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# Sound Field Analysis Toolbox

*Release 474b106*

## SFA Toolbox Developers

Sep 22, 2017

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The Sound Field Analysis Toolbox for NumPy/Python provides implementations of various techniques for the analysis of sound fields and beamforming using microphone arrays.

**Source code and issue tracker:** <https://github.com/spatialaudio/sfa-numpy>

**License:** MIT – see the file LICENSE for details.

**Quick start:**

- Install NumPy, SciPy and for the examples Matplotlib
  - git clone <https://github.com/spatialaudio/sfa-numpy.git>
  - cd sfa-numpy
  - python setup.py install --user
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## 1 Usage

### 1.1 Requirements

Obviously, you'll need Python<sup>1</sup>. We normally use Python 3.x, but it *should* also work with Python 2.x. NumPy<sup>2</sup> and SciPy<sup>3</sup> are needed for the calculations. If you also want to plot the results in the examples, you'll need matplotlib<sup>4</sup>.

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<sup>1</sup> <http://www.python.org/>

<sup>2</sup> <http://www.numpy.org/>

<sup>3</sup> <http://www.scipy.org/scipylib/>

<sup>4</sup> <http://matplotlib.org/>

Instead of installing all of them separately, you should probably get a Python distribution that already includes everything, e.g. Anaconda<sup>5</sup>.

## 1.2 How to Get Started

Various examples are located in the examples directory.

# 2 Modal Beamforming

Submodules for modal beamforming

## 2.1 Angular

`micarray.modal.angular.sht_matrix(N, azi, elev, weights=None)`

Matrix of spherical harmonics up to order N for given angles.

Computes a matrix of spherical harmonics up to order  $N$  for the given angles/grid.

$$\mathbf{Y} = \begin{bmatrix} Y_0^0(\theta[0], \phi[0]) & Y_1^{-1}(\theta[0], \phi[0]) & Y_1^0(\theta[0], \phi[0]) & Y_1^1(\theta[0], \phi[0]) & \dots \\ Y_0^0(\theta[1], \phi[1]) & Y_1^{-1}(\theta[1], \phi[1]) & Y_1^0(\theta[1], \phi[1]) & Y_1^1(\theta[1], \phi[1]) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_0^0(\theta[Q-1], \phi[Q-1]) & Y_1^{-1}(\theta[Q-1], \phi[Q-1]) & Y_1^0(\theta[Q-1], \phi[Q-1]) & Y_1^1(\theta[Q-1], \phi[Q-1]) & \dots \end{bmatrix} Y_N$$

where

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi}} \frac{(n-m)!}{(n+m)!} P_n^m(\cos \theta) e^{im\phi}$$

### Parameters

- `N (int)` – Maximum order.
- `azi ((Q,) array_like)` – Azimuth.
- `elev ((Q,) array_like)` – Elevation.
- `weights ((Q,) array_like, optional)` – Quadrature weights.

**Returns** `Ymn (((N+1)**2, Q) numpy.ndarray)` – Matrix of spherical harmonics.

`micarray.modal.angular.Legendre_matrix(N, ctheta)`

Matrix of weighted Legendre Polynomials.

Computes a matrix of weighted Legendre polynomials up to order N for the given angles

$$L_n(\theta) = \frac{2n+1}{4\pi} P_n(\theta)$$

### Parameters

- `N (int)` – Maximum order.
- `ctheta ((Q,) array_like)` – Angles.

**Returns** `Lmn (((N+1), Q) numpy.ndarray)` – Matrix containing Legendre polynomials.

`micarray.modal.angular.grid_equal_angle(n)`

Equi-angular sampling points on a sphere.

According to (cf. Rafaely book, sec.3.2)

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<sup>5</sup> <http://docs.continuum.io/anaconda/>

**Parameters** `n` (*int*) – Maximum order.

**Returns**

- `azi` (*array\_like*) – Azimuth.
- `elev` (*array\_like*) – Elevation.
- `weights` (*array\_like*) – Quadrature weights.

`micarray.modal.angular.grid_gauss(n)`

Gauss-Legendre sampling points on sphere.

According to (cf. Rafaely book, sec.3.3)

**Parameters** `n` (*int*) – Maximum order.

**Returns**

- `azi` (*array\_like*) – Azimuth.
- `elev` (*array\_like*) – Elevation.
- `weights` (*array\_like*) – Quadrature weights.

## 2.2 Radial

`micarray.modal.radial.spherical_pw(N, k, r, setup)`

Radial coefficients for a plane wave.

