  $\delta$  is the consumer's own personal psychic discount factor and  $r$  is the financial sector's opportunity cost of funds. Competition drives banks to offer earn zero-profits, leading them to offer consumption contract  $c^* = (c_0^*, c_1^*, c_2^*)$  in exchange for the consumer's original more volatile income stream  $c^* = (y_0, y_1, y_2)$  of equal monetary present value.

Let us solve this by setting up a Lagrangean

$$u(c_0) + \delta u(c_1) + \delta^2 u(c_2) - \lambda \left[ Ey - c_0 - \frac{c_1}{1+r} - \frac{c_2}{1+r} \right]$$

The first-order necessary conditions for an interior optimum are:

$$\begin{aligned} u'(c_0^*) &= \lambda \\ \delta u'(c_1^*) &= \lambda \frac{1}{(1+r)} \\ \delta^2 u'(c_2^*) &= \lambda \frac{1}{(1+r)^2} \end{aligned}$$

If it just happens that the consumer's discount factor  $\delta$  has the following relationship to the bank's opportunity cost of funds  $r$ :

$$\delta = \frac{1}{1+r}$$

Then the first order conditions collapse down to:

$$u'(c_0^*) = u'(c_1^*) = u'(c_2^*) = \lambda$$

which of course then implies that the consumer has constant consumption across periods  $c_0^* = c_1^* = c_2^*$

It's easy to establish that if the consumer is sufficiently patient and/or the return to saving is high enough then the consumer will consume more in later periods:

$$\delta > \frac{1}{1+r} \text{ then } c_0^* < c_1^* < c_2^*$$

and if the situation is reversed so the consumer is relatively impatient and/or the return to savings is low then they will consume more in earlier periods:

$$\delta < \frac{1}{1+r} \text{ then } c_0^* > c_1^* > c_2^*$$

### 1.12.2 The CRRA utility case

A Constant-Relative Risk Aversion (CRRA) felicity function is given by:

$$u(c_t) = \begin{cases} \frac{c_t^{1-\rho}}{1-\rho}, & \text{if } \rho > 0 \text{ and } \rho \neq 1 \\ \ln(c) & \text{if } \rho = 1 \end{cases}$$

if  $\rho = 1$

The Arrow-Pratt measure of **relative risk aversion**

$$R(c) = \frac{-cu''(c)}{u'(c)}$$

gives us a measure of how the curvature of the consumer's utility function – or how averse consumers are to risks – changes with wealth (consumption). As its name implies the CRRA function has a constant measure of relative risk aversion given simply by  $\rho$ .

The elasticity of intertemporal substitution  $\sigma$  (the responsiveness of the slope of the consumption path to changes in the interest rate) for a CRRA function is also constant and given simply by:

$$\sigma = \frac{1}{\rho}$$

If we plug our CRRA utility function into our earlier derived FOC we can solve to find:

$$\frac{c_1^*}{c_0^*} = \frac{c_2^*}{c_1^*} = [\delta(1+r)]^{\frac{1}{\rho}}$$

Match this to the zero profit condition and we can find a closed form solution for the consumption path.

### 1.12.3 Quasi-hyperbolic time-inconsistent preferences

A relatively simple way to model present-biased and time-inconsistent preferences is by modeling preferences using quasi-hyperbolic preferences as follows. The key idea in what follows is that the agent's preferences change from period to period.

The problem can be described in broad terms like this. When first making plans for future consumption choices – say the choice between period 1 consumption and period 2 consumption – the consumer's period 0 self wants to keep consumption smoothly balanced between those two periods. However when period 1 arrives the consumer's preferences *change* in a way that makes him/her more impatient or present biased. The new period 1 self consumer now wants to undo the optimal consumption plans laid out by her earlier period 0 self and to choose a new consumption plan that boosts present (period 1) consumption at the expense of future (period 2) consumption.

The 'sophisticated' (quasi) hyperbolic discounter anticipates his own future self's likely change of heart and this is likely to alter their original period 0 consumption plan. This sets up in effect a conflict or a struggle between the consumer's earlier and future selves. The period 0 self acts strategically, anticipating their own later period's optimal responses.

#### The demand for commitment services

Let's see this formally. The consumer's "period-0" self wants to choose consumption path  $(c_0, c_1, c_2)$  to maximize:

$$u(c_0) + \beta[\delta u(c_1) + \delta^2 u(c_2)]$$

subject to the earlier described intertemporal budget constraint.

This is as before except for the fact that we've now introduced a 'present-bias' parameter  $\beta$  (where  $0 < \beta \leq 1$ ) that tilts the consumer's preferences toward period zero consumption. Whenever  $\beta$  is strictly less than one the consumer wants to smooth preferences in a way that tilts toward period 0 consumption but seeks to keep consumption between period 1 and 2 more balanced.

When period 1 rolls around however the consumer's preferences over remaining consumption bundles **change**. The period 1 self now wants to re-arrange remaining consumption to now maximize:

$$u(c_1) + \beta \delta u(c_2)$$

While the period-0 self wanted to trade off period 1 and period 2 consumption to equalize marginal utilities like this:

$$u'(c_1^*) = \delta(1+r)u'(c_2^*)$$

the new period-1 self now prefers to tradeoff like this:

$$u'(c_1^*) = \beta \delta(1+r)u'(c_2^*)$$

The  $\beta$  on the right-hand side of the last equation means the period 1 consumer now values period 2 consumption less relative to period 1 consumption compared to his period 0 self. Compared to the period-0 self's plans the new period-1 consumer wants to 'raid-savings' and/or 'take out a new loan'.

Lets look again at the CRRA case under the special assumption that  $\delta = \frac{1}{1+r}$  and  $r = 0$  (and hence  $\delta = 1$ ). These last assumptions are without loss of generality and done only to simplify the math and spotlight the key mechanisms at work.

Let's assume also that the period zero consumer can contract with a bank *and that the bank can credibly commit to not renegotiating the terms of the contract* even when the consumer's period 1 self comes begging for the bank to do so.

In this case the problem is exactly as described in the earlier section. The period 0 consumer maximizes

$$u(c_0) + \beta[\delta u(c_1) + \delta^2 u(c_2)]$$

subject to the intertemporal budget constraint.

When  $\delta = \frac{1}{1+r}$  the period 0 consumer would like to keep consumption between period 1 and period 2 flat. The first order conditions now reduce to:

$$c_1^* = \beta^{\frac{1}{\rho}} c_0^*$$

$$c_2^* = c_1^*$$

Substituting these into the binding budget constraint yields:

$$c_0^* = \frac{E[y]}{1 + \beta^{\frac{1}{\rho}}}$$

$$c_1^* = c_2^* = \beta^{\frac{1}{\rho}} c_0^*$$

We call this the **full commitment contract**.

The financial intermediary who offers such a contract is really providing *two* services to consumer's period 0 self: they're helping the consumer to smooth consumption between period zero and later periods and they're also helping the consumer resist his period 1 self's temptation to disrupt this optimal consumption plan.

For example suppose that consumer endowment income is such that she finds it optimum to save in period 0 and then finance constant consumption out of endowment income and savings in period 1 and 2. When period 1 rolls around the consumer's period 1 self present bias would tempt them to want to 'raid savings' and/or take out a new loan to boost period 1 consumption at the expense of period 2 consumption.

To see this formally note that under the full commitment contract the consumer enters the period with contractual claims to the remaining consumption stream  $(c_1^*, c_2^*) = (\bar{c}^*, \bar{c}^*)$ . If they could however the period 1 self would re-contract with this bank (or another) to choose another contract  $(\hat{c}_1, \hat{c}_2)$  that solves:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2)$$

subject to

$$c_1 + \frac{c_2}{1+r} \leq \bar{c}^* + \frac{\bar{c}^*}{1+r}$$

The first order conditions for this problem are

$$u'(c_1) = \beta u'(c_2)$$

which in the CRRA case requires  $c_2 = \beta^{\frac{1}{\rho}} c_1$

which is clearly not satisfied along the original contract which had  $(c_1^*, c_2^*) = (\bar{c}^*, \bar{c}^*)$

Substituting the FOC into the new period 1 budget constraint we can arrive at a solution or reaction function that states that when the period 1 self enters the period with claims  $c_1, c_2$  they will want to renegotiate to:

$$\hat{c}_1 = \frac{c_1 + c_2}{1 + \beta^{\frac{1}{\rho}}}$$

$$\hat{c}_2 = \beta^{\frac{1}{\rho}} \hat{c}_1$$

If this renegotiation takes place then the period 1's welfare would increase but at the expense of period 0's welfare, since period 0's optimal consumption plan would have been undone.

From this discussion it should be clear that the period 0 self would like to have banks compete for her business in period 0 but offer a full-commitment contract which in practice means being locked into an exclusive relationship (if the relationship were not exclusive then the period 1 self would approach a second bank in period 1 and ask them to 'buy out' and then renegotiate their contract with the first bank.

### 1.12.4 More detailed analysis

Much more detail on the analysis of contracts like this can be found in our 2016 working paper (with Karna Basu) entitled "Sticking to the Contract: Nonprofit vs For-profit Banks and Hyperbolic Discounters." A github [repository](#) is available with code and jupyter notebooks describing some of that analysis. I draw on that here, but most of the details of the methods are in those other notebooks.

### 1.12.5 Commitment Savings products

Many economists believe that time-inconsistent preferences drive people to have to struggle with important issues of self-control that affects their ability to reach savings goals, and generates a demand for commitment services. Indeed some economists have gone so far as to argue that these problems are ubiquitous and that one reason for the demand for microfinance services is that they provide individuals with the discipline and commitment mechanisms that can help them with these problems of self-control.

Rotating Savings and Credit Associations (ROSCAs) for example place individuals into savings groups and then create pressures and social sanctions to help individuals achieve savings goals. Iterant savings collectors may serve a similar purpose. Some have even suggested that many microcredit loans which advance loans to individuals and then put them on tight weekly repayment schedules may be popular not so much because they are releasing the individual from a binding credit constraint as because they are helping the individual 'borrow to save.' In this type of scenario the individual wants to save up to buy say a refrigerator or other household appliance but cannot find the discipline to save

up for the item themselves. A microcredit loan allows them to buy the refrigerator outright and then places them on a strict repayment schedule that commits the individual to putting aside the fraction of weekly income toward the item that they might have otherwise struggled to do on their own.

In all these stories the availability of commitment contracts provides additional value to the consumer, value that they may be willing to pay for.

In the following section I will illustrate some of these ideas with contracts that we will solve for and illustrate for the CRRA case. The code for much of what follows is in the `Contract.py` module described in greater detail in the github repo.

```
In [5]: import Contract
```

Lets look at a competitive contracting situation where  $\beta = 0.8$  and  $\rho = 0.5$

```
In [9]: cC = Contract.Competitive(beta = 0.8)
        cC.rho = 0.5
```

Now lets give the individual an endowment income of  $(y_0, y_1, y_2) = (150, 100, 50)$

```
In [15]: cC.y = [150, 100, 50]
```

The full commitment contract is:

```
In [21]: cCF = cC.fcommit(); cCF
Out[21]: array([ 131.57894737,   84.21052632,   84.21052632])
```

Note how this involves some saving in period 0 ( $y_0 = 150$  and consumption  $c_0 = 131.6$ ) as well as additional saving for period 2.

If for some reason period 0 self agreed to this contract (sincerely believing that it would never be renegotiated) but then suddenly to everyone's surprise period 1 self had an opportunity to renegotiate the continuation of this contract, they would renegotiate to the following contract:

```
In [22]: cCR = cC.reneg(cCF); cCR
Out[22]: array([ 131.57894737,  101.44337644,   64.92376092])
```

We can see here how period 1 self 'raids' the savings that the period 0 self had intended should be passed onto period 2 consumption. The period 1 self boosts period 1 consumption from 84 to 101. Rather than pass 34 units of consumption into period 2 only 20 units are now passed.

If the bank cannot credibly commit to not renegotiate the contract then the sophisticated time-inconsistent consumer will defensively alter their consumption plans to try to thwart their future self's strategic renegotiations. The period 0 consumer will insist on **renegotiation proof contracts** which will impose an additional constraint on the problem, namely that no bank find it profitable to renegotiate the contract. In general this will be achieved by in a sense 'surrendering' to their future self's bias tilt toward period 1 consumption.

Details in the paper and in class lecture on the shape of this constraint.

The important thing to understand is that adding any constraint to the problem can only lower consumer welfare and push us away from the full-commitment optimum.

For the present setup the renegotiation proof contract can be solved for to find:

```
In [31]: cCRP = cC.reneg_proof().x; cCRP
Out[31]: array([ 122.59354576,  108.17462856,   69.23182568])
```

Compare to the full commitment contract this contract involves more savings passed from from period 0 to period 1 but much less savings passed from period 1 to period 2. Total savings (period 0 plus period 1 savings) are higher when credible commitment savings devices are available compared to when not.

### 1.12.6 Replicating Ashraf, Karlan and Yin (2006) Commitment Savings paper

Ashraf, Karlan and Yin (2006) “Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines,” \*Quarterly Journal of Economics.

