gbok Documentation

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Contents

CHAPTER 1

Rockets

A collection of things useful for rocket design.

1.1 Rocket Performance

Note: The equations presented here are derived for an isentropic rocket engine with constant-pressure combustion and steady, one-dimensional flow. For higher fidelity analysis, simulations with more realistic assumptions should be performed.

1.1.1 The Basic Things

Thermodynamic Relationships

Thermodynamic relationships have their foundation in gasses equations of state. I highly recommend going through the derivation to get to these equations. Shapiro's *The Dynamics and Thermodynamics of Compressible Fluid Flow* has an excellent explanation and derivation.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$
$$\frac{p_0}{p}^{(\gamma-1)/\gamma} = \frac{\rho_0}{\rho}^{\gamma-1} = \frac{T_0}{T}$$

Thrust

The equation for thrust can be derived from the conservation of momentum by taking a control volume around the rocket. The result is a function of exhaust velocity (u_e), mass flow rate ($\dot{m}$), exit area (A_e), exit pressure (math:*p_e*), and ambient pressure (p_a).

$$T = \dot{m} u_e + (p_e - p_a) * A_e$$

Specific Impulse

A metric that describes the efficiency of the engine. Units of s.

$$I_{sp} = \frac{T}{\dot{m} g_0}$$

Exhaust Velocity

$$u_e = \sqrt{2 c_p T_{02} \left[1 - \left(\frac{p_e}{p_a}^{(\gamma-1)/\gamma}\right)\right]}$$

Propellant Mass Flow Rate

$$\dot{m} = \frac{A^* p_{02}}{\sqrt{R T_{02}}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

Area Ratio

$$\frac{A}{A^*} = \frac{1}{M_e} \left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\gamma/(\gamma-1)}$$

1.1.2 Characteristic Velocity and Thrust Coefficient

Characteristic Velocity

The characteristic velocity is a function of the combustion chamber properties. As stated below, it is a function of ratio of specific heats (γ), specific gas constant (R), and the chamber stagnation temperature (T_0)

$$c^* = f(\gamma R T_0) = \frac{p_0 A}{\dot{m}}$$

Characteristic velocity can be written in a more verbose form,

$$c^* = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(\gamma-1)} R T_0}$$

Thrust Coefficient

The thrust coefficient is a performance metric used to describe nozzle.

$$C_T = \frac{T}{p_0 A}$$

Another form of the thrust coefficient makes the effect of nozzle performance abundantly clear.

$$C_T = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\left[1 - \left(\frac{p_e}{p_0}\right)^{(\gamma-1)/\gamma}\right]} + \frac{p_e - p_a}{p_0}\frac{A_e}{A^*}$$

Combining c^* and C_T yields an unsurprising result.

$$T = \dot{m} c^* C_T$$

1.2 Injectors

Injectors are responsible for the distribution, atomization and mixing of propellants into the combustion chamber. Engine efficiency is closely related to the efficienty of injection.

1.2.1 Design

A lot of injector design is based on historical success, however, there are several imporant factors that must be kept in mind during the design process.

Stability

Injectors are crucial for combustion stability. Injectors for larger engines commonly have baffels used to prevent large thermoacoustic waves from arrising in the chamber. Stability is closely coupled with the choice of elements, thrust per element, element arrangement, and other hydrodynamic flow characteristics.

Achieving a δp of about $20 combustion instability : chugging.$

Discharge Coefficient

One important metric in analyzing an injector is the disharge coefficient. It is common to design an injector to obtain a specific "delta-p" (pressure drop across the injector). This is important to ensure combustion stability. The discharge coefficient describes the flow restriction of the injector and therefore can be related to δp with the following equation.

$$Q = C_d A \sqrt{2\Delta p/\rho}$$

CHAPTER 2

Mathematics

List of useful general mathematical concepts that are applicable to a wide range of engineering problems.

2.1 Norms

2.1.1 l_1 Norm

2.1.2 l_∞ Norm

Also called a cheybshev approximation.

2.2 Functions

2.2.1 Linear

A functionm, $f : \mathbb{R}^n \to \mathbb{R}$, is **linear** if

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$
$$\forall x, y \in \mathbb{R}^n \text{ and } \alpha, \beta \in \mathbb{R}$$

property: f is linear if and only if $f(x) = a^T x$ for some a

2.2.2 Affine

A function, $f : \mathbb{R}^n \to \mathbb{R}$, is **affine** if

$$f(\alpha x + (1-\alpha)y) = \alpha f(x) + (1-\alpha) f(y)$$
$$\forall x, y \in \mathbb{R}^n \text{ and } \alpha \in \mathbb{R}$$

property: f is linear if and only if $f(x) = a^T x + b$ for some a, b

CHAPTER 3

Controls

3.1 Optimal Control

This document provides an introduction to optimal control theory.

3.1.1 Overview

Optimal control theory

3.2 Linear Programming

3.2.1 Overview

Linear Programming is a method by which an optimal, minimum or maximum, outcome is obtained for mathematical models formed from linear functions of decision variables subject to constraints.

Typically, a linear program (LP) takes the following form.

$$\begin{aligned} \min \sum c^T x \\ \text{subject to } Ax = b \\ l \le x \le u \end{aligned}$$

where $\sum c^T x$ is the cost function or objective function, x_j is the optimization variable, and $Ax = b$ and $l \le x \le u$ are constraints. In such a problem, $A, b, c, l,$ and u are assumed to be known parameters of the mathematical model.

3.2.2 Important Concepts

Review the following concepts.

Geometry of Linear Programs

Convex Sets

A set of points is called a *convex* set if all the points on the straight line segment joining any two points in the set belong to the set.

linear_programming/resources/convex_sets.png

Affine Sets

Affine sets: allows us to describe a set independently of system of coordinates

Note: line going through the origin defines a Subspace i.e. $y = ax$

line going through through $y(0) = b$, or defined by $y = ax + b$ defines an affine set

Affine sets let us define systems independent of the origin.

Note: Parallel Subspace To every set C, we can associate a subspace V called a "parallel subspace"

The dimension of a parallel subspace can be defined by the affinely independent vectors of the subspace !add formal definition!

Every affine set can be expressed as the set of solutions of linear equations.

References

3.2.3 Solving Linear Programs

LPs can be efficiently solved with the following numerical methods:

- simplex, dual simplex method
- interior point methods for LPs with very sparse matricies
- decomposition, dual decomposition and regularized decomposition approaches for LP's with special block structures of their coefficient matrices A

3.2.4 Stochastic Linear Programs

In many applications of Linear Programming, exact values are not known for the mathematical model; rather, expectations are used. As a result, the solution must be computed with different methods in order to achieve a desired probability distribution, rather than a known solution.

CHAPTER 4

Overview

CHAPTER 5

Indices and tables

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