Computes the radial component of the spherical harmonics expansion of a plane wave impinging on a spherical array.

$$\hat{P}_n(k) = 4\pi i^n b_n(kr)$$

**Parameters**

- `N` (*int*) – Maximum order.
- `k` ((*M*,) *array\_like*) – Wavenumber.
- `r` (*float*) – Radius of microphone array.
- `setup` ({‘open’, ‘card’, ‘rigid’}) – Array configuration (open, cardioids, rigid).

**Returns** `bn` ((*M*, *N+1*) *numpy.ndarray*) – Radial weights for all orders up to N and the given wavenumbers.

`micarray.modal.radial.spherical_ps(N, k, r, rs, setup)`

Radial coefficients for a point source.

Computes the radial component of the spherical harmonics expansion of a point source impinging on a spherical array.

$$\hat{P}_n(k) = 4\pi(-i)kh_n^{(2)}(kr_s)b_n(kr)$$

**Parameters**

- `N` (*int*) – Maximum order.
- `k` ((*M*,) *array\_like*) – Wavenumber.
- `r` (*float*) – Radius of microphone array.
- `rs` (*float*) – Distance of source.
- `setup` ({‘open’, ‘card’, ‘rigid’}) – Array configuration (open, cardioids, rigid).

**Returns** `bn` ((*M*, *N+1*) *numpy.ndarray*) – Radial weights for all orders up to N and the given wavenumbers.

```
micarray.modal.radial.weights (N, kr, setup)
```

Radial weighing functions.

Computes the radial weighting functions for different array types (cf. eq.(2.62), Rafaely 2015).

For instance for an rigid array

$$b_n(kr) = j_n(kr) - \frac{j'_n(kr)}{h_n^{(2)'}(kr)} h_n^{(2)}(kr)$$

#### Parameters

- **N** (*int*) – Maximum order.
- **kr** ((*M*,) *array\_like*) – Wavenumber \* radius.
- **setup** ({‘open’, ‘card’, ‘rigid’}) – Array configuration (open, cardioids, rigid).

**Returns** **bn** ((*M*, *N+1*) *numpy.ndarray*) – Radial weights for all orders up to N and the given wavenumbers.

```
micarray.modal.radial.regularize (dn, a0, method)
```

Regularization (amplitude limitation) of radial filters.

Amplitude limitation of radial filter coefficients, methods according to (cf. Rettberg, Spors : DAGA 2014)

#### Parameters

- **dn** (*numpy.ndarray*) – Values to be regularized
- **a0** (*float*) – Parameter for regularization (not required for all methods)
- **method** ({‘none’, ‘discard’, ‘softclip’, ‘Tikh’, ‘wng’}) – Method used for regularization/amplitude limitation (none, discard, hardclip, Tikhonov, White Noise Gain).

#### Returns

- **dn** (*numpy.ndarray*) – Regularized values.
- **hn** (*array\_like*)

```
micarray.modal.radial.diagonal_mode_mat (bk)
```

Diagonal matrix of radial coefficients for all modes/wavenumbers.

**Parameters** **bk** ((*M*, *N+1*) *numpy.ndarray*) – Vector containing values for all wavenumbers *M* and modes up to order *N*

**Returns** **Bk** ((*M*, (*N+1*)\*\*2, (*N+1*)\*\*2) *numpy.ndarray*) – Multidimensional array containing diagonal matrices with input vector on main diagonal.

## 3 Utilities

```
micarray.util.norm_of_columns (A, p=2)
```

Vector p-norm of each column of a matrix.

#### Parameters

- **A** (*array\_like*) – Input matrix.
- **p** (*int, optional*) – p-th norm.

**Returns** *array\_like* – p-norm of each column of A.

```
micarray.util.coherence_of_columns (A)
```

Mutual coherence of columns of A.

#### Parameters

- **A** (*array\_like*) – Input matrix.

- **p** (*int, optional*) – p-th norm.

**Returns** *array\_like* – Mutual coherence of columns of A.

`micarray.util.asarray_1d(a, **kwargs)`

Squeeze the input and check if the result is one-dimensional.

Returns *a* converted to a `numpy.ndarray`<sup>6</sup> and stripped of all singleton dimensions. Scalars are “upgraded” to 1D arrays. The result must have exactly one dimension. If not, an error is raised.

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<sup>6</sup> <https://docs.scipy.org/doc/numpy/reference/generated/numpy.ndarray.html#numpy.ndarray>