What follows are incomplete notes... just some pieces of the puzzle needed to replicate the paper. I hope to improve on this in time and I just offer this here so that you might form an idea on how this might be done. Note that I am using the `ipystata` library which requires Stata to be running in the background. This seems to only be possible to do on Windows machines as described in the `ipystata` library.

The Stata dataset and code replication files have been made available by the authors via the Harvard Dataverse [here](#)

Notes to self: Files in D:

Drive

\_S15

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\_011204\_1.dta” “D:

Drive

\_S15

\_011204\_080404\_1.dta” “D:

Drive

\_S15

\_080404\_1.dta”

The posted files are Stata version 14. These can be opened by python pandas but not by older versions of Stata.

```
In [2]: %matplotlib inline
import seaborn as sns
import pandas as pd
import ipystata
import statsmodels.formula.api as sm
```

#### Open the datasets

One disadvantage of proprietary software is that you often cannot open a dataset saved with a later version of the software unless you pay to upgrade the software. I’m going to load the Stata dataset into a python pandas dataframe using its `read_stata` method and then pass the dataset into the running Stata session (which on my home machine is Stata 11).

```
In [7]: #df = pd.read_stata(r"G:\GC\Dev-II\notebooks\seedanalysis_011204_1_v12.dta")
df = pd.read_stata(r"G:\GC\Dev-II\notebooks\data\savings1.dta")

In [8]: regVI_1 = 'balchange ~ treatment + marketing'
model = sm.ols(regVI_1, df)
fitted = model.fit(cov_type='HC1')
print(fitted.summary())
```



```

OLS Regression Results
=====
Dep. Variable:          balchange    R-squared:                0.002
Model:                  OLS          Adj. R-squared:           0.001
Method:                 Least Squares  F-statistic:              2.901
Date:                  Tue, 29 Mar 2016  Prob (F-statistic):       0.0553
Time:                  17:00:31       Log-Likelihood:           -16270.
No. Observations:      1777          AIC:                     3.255e+04
Df Residuals:          1774          BIC:                     3.256e+04
Df Model:               2
Covariance Type:       HC1
=====

```

	coef	std err	z	P> z	[95.0% Conf. Int.]
Intercept	40.6257	61.676	0.659	0.510	-80.257 161.509
treatment	234.6785	101.748	2.306	0.021	35.256 434.101
marketing	184.8506	146.982	1.258	0.209	-103.228 472.929

```

=====
Omnibus:                 3458.895    Durbin-Watson:           0.051
Prob(Omnibus):           0.000       Jarque-Bera (JB):        5716807.283
Skew:                    14.932       Prob(JB):                0.00
Kurtosis:                279.259      Cond. No.                4.24
=====

```

Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

```

In [33]: model = sm.ols('balchange ~ treatment', df[(df.treatment ==1) | (df.marketing ==1)])
         fitted = model.fit(cov_type='HC1')
         print(fitted.summary())

```

```

OLS Regression Results
=====
Dep. Variable:          balchange    R-squared:                0.000
Model:                  OLS          Adj. R-squared:           -0.001
Method:                 Least Squares  F-statistic:              0.1020
Date:                  Thu, 24 Mar 2016  Prob (F-statistic):       0.750
Time:                  23:43:38       Log-Likelihood:           -12115.
No. Observations:      1308          AIC:                     2.423e+04
Df Residuals:          1306          BIC:                     2.424e+04
Df Model:               1
Covariance Type:       HC1
=====

```

	coef	std err	z	P> z	[95.0% Conf. Int.]
Intercept	225.4763	133.405	1.690	0.091	-35.992 486.944
treatment	49.8279	156.027	0.319	0.749	-255.979 355.635

```

=====
Omnibus:                 2498.290    Durbin-Watson:           0.056
Prob(Omnibus):           0.000       Jarque-Bera (JB):        3202671.430
Skew:                    14.153       Prob(JB):                0.00
Kurtosis:                243.756      Cond. No.                3.11
=====

```

Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

Table 6: Impact: 6 months

## 1.13 Notes on Incentive Contracts and Corruption

Many public policy issues in developing countries be understood and modeled as asymmetric information problems where Principal - Monitor - Agent contract structures must be employed.

Consider for example a central government that launches a project either to improve rural roads (as in Olken, 2007) or perhaps instead a project to carry out a campaign to distribute anti-malaria mosquito nets.

For either type of project the government will make funds available to local communities across the country but must rely on local agents (e.g. local contractors or officials) to implement the projects. The government may have a very good idea of how much it costs to repair a kilometer or make and distribute 1000 mosquito nets but typically cannot monitor whether the local agents ‘diligently’ carry out the project to specification or not. The government will in the end see if the project has succeeded or failed, and the project is much more likely to succeed if the local contractor has carried out the contract to specification, but the government cannot directly verify whether this has been the case or not. This moral hazard situation means that the government can only put local agents on outcome-contingent and not action-contingent contracts.

For example after the local road repair project is completed the government will be able to see if the project succeeded (e.g. the bridge was not washed away this year when the river floods) or whether it failed (it did not wash away). But the government can not directly observe the agent’s level of diligence in the project – for example whether they built the bridge to specification with the allocated funds or if they perhaps substituted shoddier building materials to divert funds to private gain) because the project’s outcome is probabilistic.

Similarly in an anti-malarial campaign the local agent might claim to have made and distributed 1000 mosquito nets but in fact it only made and distributed 500. All the government can do is observe whether the number of malaria cases has been reduced (success) or increased or stayed the same compared to earlier years.

Let’s model this principal-agent problem using the same simple model we used to describe tenancy and credit contracts.

The government advances  $I$  in funds to an agent to purchase materials to carry out the project. If the agent diligently carries out the project to specification the project succeeds with probability  $p$  in which case the community receives benefit  $X_s$  or it fails with probability  $1 - p$  in which case the community receives benefits of only  $X_f < X_s$ .

If the agent is not diligent they divert project funds and/or put in less effort all of which allows them to capture private benefits  $B$ . Furthermore when the agent is non-diligent the project succeeds with only probability  $q$  (in which case the community again receives benefit  $X_s$ ) or it fails with probability  $1 - q$  (in which case the community receives benefits of only  $X_f < X_s$ ).

Since the government can only observe project outcomes and not the choice of diligence the principal can only get the agent to be diligent by offering a contract that rewards the agent sufficiently more for project successes as for failures, in order to want to make the agent want to on their own raise the probability of success via their choice of diligence.

The government aims to maximize the expected value of community benefits minus the cost of the investment funds and the cost of renumerating the agent:

$$\max_{c_s, c_f} p(X_s - c_s) + (1 - p)(X_f - c_f) - I$$

subject to the agent’s participation (PC) constraint:

$$pc_s + (1 - p)c_f \geq \bar{u}$$

Without loss of generality in what follows we will set  $\bar{u} = 0$ , so in the ideal of circumstances the government would be able to hire the agent away from their next best opportunity by paying them amount normalized to zero.

and an incentive compatibility (IC) constraint:

$$pc_s + (1 - p)c_f \geq qc_s + (1 - q)c_f + \bar{B}$$

Note that we will at times write this problem compactly as:

$$\max_{c_s, c_f} E(X|p) - E(c|p) - I$$

s.t.

$$E(c|p) \geq \bar{u}$$

$$E(c|p) \geq E(c|q) + \bar{B}$$

The IC constraint can be rewritten:

$$c_s \geq c_f + \frac{\bar{B}}{p - q}$$

This can be satisfied at minimum cost when this constraint binds. This tells us that in the event of project success the agent must receive a ‘bonus’ of  $\frac{\bar{B}}{p-q}$  over what they get paid for failure outcomes. This higher reward for success compared to failure is what induces the agent to want to be diligent and increase the probability of success from  $q$  to  $p$ . The contractual cost of this remuneration strategy is then  $pc_s + (1 - p)c_f$  or:

$$E(c|p) = c_f + p \frac{\bar{B}}{\Delta}$$

where  $\Delta = p - q$  which then means that the expected net benefit of the government project is:

$$E(X|p) - I - c_f - p \frac{\bar{B}}{\Delta}$$

Note that we earlier normalized the agent’s next best employment opportunity to a remuneration of zero. If the government could get local agents to competitively bid against each other for the government contract the agent’s contract could be made to bind, but this in turn would require:

$$c_f = -p \frac{\bar{B}}{\Delta} + \bar{u}$$

$$c_s = (1 - p) \frac{\bar{B}}{\Delta} + \bar{u}$$

One way to think of this is that the agent is made to pay a fine of  $-p \frac{\bar{B}}{\Delta}$  when the project fails while if the project succeeds she earns a reward of  $(1 - p) \frac{\bar{B}}{\Delta}$

A possible problem with this type of project is that it may be difficult for the government to impose a penalty on agents when the project fails (e.g. the local contractor leaves town when the bridge collapses or the incidence of malaria cases surges). One way to try to resolve that problem is by asking local contractors to post a **bond** but this solution may be hard to implement particularly in poor communities where the agents are poor to start with.

The consequence of not being able to impose a fine when the project fails is that we have to now impose yet another constraint on the contract design problem, a **limited liability** constraint of the form

$$c_f \geq 0$$

for example if the heaviest fine that can be imposed is to pay the local agent nothing when the project fails. The lowest cost way to remunerate the agent will be for this limited liability constraint and the incentive compatibility constraints to bind (to set the punishment as high as possible and the bonus as low as possible, compatible with maintaining incentives. With  $c_f = 0$  an extra bonus must now be paid following success outcomes to continue to satisfy the incentive constraint. But this increases the expected cost of remuneration and reduces expected benefits from the project to:

$$E(X|p) - I - p \frac{\bar{B}}{\Delta}$$

The last term  $p \frac{\bar{B}}{\Delta}$  is sometimes referred to as an ‘information rent’ that must be paid to the agent that arises due to the asymmetric information problem.

### 1.13.1 An Example

```
In [1]: %matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
from ipywidgets import interact, fixed
```

```
In [97]: def E(xs,xf,p):
        """Expectation operator """
        return p*xs + (1-p)*xf
```

Consider a project with the following characteristics:

```
In [100]: I = 75          # Lump sum investment to start project
          Xs = 100         # project success return
          Xf = 0           # project failure return
          p = 0.95         # probability of success when diligent
          q = 0.50         # probability of success when non-diligent
          EX = E(Xs,Xf,p) # Gross expected project return
          ubar = 5         # Consumer reservation income
          B = 10          # private benefits to being non-diligent
```

```
In [84]: print('Expected returns Diligent (p): {}, Non-diligent (q): {}'.format(E(Xs,Xf,p), E(Xs,Xf,q)))
```

Expected returns Diligent (p): 95.0, Non-diligent (q): 50.0

This project fails only 1 percent of the time when the agent is non-diligent (corrupt) but fails 50 percent of the time when they are non-diligent (corrupt). We associate non-diligence with an opportunity to divert  $\bar{B}$  in funds to private uses.

```
In [85]: B = 10
```

As derived above the optimal remuneration contract calls for the agent to pay a big fine for failure and earn a positive reward for success:

```
In [111]: cf = -p*B/(p-q) + ubar
          cs = (1-p)*B/(p-q) + ubar

In [112]: print('(c_f, c_s) = ({:5.1f}{:5.1f})'.format(cf, cs))
```

(c\_f, c\_s) = (-16.1 6.1)

```
In [113]: print('consumer and bank expected payments:')
          E(cs,cf,p), E(Xs-cs, Xf-cf,p) - I
```

consumer and bank expected payments:

```
Out[113]: (5.0, 15.0)
```

In expectation this covers the agent's opportunity cost of funds  $\bar{u}$ . Since the incentive compatibility constraint is met (by construction) when she is diligent:

#### Diagram

```
In [89]: def zeroprofit(c):
        return EX/p - ((1-p)/p)*c - I/p

        def IC(c):
            return c + B/(p-q)

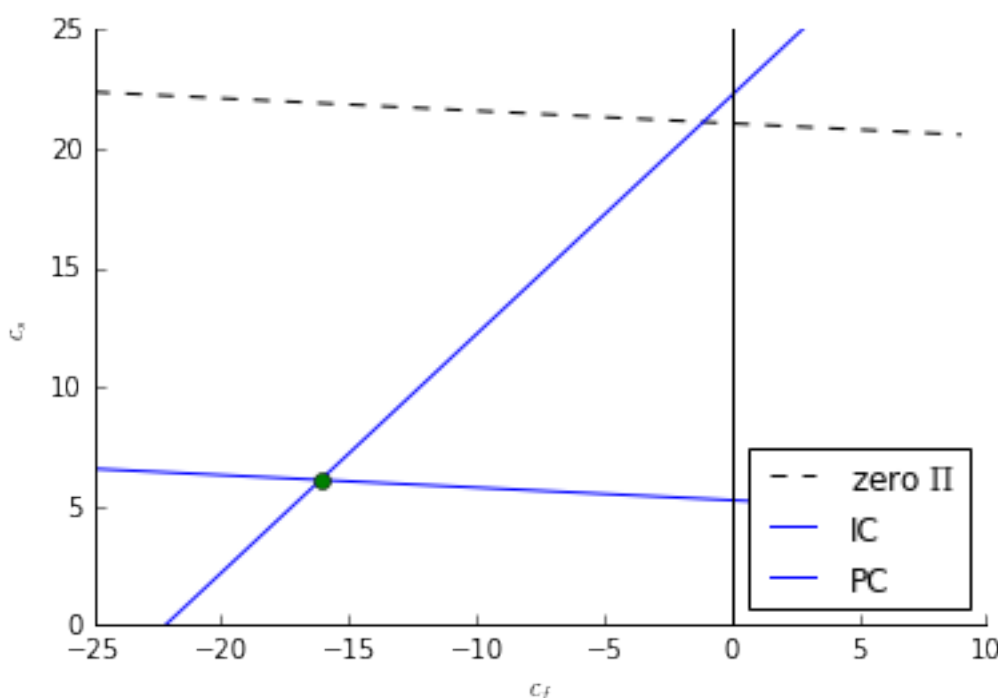
        def BPC(c,ubar):
            return ubar/p - ((1-p)/p)*c
```

```

In [90]: cf_min, cf_max = -25, 10
         c = np.arange(cf_min, cf_max)

In [102]: ax = plt.subplot(111)
          ax.plot(c, zeroprofit(c), 'k--', label='zero  $\Pi$ ')
          ax.plot(c, IC(c), label='IC')
          ax.plot(c, BPC(c, ubar), label='PC', color='b')
          ax.plot(cf, cs, marker='o')
          ax.legend(loc='lower right')
          ax.set_xlabel('$c_f$'), ax.set_ylabel('$c_s$')
          ax.axvline(0, color='k')
          ax.set_ylim(0, 25)
          ax.spines['right'].set_visible(False)
          ax.spines['top'].set_visible(False)
          ax.yaxis.set_ticks_position('left')
          ax.xaxis.set_ticks_position('bottom')

```



The principal extracts

```
In [110]: EX - I - ubar
```

```
Out[110]: 15.0
```

Under this contract the agent will be diligent even though they cannot be observed. Had they been non-dilig

```
In [116]: q*(Xs- cs) + (1-q)*(Xf-cf) - I
```

```
Out[116]: -20.0
```

### Limited Liability constraints

In the example above the agent is asked to pay the principal in the event of failure ( $c_f < 0$ ). Suppose however that the agent cannot be made to pay the fine (e.g. they cannot post bond and run away before paying the fine).

Suppose in fact that the worst fine we can impose is that they pay  $c_f = -5$ .

When that is the case the cheapest way to satisfy the incentive compatibility constraint is to set:

```
In [117]: cf = -5
          cs = -5 + B/(p-q)
```

Which then means that in expectation the agent earns an information rent of

```
In [118]: E(cs, cf, p)
Out[118]: 16.111111111111114
```

This is above their reservation utility  $\bar{u}$  and this contract is therefore quite a bit more expensive to the principal. Their net return is now

$$E(X, p) - I - E(C, p)$$

$$E(X, p) - I - p/(p - q)$$

And the net benefits to the project are greatly reduced to:

```
In [119]: EX - I - E(cs, cf, p)
Out[119]: 3.8888888888888857
```

The net benefits have been reduced by the heavy cost of incentive contract to keep the agent diligent or non-corrupt.

But this is still better than allowing the agent to be corrupt.

```
In [121]: q*(XS-cs) + (1-q)*(Xf-cf) - I
Out[121]: -31.111111111111114
```

As we can see from the diagram above the principal can contract with agents who face limited liability but they earn less from agents where the LL constraint binds. The limited liability constraint means the agent must earn a rent in excess of their reservation utility.

Suppose the most we can take away from the agent is an amount  $A$ , equal for example to the amount of resources that can be seized or that they posted as bond or collateral.

The cheapest way to satisfy the IC is then:

$$c_f = -A$$

$$c_s = -A + B/(p - q)$$

Which implies the agent's expected repayment is:

$$E(c|p) = -A + \frac{pB}{p - q}$$

which will be more than their reservation wage  $\bar{u}$  as long as  $A < p \frac{B}{p-q} \bar{u}$

## Minimum collateral requirement

What is the minimum collateral requirement below which the contract cannot both satisfy the incentive compatibility constraint and guarantee at least zero profits to the principal?

Substitute the expected repayment under limited liability (above) into the principal's zero profit condition and solve for  $A$  (on the diagram above this is the  $c_f$  at the intersection of the IC constraint and the principal's zero profit condition:

$$E(X|p) - E(c|p) - I = 0$$

$$E(X|p) + A - \frac{pB}{p-q} - I = 0$$

$$\underline{A} = \frac{pB}{p-q} - [E(X|p) - I]$$

For our running example this minimum collateral requirement is:

```
In [123]: Amc = p*B/(p-q) - (EX - I)
          Amc
```

```
Out[123]: 1.1111111111111143
```

This is an important expression. This tells us that unless the agent can post a minimum bond or collateral of this amount then the principal cannot provide them with strong enough incentives for them to be diligent and still allow the principal to break even on the transaction.

The take away lesson is that sometimes in asymmetric information situations one has to pay employees a rent (expected payment in excess of their next best option) in order to motivate their behavior. It also means however that if the principal (employer, lender, etc) has a choice of agent to deal with they will prefer to deal with those who can post collateral.

### 1.13.2 Monitoring by an intermediary

Suppose an intermediary can ‘monitor’ the project. By expending resources  $m$  the monitor can reduce the agent’s private benefits from non-diligence from  $\bar{B}$  to  $\bar{B}(m) < \bar{B}$ .

For example the intermediary might visit the agent at random times to check up on the progress of the project. This does not completely eliminate the scope for corruption but limits how much can be privately captured (perhaps because the agent now has to spend additional resources hiding her diversions of effort and funds).

The obvious advantage of this is that it reduces the size of the information rent to:

$$\frac{\bar{B}(m)}{\Delta}$$

And this in turn will reduce the total cost of remunerating the agent. Intuitively, since the private benefit that can be captured has been directly reduced by monitoring the contract does not have to rely so much on costly bonus payments to motivate diligence.

Now of course the Principal will have to pay the intermediary to compensate them for their expense  $m$  and this will add to the cost. But so long as this extra cost is smaller than the reduction in the cost of remunerating the agent, net project benefits will improve.

Under the assumption that Principal can specify and observe the intermediary’s monitoring effort the net benefits from the project will now be:

$$E(X|p) - I - p \frac{\bar{B}(m)}{\Delta} - m$$

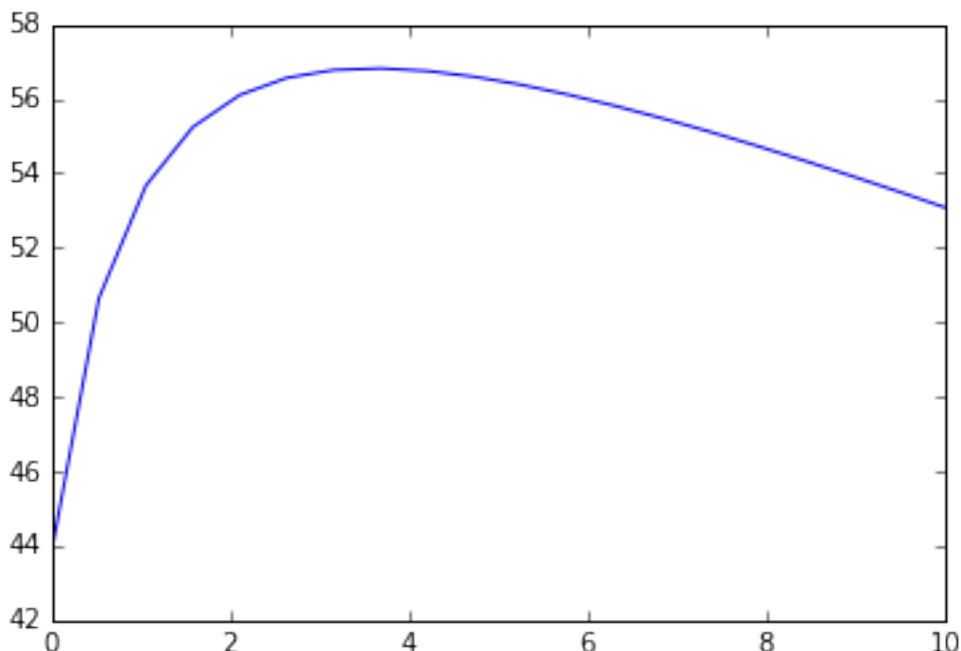
To take a concrete example suppose that the ‘monitoring function’ where given by:

$$\bar{B}(m) = \frac{\bar{B}}{1+m}$$

then the total expected cost of remunerating the agent and the monitoring intermediary would look as follows as a function of  $m$ :

```
In [12]: m = np.linspace(0,10,20)
          plt.plot(m, EX - I - p*(B/(1+m))/D - m)

Out[12]: [<matplotlib.lines.Line2D at 0x759e8d0>]
```



Which shows that over a range monitoring by the intermediary lowers the information rent that must be left with the agent faster than the cost of monitoring but eventually diminishing returns to this activity kick in (at somewhat less than 4 units of monitoring).

### Who monitors the monitor? Two-layered moral hazard

More likely the principal cannot directly contract on the intermediary's level of monitoring. The intermediary is supposed to spend resources  $m$  to monitor the agent but if the government has no way to directly verify if this is happening or not, the intermediary may well be tempted to monitor at expense of zero but claim that it has monitored at expense  $m$ .

The only way for the government to avoid this from happening is to put also put the intermediary on an incentive contract. The way to do this is to make the intermediary share in the agent's success and failures.

The Principal's contract design problem is to choose remuneration packages  $(c_s, c_f)$  and  $(w_s, w_f)$  to maximize:

$$E(X|p) - E(c|p) - E(m|p) - I$$

subject to participation constraints for both the agent and the intermediary

$$E(c|p) \geq 0$$

$$E(w|p) \geq 0$$

the (now modified) incentive compatibility constraint for the agent:

$$E(c|p) \geq E(c|q) + \bar{B}(m)$$

and an incentive compatibility constraint for the intermediary monitor:

$$E(w|p) \geq E(w|q) + m$$

As was the case with the agent the cost of providing incentives to the intermediary monitor will depend on whether the intermediary can be made to lose money when the project fails or not.



Let's first consider the case when this is not the case. In that event the intermediary is paid 0 when the project fails and

$$w_s = \frac{m}{\Delta}$$

when the project succeeds. Note this is very much like the expression we derived for the bonus that had to be paid to the agent. The expected cost of this intermediary remuneration contract (when  $w_f = 0$ ) is then:

$$E(w|p) = p \frac{m}{\Delta}$$

which is always larger than  $m$  so long as  $p > q$ . This suggests that the intermediary will also earn an information rent equal to

$$E(w|p) - m = p \frac{m}{\Delta} - m > 0$$

since the monitor has to pay expenses  $m$  while monitoring.

If on the other hand we can assume that intermediary's can be made to bear liability for the projects that they monitor and fail then this rent can be eliminated. Consider the case of competitive bidding for the intermediary monitor job. Different firms will compete to offer their services until the expected return to being an intermediary monitor is equal to what they could earn in their next best occupation which we assume to be zero. Then

$$E(w|p) = m$$

which implies

$$w_f + p \frac{m}{\Delta} = m$$

or

$$w_f = -p \frac{m}{\Delta} + m$$

(which then implies  $w_s = (1-p) \frac{m}{\Delta + m}$ )

One way to think of this is that the principal asks the intermediary put up portion

$$I_m = p \frac{m}{\Delta} - m$$

of the total cost  $I$  of the project while the uninformed principal puts up the remainder

$$I_u = I - I_m$$

Then if the project fails the intermediary loses  $I_m + m$  (their investment and their monitoring cost). If the project succeeds the intermediary pockets net of their monitoring cost:

$$w_s = (1 - p) \frac{m}{\Delta} - m$$

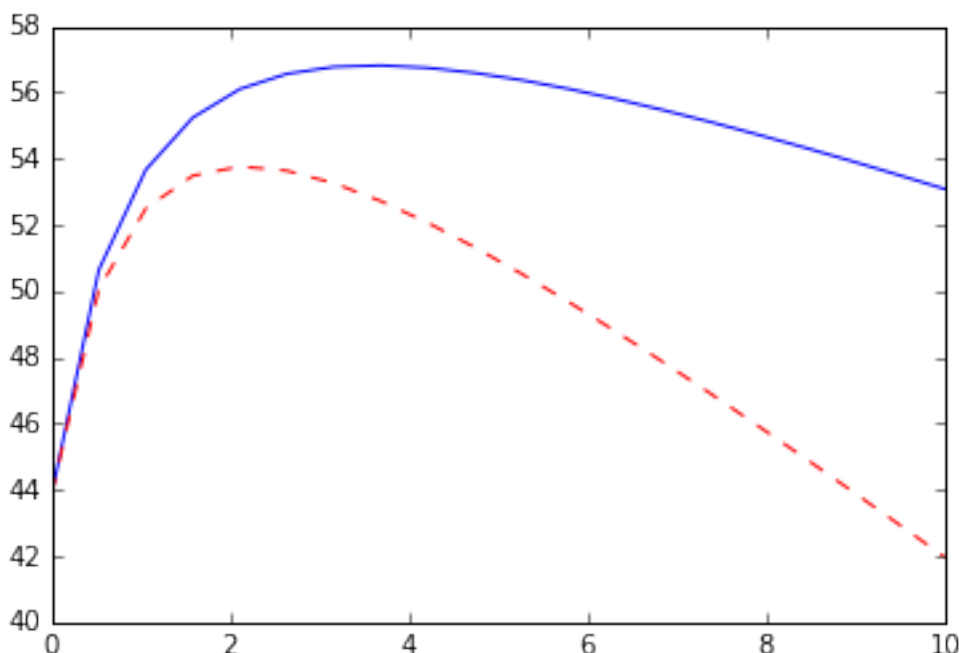
For a zero profit expected return.

In this last competitive scenario the cost to the Principal of adding the intermediary to the contract is just the monitoring cost  $m$  that must be compensated and not the larger information rent  $p \frac{m}{\Delta}$  that we saw in the non-competitive case.

The diagram below depicts net benefits under the competitive (solid line) and non-competitive (dashed line) scenarios:

```
In [13]: plt.plot(m, EX - I - p*(B/(1+m))/D - m)
         plt.plot(m, EX - I - p*(B/(1+m))/D - p*m/D, 'r--')
```

```
Out[13]: [<matplotlib.lines.Line2D at 0x7469160>]
```



In the competitive scenario more monitoring is employed but this monitoring is effective at bringing down the total cost of implementation, leaving more net project benefits. When the market for intermediaries is not competitive (and/or intermediaries are subject to limited liability constraints themselves) then monitoring still works (given our parameter assumptions in this example) but more of the project returns must be used for costly contractual bonus payments to motivate both the agent and the intermediary. Less monitoring will be employed and the net project returns will be reduced

Keep in mind that net project returns could be even larger if agents themselves could be made to post bond (credibly pay fines) in the event of project failure. Then no monitoring at all would be required and the agent could be paid the reservation wage (of zero in this example) and maximum project returns of  $E(X|p) - I$  or

```
In [14]: EX - I
```

```
Out[14]: 65.0
```

Could be achieved.

### 1.13.3 Extensions

This is the type of model that made Jean-Jacques Laffont and Jean Tirole famous. See for example their book on *A Theory of Incentives in Procurement and Regulation*. Jean Tirole powers much of his other important book *Modern Corporate Finance* with similar double-moral hazard models as well (and come to think of it his famous *The Theory of Industrial Organization* features this type of structure as well. Many problems of fiscal federalism with Central, State and Local governments can also be modeled this way.

Holmstrom and Tirole (1993) and Conning (1997) working with variations on an idea first introduced by Diamond (1984) note that if the intermediary monitors several independent (or at least not perfectly correlated) projects then the problem is like a multi-task principal - agent problem (except here between the principal and the intermediary) and it turns out that one can reduce the money that the intermediary has to put at risk in the event of failure by making their 'bonus' reward depend on success across several projects. For example if the intermediary is monitoring two agents on projects with independent projects it may be cheaper to pay zero or low compensation to the intermediary when either or both projects fails but a healthy bonus if they both succeed, then to structure the two remuneration projects separately.

A variation on the above idea is also one way to understand joint liability contracts and ‘peer monitoring’ although in this case the problem is a bit more tricky. For example suppose that we had two projects and two agents and we engaged each agent to work on their own project (say building bridges as described above) and also as a monitor of the other agent. The challenge is to then design a contract that then induces the two agents to choose effort on their bridge building and monitoring of the other agent strategically. This can be solved as a mechanism design problem: the principal determines the terms of a joint liability contract implement the desired levels of monitoring and project effort as the solution to a Nash-game played between the agents, guarding against the possibility that the agents might collude with each other against the principal.

Depending on the assumptions one makes about monitoring technologies and credible enforcement within groups, one can derive several interesting results that are relevant to understanding group versus individual liability loans as well as questions as to whether it might be preferable to monitor projects subject to corruption with ‘community’ monitoring or outside monitoring.

Local community agents may have better information or monitoring technologies which might appear to make them better monitors, but they may have fewer resources to put up as bond and/or they may be more likely to ‘collude’ against the outside principal.

In [ ] :

## 1.14 Research Discontinuity Designs (in R)

This is a jupyter notebook with an [R kernel](#) running in the background to execute R code.

The content below draws heavily upon

- Cattaneo, Idrobo and Titiunik (2008) “A Practical Introduction to Regression Discontinuity Designs: Part I,” in *Cambridge Elements: Quantitative and Computational Methods for Social Science*, Cambridge University Press. See also “Part II” paper.
- Meyersson (2014): [Islamic Rule and the Empowerment of the Poor and Pious](#), *Econometrica* 82(1): 229-269.

Links to these papers and data and Stata and R code for replication at [RD Software Packages site](#).

### 1.14.1 Sharp RD

Assignment to treatment status is a deterministic and discontinuous function of a covariate (running variable or forcing variable)  $X_i$ .

$$D_i = \begin{cases} 1 & \text{if } X_i \geq X_0 \\ 0 & \text{if } X_i < X_0 \end{cases}$$

where  $X_0$  is a threshold or cutoff.

**Example:** National Merit scholarship awarded to all students with PSAT score above some threshold.

$Y_i(1)$  and  $Y_i(0)$  are potential outcomes observed under treatment or control, respectively.  $E[Y_i(1)|X_i]$  observed only to right of cutoff.  $E[Y_i(0)|X_i]$  only to left.

Treatment effect estimated as

$$\tau_{SRD} = E[Y_i(1) - Y_i(0)|X_i = X_0]$$

Local treatment effect at/near the cutoff. In practice SRD estimate estimated as difference between weighted average of outcomes on the either side of the discontinuity.

Conditional independence:

$$E[Y_i(0)|X_i, D_i] = E[Y_i(0)|X_i]$$

Figure 2.1: Conditional Probability of Receiving Treatment in Sharp RD Design

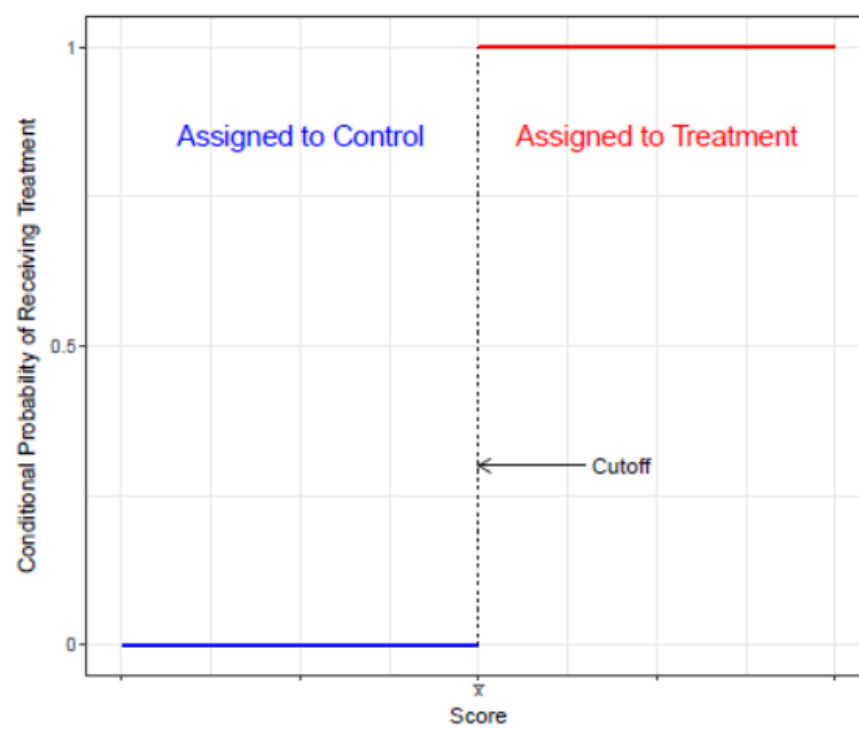


Fig. 1: sharpRD

Figure 2.2: RD Treatment Effect in Sharp RD Design

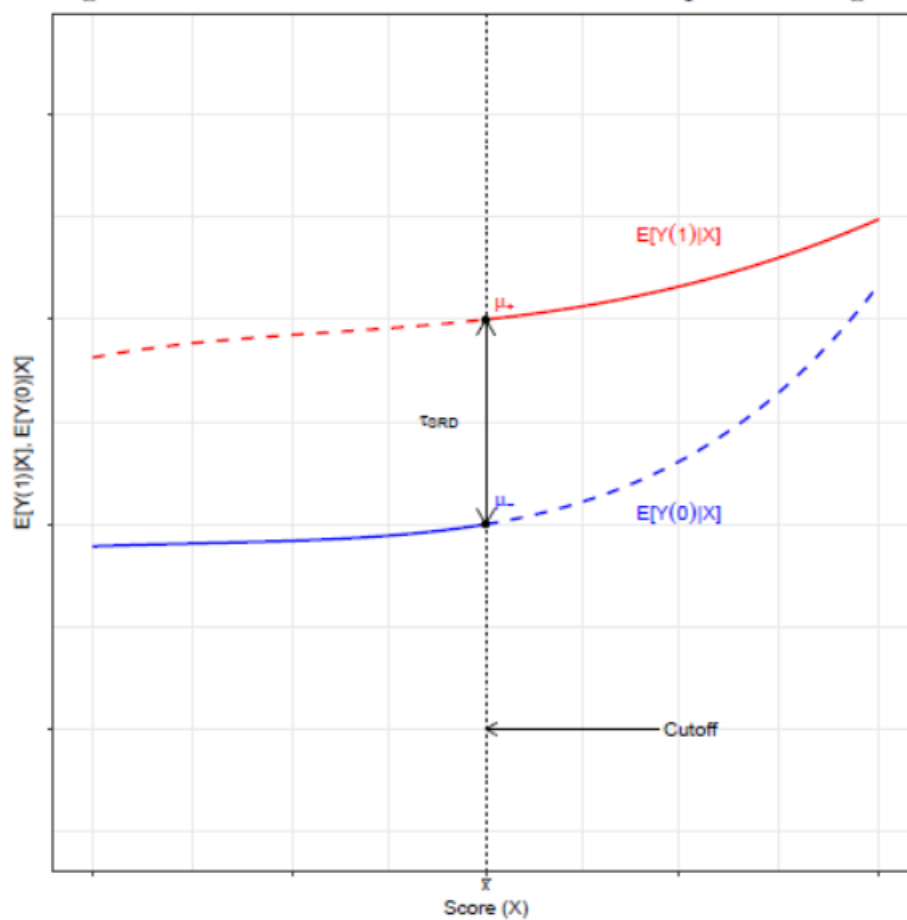


Fig. 2: sharpRD2

once we control for cofounder  $X_i$ , treatment assignment as good as random.

For observations very close to the discontinuity we effectively have an experiment.

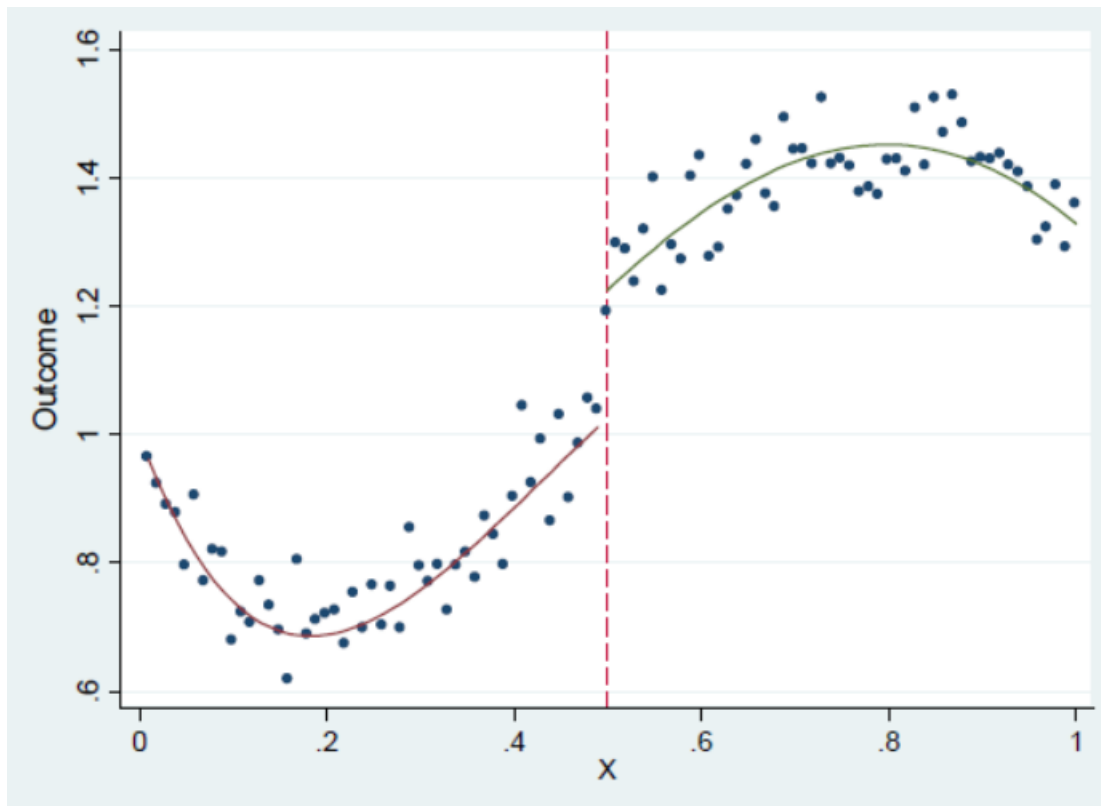


Fig. 3: rdd1

Non-linearity must not be mistaken for a discontinuity. In this example with linear fits on each side, estimate a positive ‘treatment’ effect. More likely relationship is non-linear with zero treatment effect.

### 1.14.2 Meyersson (2014)

### 1.14.3 Identifying impacts of municipal election victories by Islamic political parties on educational attainment of women

$X$  - running variable

$Y$  - outcome women’s high-school completion rate

$T$  - treatment Islamic party comes to power in municipal government

```
In [41]: #control the size of R plots in jupyter
options(repr.plot.width=5, repr.plot.height=5)

In [42]: # Load R libraries
library(foreign)
library(ggplot2)
library(lpdensity)
library(rddensity)
library(rdrobust)
```

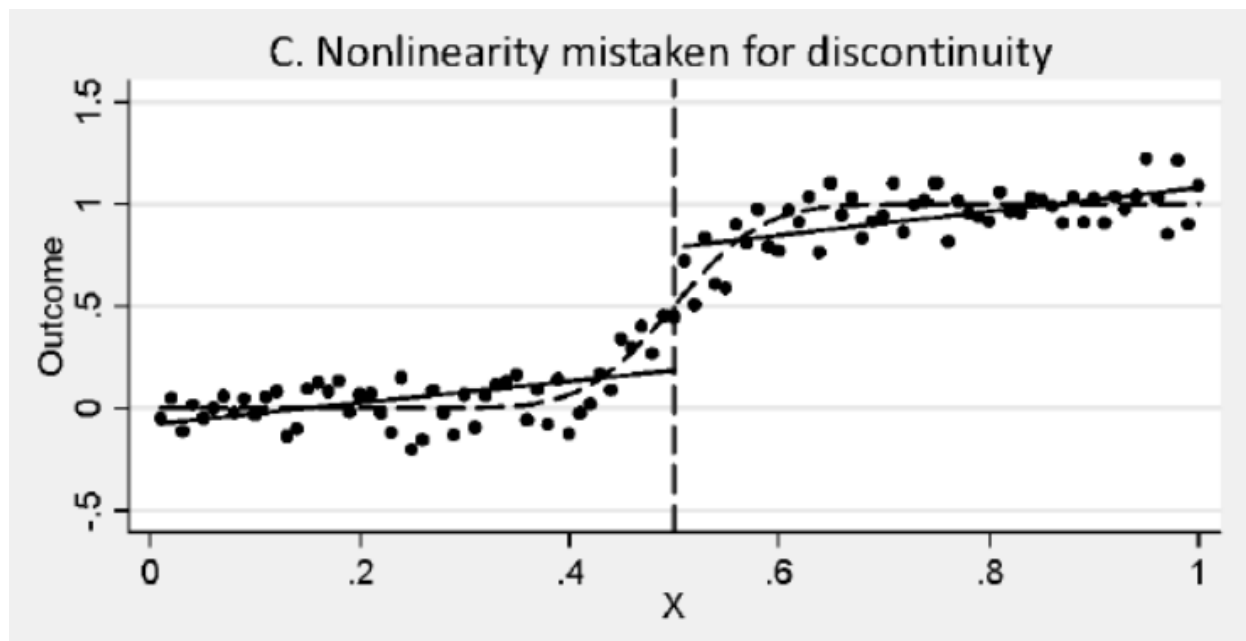


Fig. 4: Non Linear relationship

```
library(rdlocrand)
library(TeachingDemos)
```

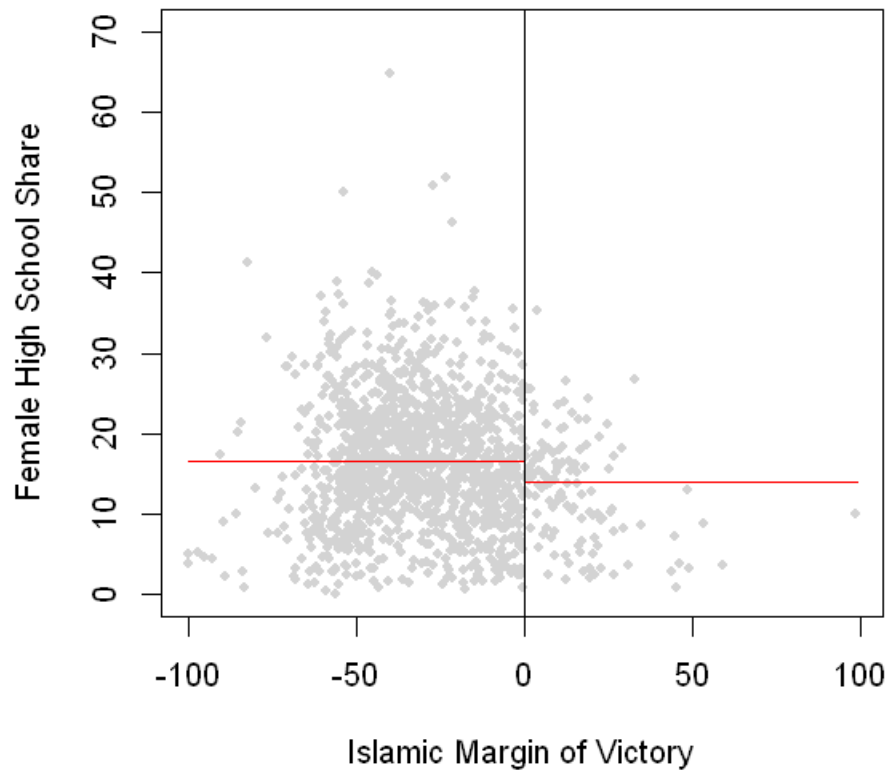
### R Code

```
In [43]: data = read.dta("CIT_2018_Cambridge_polecon.dta")
In [44]: Y = data$Y
         X = data$X
         T = data$T
         T_X = T*X
```

### Raw Comparison of means (Figure 2.3a)

If we simply compare outcomes in treated (Islamic major) and non-treated areas (secular major), we find lower female educational attainment in areas where Islamic parties won.

```
In [45]: rdplot(Y, X, nbins = c(2500, 500), p = 0, col.lines = "red", col.dots = "lightgray", title =
         x.label = "Islamic Margin of Victory", y.label = "Female High School Share", y.lim =
```



### Local comparison of means: Narrowing the bandwidth

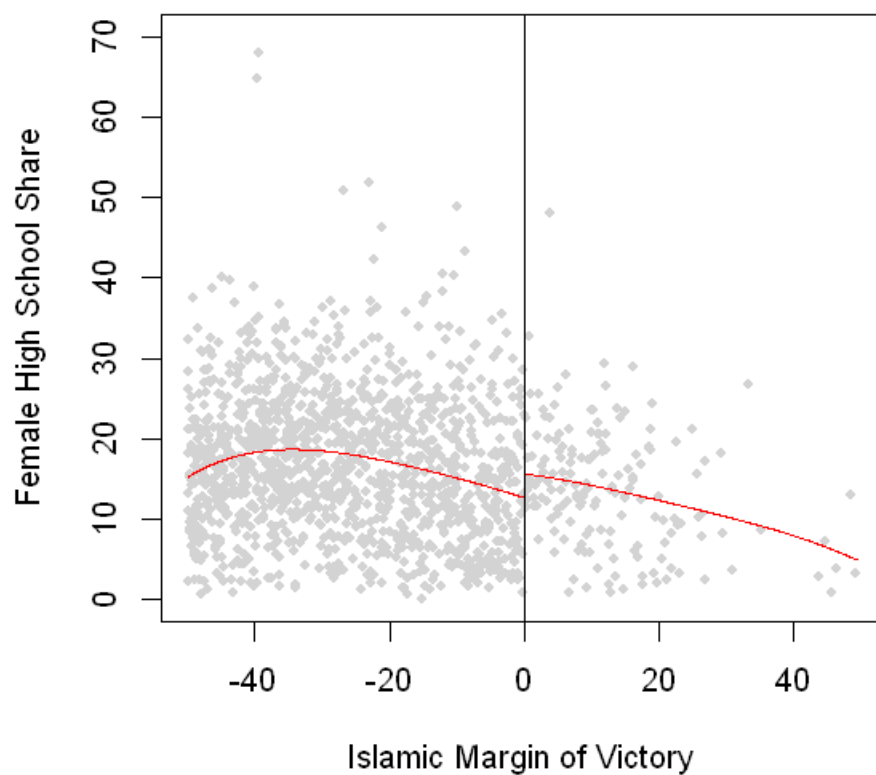
In last plot we compared mean value in treated and control group. But these two groups differ considerably.

Below we focus on more ‘local’ effects (closer to just above and below the cutoff).

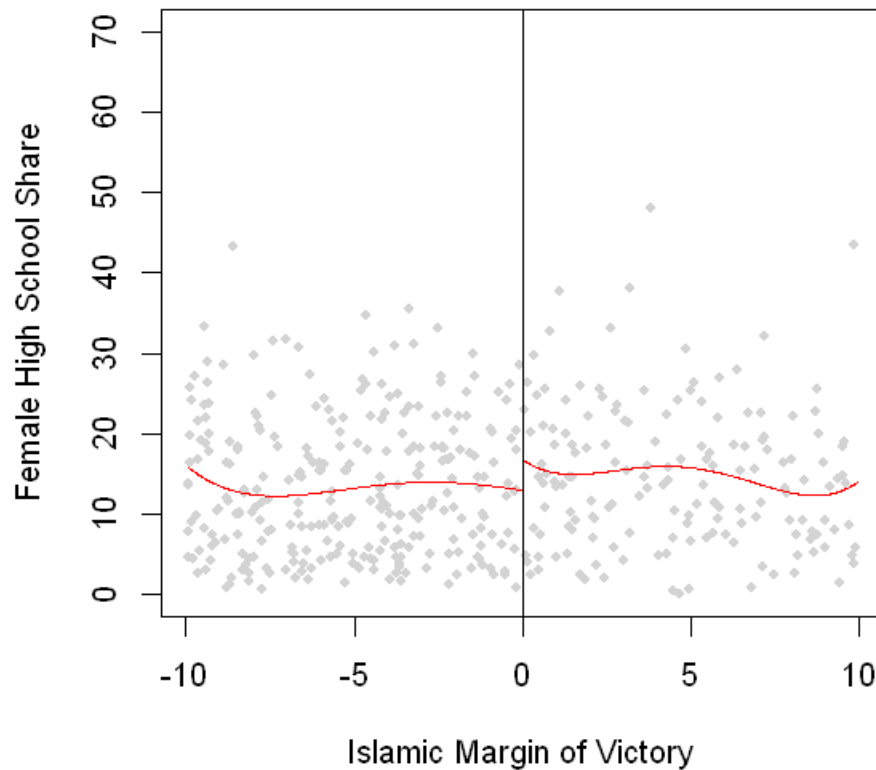
In next plot we fit a 4th degree polynomial on each side but now limit bandwidth to ‘closer’ contests where absolute victory margin was within 50 points.

```
In [46]: rdplot(Y[abs(X) <= 50], X[abs(X) <= 50], nbins = c(2500, 500), p = 4, col.lines = "red",
               col.dots = "lightgray", title = "", x.label = "Islamic Margin of Victory",
               y.label = "Female High School Share", y.lim = c(0,70))
```





```
In [47]: rdplot(Y[abs(X) <= 10], X[abs(X) <= 10], nbins = c(2500, 500), p = 4, col.lines = "red", col.points = "grey",  
               x.label = "Islamic Margin of Victory", y.label = "Female High School Share", y.lim = c(0, 70))
```



### RD plots

- scatter plots (as above) make it hard to discern ‘jumps’
- RD plots aggregate or ‘smooth’ data before plotting.
  - local sample means (from bins) represented by dots
  - polynomial (or local linear) fits

Plenty of variation in bin and bandwidth selection and curve fitting based on these concepts.

```
In [48]: rdplot(Y, X, nbins = c(20,20), binselect = 'esmv', x.label = 'Running Variable', y.label =  
          title = '', y.lim = c(0,25))
```

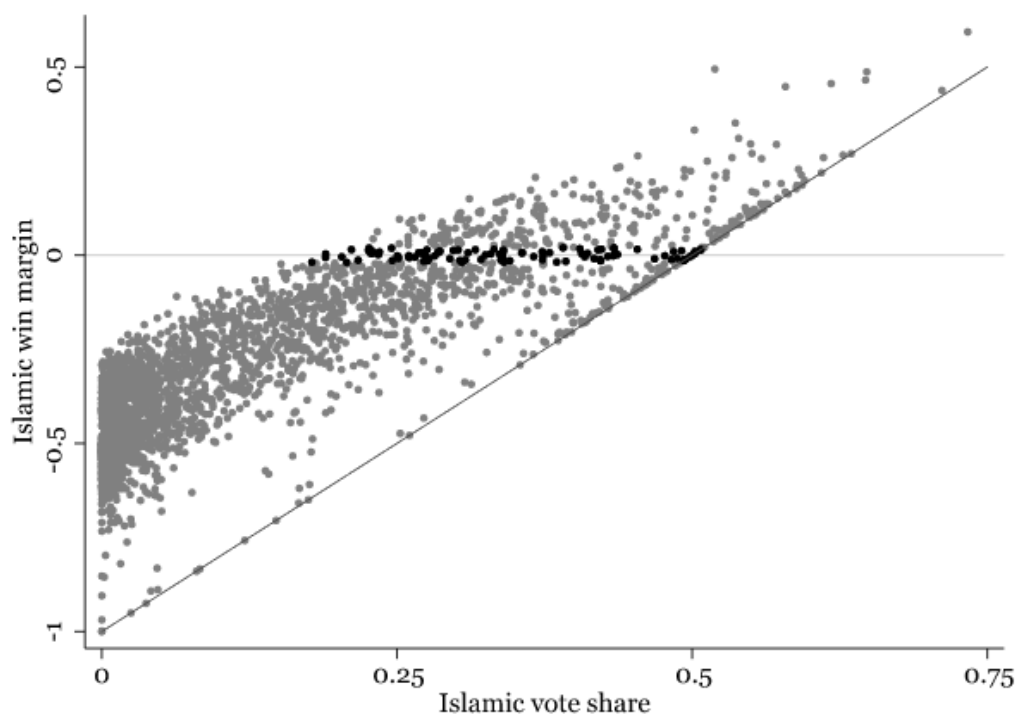
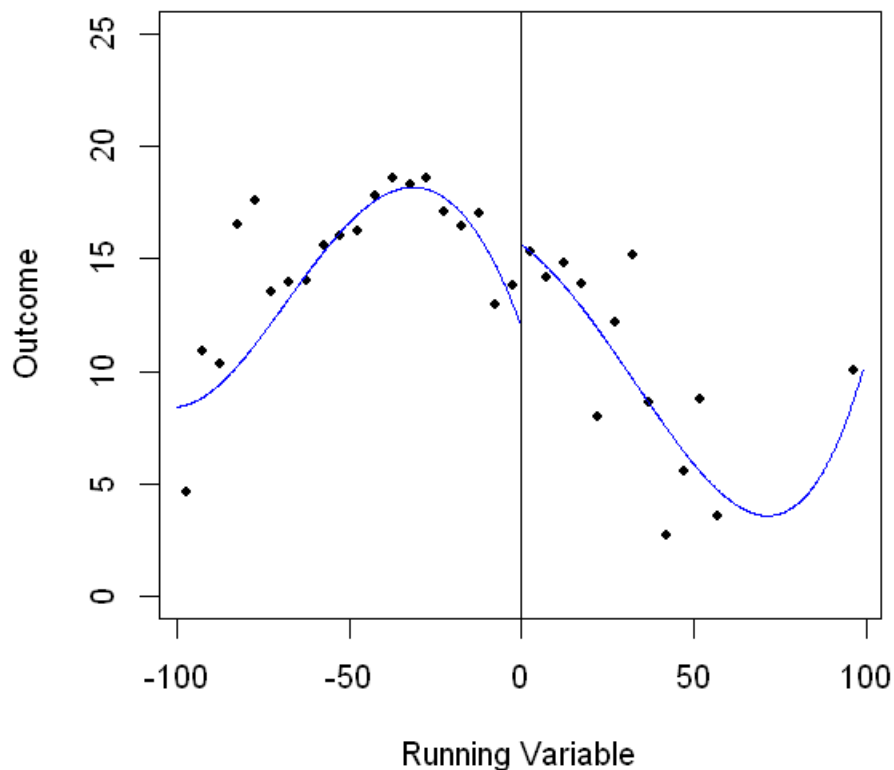


FIGURE 1.—Islamic win margin and Islamic vote share in 1994. The graph shows the total vote share for the Islamic party plotted against the Islamic win margin—the difference between the Islamic party’s vote share and the largest secular party’s vote share—both in 1994. Observations within 2 percentage points of the threshold at zero are in black. The diagonal line is the hypothetical one-to-one relationship between the two variables in an election with only two parties.

Fig. 5: Meyersson



The estimating equation:

$$Y_i(0) = f(X_i) + \epsilon_i$$

$$Y_i(1) = Y_i(0) + \beta$$

All in one:

$$Y_i = f(X_i) + \beta D_i + \epsilon_i$$

The function  $f(X_i)$  (e.g. a polynomial) must be continuous at  $X_0$

The RD estimate is difference between weighted average of outcomes on the either side of the discontinuity. Fitting a high order polynomial can mean these weighted averages are driven by observations far from the threshold.

Parametric (polynomial on full data) or non-parametric (local regression closer to cutoff) approaches?

Andrew Gelman's [blog post from 8/13](#) discusses this issue. See also [Development Impact Blog](#)

Chen et al (2013) "Evidence on the impact of sustained exposure to air pollution on life expectancy from China's Huai River policy," *PNAS*.

**Policy discontinuity:** North of China's Huai river free coal for heating is distributed in winter. None to the south. Using RDD method find total suspended particulates (TSPs) air pollution 55% higher just North of river compared to just south. Estimates China's coal-burning was reducing lifespan by 5 years for half a billion people.

Are observations far from threshold affecting polynomial fit, driving results? Smaller estimates with linear fits?

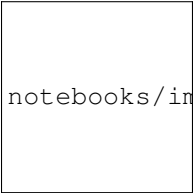


notebooks/images/ChinaRDD.png



notebooks/images/ChinaRDD2.PNG

## Fuzzy RD



notebooks/images/fuzzy.PNG

A fuzzy RD is like an RD with imperfect compliance or non-compliance. Assignment to Treatment is as before summarized by indicator  $I(X_i \geq X_0)$  but compliance is now imperfect. Like an Intent-to-Treat (ITT) estimator, the measured jump at the cutoff in the outcome needs to be rescaled.

Structural equation:  $Y_i = f(X_i) + \beta D_i + \epsilon_i$

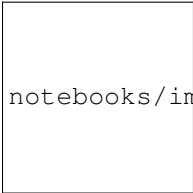
2nd stage reduced form:  $Y_i = f(X_i) + \pi_2 I(X_i \geq X_0) + \xi_{2i}$

First stage:  $D_i = g(X_i) + \pi_1 I(X_i \geq X_0) + \xi_{1i}$

Standard IV:  $\beta =$

## Multi-Score RD design

Example: Third graders have to go to summer school if score below  $X_{10}$  in reading test OR below  $X_{20}$  in math test. Does summer school raise test performance next year?



notebooks/images/2d\_distance.PNG

From Zamosc 2012

### 1.14.4 Polynomial Fit in 2 dimensions

With two forcing variables - The cutoff is now a boundary - Heterogenous effects, depending on what segment of the boundary - with right assumptions (lots of data), can identify one treatment effect for every point b on boundary. Treatment effect curve  $\tau(b)$  - Average Treatment Effect (ATE) would average over boundary

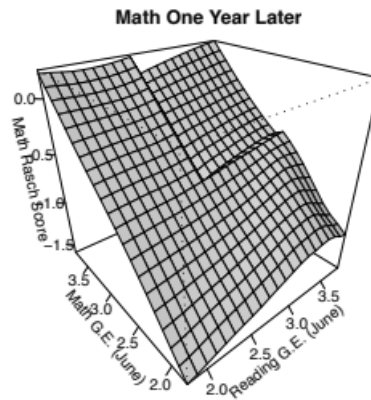
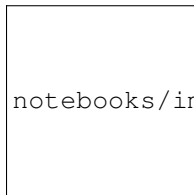


Fig. 6: 2d polyfit

### Geographic RD designs

- Latitude, Longitude forcing variables. Collapse to one-dimensional distance (e.g. ‘distance to border’) problematic.



notebooks/images/2d\_distance.PNG

Keele & Titiunik (2014)

### Application:

#### 1.14.5 Dell (2010) Persistent Effects of Peru’s Mining *Mita*

Dell, Melissa. 2010. “The Persistent Effects of Peru’s Mining ‘Mita.’” *Econometrica* 78 (6): 1863–1903.

Continue to slide notes.

## 1.15 Data APIs and pandas operations

Several of the notebooks we’ve already explored loaded datasets into a python pandas dataframe for analysis. Local copies of some of these datasets had been previously saved to disk in a few cases we read in the data directly from an online sources via a data API. This section explains how that is done in a bit more depth. Some of the possible advantages of reading in the data this way is that it allows would-be users to modify and extend the analysis, perhaps focusing on different time-periods or adding in other variables of interest.

Easy to use python wrappers for data APIs have been written for the World Bank and several other online data providers (including FRED, Eurostat, and many many others). The [pandas-datareader](#) library allows access to several databases from the [World Bank’s datasets](#) and other sources.

If you haven’t already installed the pandas-datareader library you can do so directly from a jupyter notebook code cell:

```
!pip install pandas-datareader
```

Once the library is in installed we can load it as:

```
In [1]: %matplotlib inline
import seaborn as sns
import warnings
import numpy as np
import statsmodels.formula.api as smf
import datetime as dt

In [2]: from pandas_datareader import wb
```

### 1.15.1 Data on urban bias

Our earlier analysis of the Harris\_Todaro migration model suggested that policies designed to favor certain sectors or labor groups

Let's search for indicators (and their identification codes) relating to GDP per capita and urban population share. We could look these up in a book or from the website <http://data.worldbank.org/> but we can also search for keywords directly.

First lets search for series having to do with gdp per capita

```
In [3]: wb.search('gdp.*capita.*const')[['id','name']]

Out[3]:
```

	id	name
685	6.0.GDPpc_constant	GDP per capita, PPP (constant 2011 internation...
7588	NY.GDP.PCAP.KD	GDP per capita (constant 2010 US\$)
7590	NY.GDP.PCAP.KN	GDP per capita (constant LCU)
7592	NY.GDP.PCAP.PP.KD	GDP per capita, PPP (constant 2011 internation...

We will use NY.GDP.PCAP.KD for GDP per capita (constant 2010 US\$).

You can also first browse and search for data series from the World Bank's DataBank page at <http://databank.worldbank.org/data/>. Then find the 'id' for the series that you are interested in in the 'metadata' section from the webpage

Now let's look for data on urban population share:

```
In [4]: wb.search('Urban Population')[['id','name']].tail()

Out[4]:
```

	id	name
9999	SP.URB.GROW	Urban population growth (annual %)
10000	SP.URB.TOTL	Urban population
10001	SP.URB.TOTL.FE.ZS	Urban population, female (% of total)
10002	SP.URB.TOTL.IN.ZS	Urban population (% of total)
10003	SP.URB.TOTL.MA.ZS	Urban population, male (% of total)

Let's use the ones we like but use a python dictionary to rename these to shorter variable names when we load the data into a python dataframe:

```
In [5]: indicators = ['NY.GDP.PCAP.KD', 'SP.URB.TOTL.IN.ZS']
```

Since we are interested in exploring the extent of 'urban bias' in some countries, let's load data from 1980 which was toward the end of the era of import-substituting industrialization when urban-biased policies were claimed to be most pronounced.

```
In [6]: dat = wb.download(indicator=indicators, country = 'all', start=1980, end=1980)
```

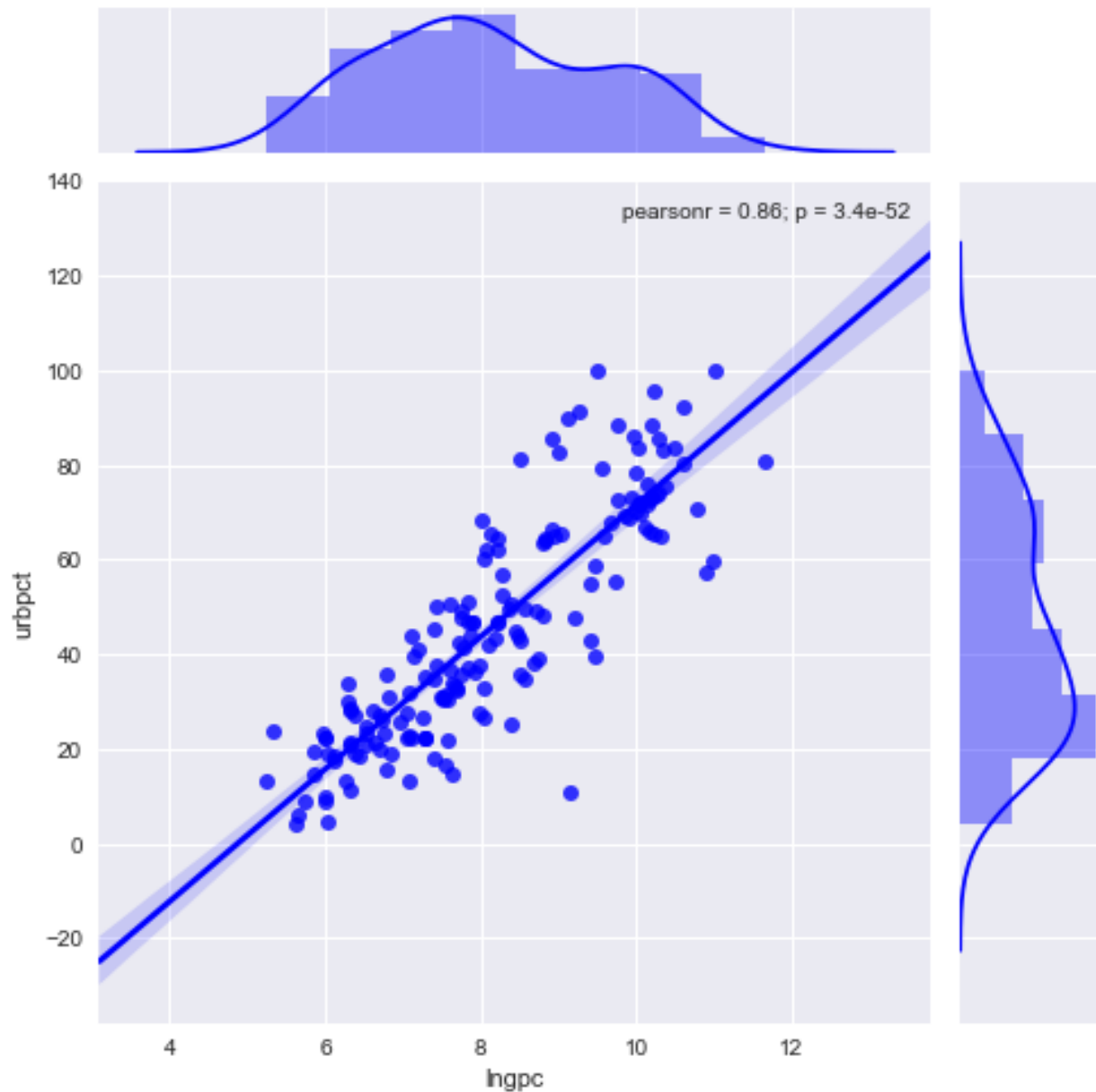
```
In [7]: dat.columns
```

```
Out[7]: Index(['NY.GDP.PCAP.KD', 'SP.URB.TOTL.IN.ZS'], dtype='object')
```

Let's rename the columns to something shorter and then plot and regress log gdp per capita against urban extent we get a pretty tight fit:

```
In [8]: dat.columns = [['gdppc', 'urbpct']]
        dat['lngpc'] = np.log(dat.gdppc)

In [9]: g = sns.jointplot("lngpc", "urbpct", data=dat, kind="reg",
                          color="b", size=7)
```



That is a pretty tight fit: urbanization rises with income per-capita, but there are several middle income country outliers that have considerably higher urbanization than would be predicted. Let's look at the regression line.

```
In [10]: mod = smf.ols("urbpct ~ lngpc", dat).fit()
In [11]: print(mod.summary())
```

```

                        OLS Regression Results
=====
Dep. Variable:          urbpct      R-squared:          0.744
Model:                  OLS        Adj. R-squared:       0.743
Method:                 Least Squares  F-statistic:       494.3
```



```

Date:                Wed, 03 May 2017    Prob (F-statistic):      3.43e-52
Time:                15:26:52           Log-Likelihood:         -670.94
No. Observations:    172                AIC:                   1346.
Df Residuals:        170                BIC:                   1352.
Df Model:            1
Covariance Type:     nonrobust

```

```

=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept    -68.0209      5.185    -13.120      0.000    -78.255    -57.787
lngpc         13.9583      0.628     22.233      0.000     12.719     15.198
=====
Omnibus:            8.555    Durbin-Watson:           2.204
Prob(Omnibus):      0.014    Jarque-Bera (JB):        17.062
Skew:               -0.027    Prob(JB):               0.000197
Kurtosis:           4.542    Cond. No.                47.3
=====

```

Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Now let's just look at a list of countries sorted by the size of their residuals in this regression line. Countries with the largest residuals had urbanization in excess of what the model predicts from their 1980 level of income per capita.

Here is the sorted list of top 15 outliers.

```
In [12]: mod.resid.sort_values(ascending=False).head(15)
```

```

Out[12]: country      year
Singapore    1980    35.469714
Chile        1980    30.563014
Malta        1980    30.448467
Hong Kong SAR, China  1980    29.957702
Uruguay      1980    29.143223
Argentina    1980    25.369855
Cuba         1980    24.148529
Belgium      1980    20.731859
Israel       1980    20.456625
Iraq         1980    20.262678
Peru         1980    17.810693
Myanmar      1980    17.719561
Bulgaria     1980    17.360914
Jordan       1980    15.706878
Colombia     1980    15.258744
dtype: float64

```

This is of course only suggestive but (leaving aside the island states like Singapore and Hong-Kong) the list is dominated by southern cone countries such as Chile, Argentina and Peru which in addition to having legacies of heavy political centralization also pursued ISI policies in the 60s and 70s that many would associate with urban biased policies.

## Panel data

Very often we want data on several indicators and a whole group of countries over a number of years. we could also have used datetime format dates:

```

In [13]: countries = ['CHL', 'USA', 'ARG']
         start, end = dt.datetime(1950, 1, 1), dt.datetime(2016, 1, 1)
         dat = wb.download(
             indicator=indicators,

```

```
country = countries,
start=start,
end=end).dropna()
```

Lets use shorter column names

```
In [14]: dat.columns
```

```
Out[14]: Index(['NY.GDP.PCAP.KD', 'SP.URB.TOTL.IN.ZS'], dtype='object')
```

```
In [15]: dat.columns = [['gdppc', 'urb']]
```

```
In [16]: dat.head()
```

```
Out[16]:
```

		gdppc	urb
country	year		
Argentina	2015	10501.660269	91.751
	2014	10334.780146	91.604
	2013	10711.229530	91.452
	2012	10558.265365	91.295
	2011	10780.342508	91.133

Notice this has a two-level multi-index. The outer level is named ‘country’ and the inner level is ‘year’

We can pull out group data for a single country like this using the `.xs` or cross section method.

```
In [17]: dat.xs('Chile', level='country').head(3)
```

```
Out[17]:
```

	gdppc	urb
year		
2015	14660.505335	89.530
2014	14479.763258	89.356
2013	14364.140970	89.175

(Note we could have also used `dat.loc['Chile'].head()`)

And we can pull a ‘year’ level cross section like this:

```
In [18]: dat.xs('2007', level='year').head()
```

```
Out[18]:
```

	gdppc	urb
country		
Argentina	9830.759871	90.445
Chile	12223.484611	87.926
United States	49979.533843	80.269

Note that what was returned was a dataframe with the data just for our selected country. We can in turn further specify what column(s) from this we want:

```
In [19]: dat.loc['Chile']['gdppc'].head()
```

```
Out[19]: year
2015    14660.505335
2014    14479.763258
2013    14364.140970
2012    13963.665402
2011    13385.131216
Name: gdppc, dtype: float64
```

## Unstack data

The `unstack` method turns index values into column names while `stack` method converts column names to index values. Here we apply `unstack`.

```
In [20]: datyr = dat.unstack(level='country')
         datyr.head()
```

```
Out[20]:
```

	gdppc			urb	
country	Argentina	Chile	United States	Argentina	Chile
year					
1960	5605.191722	3630.391789	17036.885170	73.611	67.836
1961	5815.233002	3692.112435	17142.193767	74.217	68.660
1962	5675.060043	3796.728684	17910.278790	74.767	69.435
1963	5290.764999	3939.263166	18431.158404	75.309	70.200
1964	5738.598474	3955.026738	19231.171859	75.844	70.955

```
country United States
year
1960          69.996
1961          70.377
1962          70.757
1963          71.134
1964          71.508
```

We can now easily index a 2015 cross-section of GDP per capita like so:

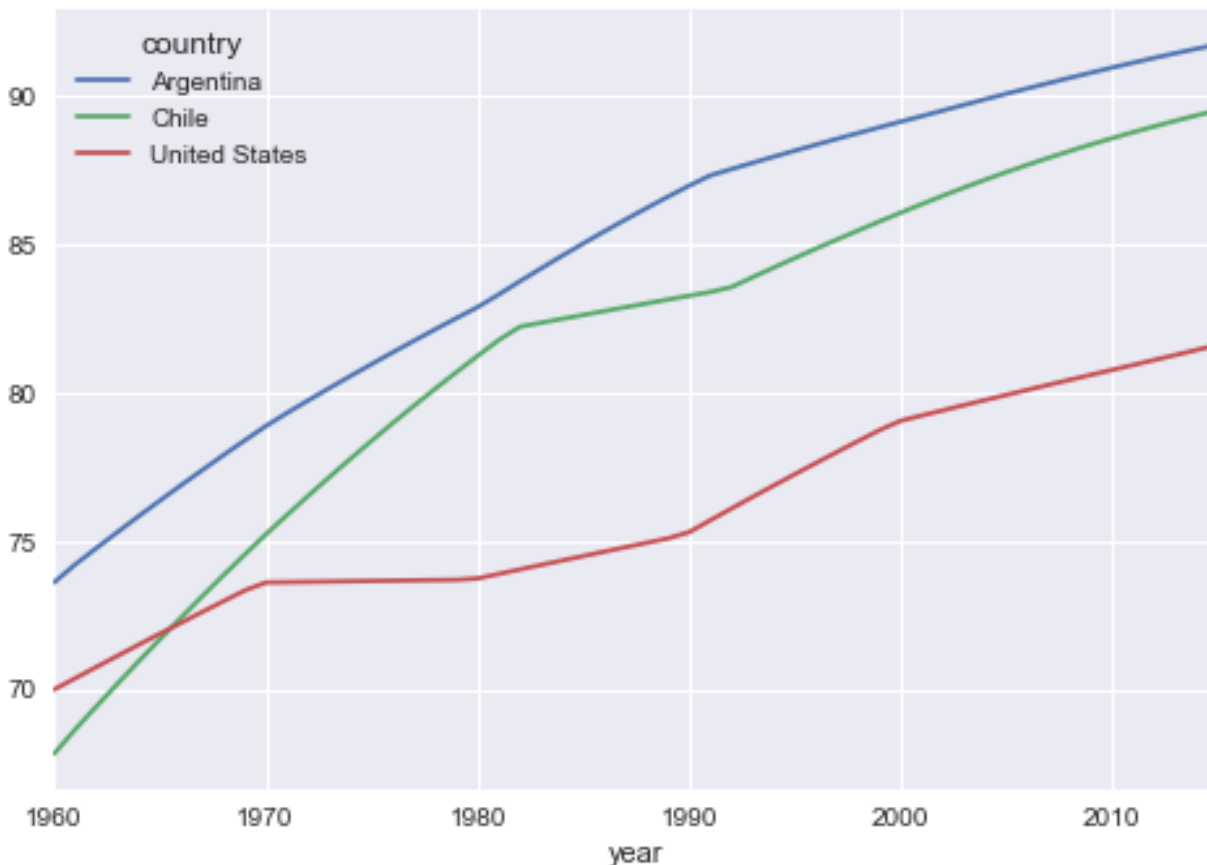
```
In [21]: datyr.xs('1962')['gdppc']
```

```
Out[21]: country
         Argentina      5675.060043
         Chile        3796.728684
         United States  17910.278790
         Name: 1962, dtype: float64
```

We'd get same result from `datyr.loc['2015']['gdppc']`

We can also easily plot all countries:

```
In [22]: datyr['urb'].plot(kind='line');
```



## 1.16 Stata and R in a jupyter notebook

The jupyter notebook project is now designed to be a ‘language agnostic’ web-application front-end for any one of many possible software language kernels. We’ve been mostly using python but there are in fact several dozen [other language kernels](#) that can be made to work with it including Julia, R, Matlab, C, Go, Fortran and Stata.

The ecosystem of libraries and packages for scientific computing with python is huge and constantly growing but there are still many statistics and econometrics applications that are available as built-in or user-written modules in Stata that have not yet been ported to python or are just simply easier to use in Stata. On the other hand there are some libraries such as python pandas and different visualization libraries such as seaborn or matplotlib that give features that are not available in Stata.

Fortunately you don’t have to choose between using Stata or python, you can use them both together, to get the best of both worlds.

## 1.17 Jupyter running an R kernel

R is a powerful open source software environment for statistical computing. R has [R markdown](#) which allows you to create R-markdown [notebooks](#) similar in concept to jupyter notebooks. But you can also run R inside a jupyter notebook (indeed the name ‘Jupyter’ is from **J**ulia, **iP**ython and **R**).

See my notebook with [notes on Research Discontinuity Design](#) for an example of a jupyter notebook running R. To install an R kernel see the [IRkernel](#) project.

## 1.18 Jupyter with a Stata Kernel

Kyle Barron has created a [stata\\_kernel](#) that offers several useful features including code-autocompletion, inline graphics, and generally fast responses.

For this to work you must have a working licensed copy of Stata version 14 or greater on your machine.

## 1.19 Python and Stata combined in the same notebook

Sometimes it may be useful to combine python and Stata in the same notebook. Ties de Kok has written a nice python library called [ipystata](#) that allows one to execute Stata code in codeblocks inside an ipython notebook when preceded by a `%%stata` magic command.

This workflow allows you to pass data between python and Stata sessions and to display Stata plots inline. Compared to the [stata\\_kernel](#) option the response times are not quite as fast.

The remainder of this notebook illustrates the use of ipystata.

### 1.19.1 A sample ipystata session

For more details see the [example notebook](#) and documentation on the ipystata repository.

```
In [1]: %matplotlib inline
import seaborn as sns
import pandas as pd
import statsmodels.formula.api as smf
import ipystata
```

The following opens a Stata session where we load a dataset and summarize the data. The `-o` flag following the `'%%Stata''` magic instructs it to output or return the dataset in Stata memory as a pandas dataframe in python.

```
In [2]: %%stata -o life_df
sysuse lifeexp.dta
summarize
```

(Life expectancy, 1998)

Variable	Obs	Mean	Std. Dev.	Min	Max
region	68	1.5	.7431277	1	3
country	0				
popgrowth	68	.9720588	.9311918	-.5	3
lexp	68	72.27941	4.715315	54	79
gnppc	63	8674.857	10634.68	370	39980
safewater	40	76.1	17.89112	28	100

Let's confirm the data was returned as a pandas dataframe:

```
In [3]: life_df.head(3)
```

```
Out[3]:
```

	region	country	popgrowth	lexp	gnppc	safewater
0	Eur & C.Asia	Albania	1.2	72	810.0	76.0
1	Eur & C.Asia	Armenia	1.1	74	460.0	NaN
2	Eur & C.Asia	Austria	0.4	79	26830.0	NaN

A simple generate variable command and ols regression in Stata:

```
In [4]: %%stata -o life_df
        gen lngnppc = ln(gnppc)
        regress lexp lngnppc
```

(5 missing values generated)

Source	SS	df	MS	Number of obs	=	63
Model	873.264865	1	873.264865	F(1, 61)	=	97.09
Residual	548.671643	61	8.99461709	Prob > F	=	0.0000
				R-squared	=	0.6141
				Adj R-squared	=	0.6078
Total	1421.93651	62	22.9344598	Root MSE	=	2.9991

lexp	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lngnppc	2.768349	.2809566	9.85	0.000	2.206542	3.330157
_cons	49.41502	2.348494	21.04	0.000	44.71892	54.11113

And the same regression using statsmodels and pandas:

```
In [5]: model = smf.ols(formula = 'lexp ~ lngnppc',
                        data = life_df)
        results = model.fit()
        print(results.summary())
```

```

                        OLS Regression Results
=====
Dep. Variable:          lexp      R-squared:                0.614
Model:                  OLS      Adj. R-squared:           0.608
Method:                 Least Squares      F-statistic:        97.09
Date:                   Tue, 08 May 2018    Prob (F-statistic):    3.13e-14
Time:                   14:25:48           Log-Likelihood:       -157.57
No. Observations:       63              AIC:                  319.1
Df Residuals:           61              BIC:                  323.4
Df Model:                1
Covariance Type:        nonrobust
=====
                        coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept              49.4150      2.348      21.041      0.000      44.719      54.111
lngnppc                 2.7683      0.281       9.853      0.000       2.207       3.330
=====
Omnibus:                21.505      Durbin-Watson:        1.518
Prob(Omnibus):           0.000      Jarque-Bera (JB):     46.140
Skew:                   -1.038      Prob(JB):              9.57e-11
Kurtosis:                6.643      Cond. No.              52.7
=====
```

Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

## Back to ipython

Let's change one of the variables in the dataframe in python:

```
In [6]: life_df.popgrowth = life_df.popgrowth * 100
```

```
In [7]: life_df.popgrowth.mean()
```

```
Out[7]: 97.20587921142578
```

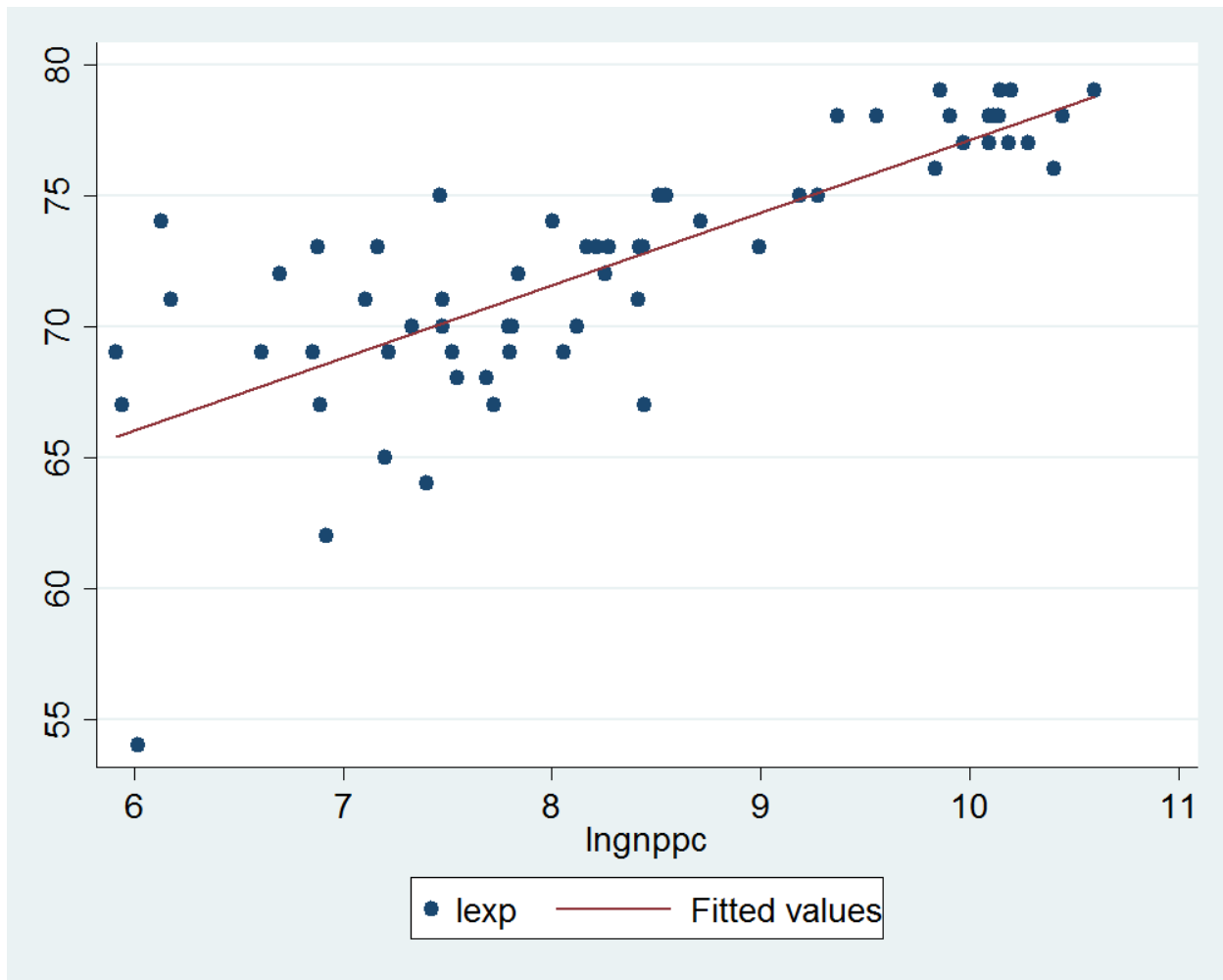
And now let's push the modified dataframe into the Stata dataset with the `-d` flag:

```
In [8]: %%stata -d life_df
        summarize
```

Variable	Obs	Mean	Std. Dev.	Min	Max
index	68	33.5	19.77372	0	67
region	68	.5	.7431277	0	2
country	0				
popgrowth	68	97.20588	93.11918	-50	300
lexp	68	72.27941	4.715315	54	79
gnppc	63	8674.857	10634.68	370	39980
safewater	40	76.1	17.89112	28	100
lngnppc	63	8.250023	1.355677	5.913503	10.59613

A Stata plot:

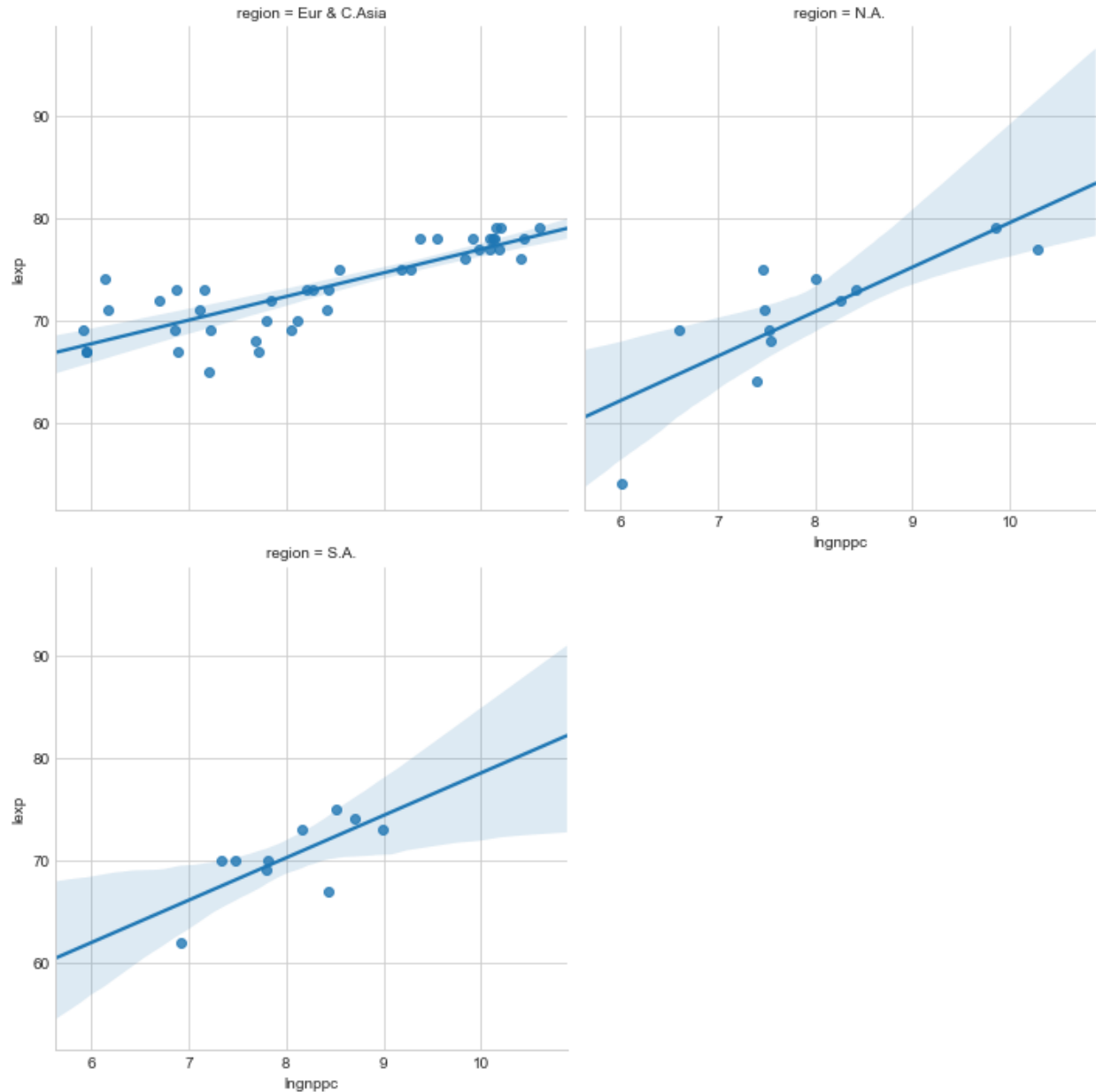
```
In [9]: %%stata -d life_df --graph
        graph twoway (scatter lexp lngnppc) (lfit lexp lngnppc)
```



Now on the python side use `lmplot` from the `seaborn` library to graph a similar scatter and fitted line but by region.

```
In [10]: sns.set_style("whitegrid")
          g=sns.lmplot(y='lnexp', x='lngnppc', col='region', data=life_df,col_wrap=2)
```





You can view the notebooks there or clone/download them to run them on a jupyter server on your own computer (see [how here](#)). The github content is also auto-rendereed as (1) a styled but static [webpages](#) and 2) a [PDF book](#) using Sphinx and the readthedocs service. The github repo is also cloned to a Microsoft [Azure notebook library](#) where you can run the notebooks on a cloud-based jupyter notebook server.

Some of this is still quite rough/preliminary content. Over time I'll try to edit and improve. If you spot errors or would like to suggest edits or contribute material please do so via the github repository (start an 'issue ticket'):